2.4 Statistics: Statistical Distributions (Binomial/Normal)

EDEXCEL 9MA0 · Calculator allowed · about 230 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Fatima is an engineer testing a large batch of light bulbs. It is known that 15% of bulbs in the batch are defective. She takes a random sample of 14 bulbs. Let X be the number of defective bulbs in the sample, so X ~ B(14, 0.15).

(a) Write down the values of E(X) and Var(X). **(2)**

(b) Calculate P(X ≤ 2). **(2)**

(c) Calculate the probability that at least 4 bulbs in the sample are defective. **(2)**

(Total for Question 1 is 6 marks)

2 Aaliyah is analysing height data. Heights of adult women in the UK, X cm, are modelled by $X \sim N(163, 6.5^2)$.

(a) Calculate $P(X > 170)$. **(3)**

(b) Calculate $P(155 < X < 170)$. **(3)**

(c) Give one criticism of using a normal distribution to model adult female height. **(1)**

(Total for Question 2 is 7 marks)

3 Grace is the quality control manager at a factory in Leeds that fills bags of sugar. The mass of sugar in a bag, W grams, is modelled by $W \sim N(1010, \sigma^2)$. It is known that 10% of bags weigh less than 1000 g.

(a) State the value of z such that $P(Z < z) = 0.10$. **(1)**

(b) Find the value of σ, giving your answer to 3 significant figures. **(3)**

(c) Using this value of σ, calculate the probability that a randomly selected bag weighs more than 1020 g. **(2)**

(d) Calculate $P(995 < W < 1020)$. **(3)**

(Total for Question 3 is 9 marks)

4 Tom works at a call centre and makes 25 sales calls each day. The probability that any call results in a sale is 0.24, independently of other calls. Let X be the number of sales Tom makes in a day, so X ~ B(25, 0.24). Tom's daily bonus, in pounds, is L = 8X + 15.

(a) Find E(X) and Var(X). **(2)**

(b) Find E(L) and Var(L). **(4)**

(c) Find the standard deviation of Tom's daily bonus, giving your answer to 3 significant figures. **(2)**

(Total for Question 4 is 8 marks)

5 Zara, a statistics student, is investigating a binomial model $X \sim B(n, p)$ where $n = 48$. She is given that $\text{Var}(X) = 0.75 \times E(X)$.

(a) Show that $p = 0.25$. **(4)**

(b) State the values of np and $n(1-p)$, and explain why a normal approximation to X would be appropriate. **(2)**

(c) Using the normal approximation to X, with a continuity correction, estimate $P(X \leq 8)$. **(4)**

(Total for Question 5 is 10 marks)

6 Mr Hussain, a maths tutor, asks his class to investigate the shape of the binomial distribution $X \sim B(25, 0.4)$.

(a) Show that, for $r = 1, 2, ..., 25$, $P(X=r) / P(X=r-1) = [(26-r)/r] \times (2/3)$. **(3)**

(b) Hence determine the mode of X, justifying your answer. **(3)**

(c) Using your answer to part (a), explain, without further calculation, why $P(X=25)$ is much smaller than $P(X=10)$. **(2)**

(Total for Question 6 is 8 marks)

7 A fair-looking spinner is spun 16 times and a success is a spin landing on blue. The probability of success on each trial is 0.45. Let X be the number of successes, so X ~ B(16, 0.45).

(a) Find P(X = 5). **(2)**

(b) Find P(X ≥ 5). **(2)**

(c) Find E(X) and Var(X), and hence the standard deviation of X to 3 significant figures. **(3)**

(d) Find P(4 ≤ X ≤ 6). **(2)**

(Total for Question 7 is 9 marks)

8 The random variable X has distribution B(50, 0.4).

(a) Explain why X may be approximated by a normal distribution, and state the parameters of this approximating distribution. **(2)**

(b) Use a normal approximation to find $P(X \geq 20)$, to 3 significant figures. **(3)**

(c) Calculate the exact value of $P(X \geq 20)$ and the difference from the approximation, to 3 significant figures. **(3)**

(Total for Question 8 is 8 marks)

9 A fair-looking spinner is spun 27 times and a success is a spin landing on blue. The probability of success on each trial is 0.45. Let X be the number of successes, so X ~ B(27, 0.45).

(a) Find P(X = 12). **(2)**

(b) Find P(X ≥ 12). **(2)**

(c) Find E(X) and Var(X), and hence the standard deviation of X to 3 significant figures. **(3)**

(d) Find P(11 ≤ X ≤ 13). **(2)**

(Total for Question 9 is 9 marks)

10 The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $P(X < 93) = 0.1587$ and $P(X < 108) = 0.6915$.

(a) Find the z values corresponding to the two probabilities, using the inverse normal distribution function, to 2 decimal places. **(3)**

(b) Hence write two equations in μ and σ and solve them to find μ and σ, each to 1 decimal place. **(4)**

(Total for Question 10 is 7 marks)

11 The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $P(X < 47) = 0.1587$ and $P(X < 63) = 0.8413$.

(a) Find the z values corresponding to the two probabilities, using the inverse normal distribution function, to 2 decimal places. **(3)**

(b) Hence write two equations in μ and σ and solve them to find μ and σ, each to 1 decimal place. **(4)**

(Total for Question 11 is 7 marks)

12 The times, in minutes, taken by customers to be served at a bank are modelled by $X \sim N(164, 2^2)$.

(a) Find $P(X > 166)$. **(3)**

(b) Find $P(161 < X < 166)$. **(3)**

(c) Find the value of a such that $P(X < a) = 0.95$, to 3 significant figures. **(3)**

(Total for Question 12 is 9 marks)

13 The random variable X has distribution B(50, 0.6).

(a) Explain why X may be approximated by a normal distribution, and state the parameters of this approximating distribution. **(2)**

(b) Use a normal approximation to find $P(X \geq 25)$, to 3 significant figures. **(3)**

(c) Calculate the exact value of $P(X \geq 25)$ and the difference from the approximation, to 3 significant figures. **(3)**

(Total for Question 13 is 8 marks)

14 The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $P(X < 40) = 0.3085$ and $P(X < 46) = 0.8413$.

(a) Find the z values corresponding to the two probabilities, using the inverse normal distribution function, to 2 decimal places. **(3)**

(b) Hence write two equations in μ and σ and solve them to find μ and σ, each to 1 decimal place. **(4)**

(Total for Question 14 is 7 marks)

15 The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $P(X < 60) = 0.0668$ and $P(X < 66) = 0.3085$.

(a) Find the z values corresponding to the two probabilities, using the inverse normal distribution function, to 2 decimal places. **(3)**

(b) Hence write two equations in μ and σ and solve them to find μ and σ, each to 1 decimal place. **(4)**

(Total for Question 15 is 7 marks)

16 The masses, in grams, of apples from an orchard are modelled by $X \sim N(81, 5^2)$.

(a) Find $P(X > 91)$. **(3)**

(b) Find $P(78.5 < X < 91)$. **(3)**

(c) Find the value of a such that $P(X < a) = 0.8$, to 3 significant figures. **(3)**

(Total for Question 16 is 9 marks)

17 The random variable X has distribution B(80, 0.3).

(a) Explain why X may be approximated by a normal distribution, and state the parameters of this approximating distribution. **(2)**

(b) Use a normal approximation to find $P(X \geq 26)$, to 3 significant figures. **(3)**

(c) Calculate the exact value of $P(X \geq 26)$ and the difference from the approximation, to 3 significant figures. **(3)**

(Total for Question 17 is 8 marks)

18 A machine produces components, and each is defective with a fixed probability, independently of the others. A sample of 15 is taken. The probability of success on each trial is 0.25. Let X be the number of successes, so X ~ B(15, 0.25).

(a) Find P(X = 5). **(2)**

(b) Find P(X ≥ 5). **(2)**

(c) Find E(X) and Var(X), and hence the standard deviation of X to 3 significant figures. **(3)**

(d) Find P(4 ≤ X ≤ 6). **(2)**

(Total for Question 18 is 9 marks)

19 The times, in minutes, taken by customers to be served at a bank are modelled by $X \sim N(56, 8^2)$.

(a) Find $P(X > 60)$. **(3)**

(b) Find $P(52 < X < 60)$. **(3)**

(c) Find the value of a such that $P(X < a) = 0.05$, to 3 significant figures. **(3)**

(Total for Question 19 is 9 marks)

20 The daily rainfall, in mm, at a weather station in the wet season is modelled by $X \sim N(117, 8^2)$.

(a) Find $P(X > 129)$. **(3)**

(b) Find $P(105 < X < 129)$. **(3)**

(c) Find the value of a such that $P(X < a) = 0.9$, to 3 significant figures. **(3)**

(Total for Question 20 is 9 marks)

21 Priya is a marketing analyst. A company claims that the probability a customer makes a purchase within a week of receiving a promotional email is 0.3. After a new campaign, Priya suspects this probability has increased. She takes a random sample of 20 customers who receive the email and finds that 10 make a purchase within a week. Let X be the number of customers, out of 20, who make a purchase, and let p be the true probability of a purchase under the new campaign.

(a) State suitable null and alternative hypotheses for Priya's test. **(2)**

(b) Find the critical region for a test at the 5% significance level, in the form $X \geq c$, showing that your value of c is correct. **(4)**

(c) Using the observed value $X = 10$, state the conclusion of the test in context. **(2)**

(d) State the actual significance level of this test. **(1)**

(Total for Question 21 is 9 marks)

22 Mrs Chen grows pea plants on her allotment in Bristol. The probability that a planted seed germinates is 0.85, independently of other seeds. She plants 16 seeds. Let X be the number of seeds that germinate, and let L be the pod length, in cm, of a germinated plant, modelled by $L \sim N(9.2, 1.1^2)$.

(a) State the distribution of X, including its parameters. **(1)**

(b) Calculate the probability that all 16 seeds germinate. **(2)**

(c) Calculate the probability that at least 14 seeds germinate. **(3)**

(d) For a randomly selected germinated plant, calculate $P(L > 10.5)$. **(3)**

(e) State one assumption made in modelling L using a normal distribution that may not hold in practice. **(1)**

(Total for Question 22 is 10 marks)

Mark scheme · 2.4 Statistics: Statistical Distributions (Binomial/Normal)

Question 1

(a) E(X) = 14 x 0.15 = 2.1, cao **B1**

(a) Var(X) = 14 x 0.15 x 0.85 = 1.785, cao **B1**

(a) Answer: E(X) = 2.1, Var(X) = 1.785

(b) sums P(X=0)+P(X=1)+P(X=2) using the binomial formula or cumulative tables **M1**

(b) P(X≤2) = awrt 0.648 **A1**

(b) Answer: P(X≤2) = awrt 0.648

(c) uses P(X≥4) = 1 - P(X≤3), with P(X=3) = C(14,3) x 0.15^3 x 0.85^{11} found and added to part (b) **M1**

(c) P(X≥4) = awrt 0.147 **A1**

(c) Answer: P(X≥4) = awrt 0.147

Question 2

(a) standardises: z = (170-163)/6.5, oe **M1**

(a) z = 1.08 (or better) and uses the normal distribution correctly **A1**

(a) P(X>170) = awrt 0.141 (allow 0.140 if z is rounded to 2 d.p. and tables are used) **A1**

(a) Answer: P(X>170) = awrt 0.141 (0.140 if z is rounded to 1.08)

(b) finds both z-values: z1=(155-163)/6.5=-1.23, z2=(170-163)/6.5=1.08 **M1**

(b) uses P(155<X<170) = Phi(1.08) - Phi(-1.23), oe **A1**

(b) P(155<X<170) = awrt 0.750 (allow 0.751 if z-values are rounded to 2 d.p. and tables are used) **A1**

(b) Answer: P(155<X<170) = awrt 0.750 (0.751 if z-values are rounded to 2 d.p.)

(c) the normal distribution is defined for all real values (including negative values), but height cannot be negative, oe; or the true distribution may have a natural upper/lower limit not reflected by the normal model **B1**

(c) Answer: The normal model allows impossible negative heights, so it can only be an approximation

Question 3

(a) z = -1.2816 (or -1.28), using the fact that P(Z<1.2816)=0.90 **B1**

(a) Answer: z = -1.2816

(b) forms the equation (1000-1010)/σ = -1.2816, oe **M1**

(b) rearranges to σ = 10/1.2816 **M1**

(b) σ = awrt 7.80 **A1**

(b) Answer: σ = awrt 7.80

(c) standardises: z = (1020-1010)/7.80 = 1.28 (ft their σ) **M1ft**

(c) P(W>1020) = awrt 0.100 **A1ft**

(c) Answer: P(W>1020) = awrt 0.100

(d) finds both z-values: z1=(995-1010)/7.80=-1.92, z2=(1020-1010)/7.80=1.28 **M1ft**

(d) uses P(995<W<1020) = Phi(1.28) - Phi(-1.92), oe **A1ft**

(d) P(995<W<1020) = awrt 0.873 (allow 0.872 if z-values are rounded to 2 d.p. and tables are used) **A1ft**

(d) Answer: P(995<W<1020) = awrt 0.873 (0.872 if z-values are rounded to 2 d.p.)

Question 4

(a) E(X) = 25 x 0.24 = 6, cao **B1**

(a) Var(X) = 25 x 0.24 x 0.76 = 4.56, cao **B1**

(a) Answer: E(X) = 6, Var(X) = 4.56

(b) uses E(L) = 8E(X) + 15 **M1**

(b) E(L) = 63, cao **A1**

(b) uses Var(L) = 8^2 Var(X) **M1**

(b) Var(L) = 291.84, cao **A1**

(b) Answer: E(L) = 63, Var(L) = 291.84

(c) uses SD(L) = $\sqrt{\text{Var(L)}}$ **M1**

(c) SD(L) = awrt 17.1 (pounds) **A1**

(c) Answer: SD(L) = awrt £17.1

Question 5

(a) states E(X) = 48p **M1**

(a) states Var(X) = 48p(1-p) **M1**

(a) forms 48p(1-p) = 0.75 x 48p and divides both sides by 48p (p not 0) to get 1-p = 0.75 **M1**

(a) p = 0.25, cso **A1**

(a) Answer: p = 0.25

(b) np = 12 and n(1-p) = 36 **B1**

(b) both values are greater than 5, so X can be well approximated by a normal distribution, oe **B1**

(b) Answer: np=12, n(1-p)=36; both exceed 5, so normal approximation is appropriate

(c) states Y ~ N(12, 9), since Var(X)=9 from part (a) **B1**

(c) applies continuity correction: P(X≤8) approx P(Y<8.5) **M1**

(c) standardises: z = (8.5-12)/3 = -1.17 **M1**

(c) P(X≤8) = awrt 0.122 (allow 0.121 if z is rounded to -1.17 and tables are used) **A1**

(c) Answer: P(X≤8) = awrt 0.122 (0.121 if z is rounded to -1.17)

Question 6

(a) writes the ratio of binomial coefficients C(25,r)/C(25,r-1) = (25-r+1)/r = (26-r)/r **M1**

(a) writes the ratio of probability terms as (p/(1-p)) = 0.4/0.6 = 2/3 **M1**

(a) combines to give P(X=r)/P(X=r-1) = [(26-r)/r] x (2/3), cso **A1**

(a) Answer: P(X=r)/P(X=r-1) = [(26-r)/r] x (2/3)

(b) sets [(26-r)/r] x (2/3) ≥ 1 and solves to find r ≤ 10.4 **M1**

(b) deduces P(X=r) > P(X=r-1) for r ≤ 10, and P(X=r) < P(X=r-1) for r ≥ 11 **A1**

(b) mode of X = 10, cao **A1**

(b) Answer: Mode = 10

(c) for r>10.4 the ratio [(26-r)/r]x(2/3) is less than 1, so probabilities decrease steadily as r increases from 11 to 25 **B1**

(c) at r=25 the ratio [(26-25)/25]x(2/3) = 2/75, a very small factor, so after many successive decreasing multiplications P(X=25) is far smaller than the peak value P(X=10), oe **B1**

(c) Answer: The ratio [(26-r)/r]x(2/3) falls well below 1 for large r (e.g. 2/75 at r=25), so repeated multiplication by small factors makes P(X=25) far smaller than P(X=10)

Question 7

(a) $16C5 \times 0.45^5 \times 0.55^{11}$ **M1**

(a) awrt 0.112 **A1**

(a) Answer: awrt 0.112

(b) $1 - P(X \leq 4)$ **M1**

(b) awrt 0.915 **A1**

(b) Answer: awrt 0.915

(c) $E(X) = 16 \times 0.45 = 7.2$ **B1**

(c) $Var(X) = 16 \times 0.45 \times 0.55 = 3.960$ **B1**

(c) standard deviation = 1.99 **B1**

(c) Answer: E(X) = 7.2, Var(X) = 3.960, sd 1.99

(d) $P(X \leq 6) - P(X \leq 3)$ **M1**

(d) awrt 0.338 **A1**

(d) Answer: awrt 0.338

Question 8

(a) n is large and p is close enough to 0.5 (np = 20 and n(1 - p) = 30 both large) **B1**

(a) mean 20, variance 12.00 **B1**

(a) Answer: N(20, 12.00)

(b) continuity correction: P(Y > 19.5) **M1**

(b) $z = (19.5 - 20)/3.4641 = -0.1443$ **M1**

(b) 0.557 **A1**

(b) Answer: 0.557

(c) $1 - P(X \leq 19)$ with the binomial distribution **M1**

(c) 0.554 **A1**

(c) difference 0.0039 **A1**

(c) Answer: exact 0.554, difference 0.0039

Question 9

(a) $27C12 \times 0.45^{12} \times 0.55^{15}$ **M1**

(a) awrt 0.153 **A1**

(a) Answer: awrt 0.153

(b) $1 - P(X \leq 11)$ **M1**

(b) awrt 0.597 **A1**

(b) Answer: awrt 0.597

(c) $E(X) = 27 \times 0.45 = 12.15$ **B1**

(c) $Var(X) = 27 \times 0.45 \times 0.55 = 6.6825$ **B1**

(c) standard deviation = 2.59 **B1**

(c) Answer: E(X) = 12.15, Var(X) = 6.6825, sd 2.59

(d) $P(X \leq 13) - P(X \leq 10)$ **M1**

(d) awrt 0.437 **A1**

(d) Answer: awrt 0.437

Question 10

(a) inverse normal of 0.1587 **M1**

(a) z = -1.00 **A1**

(a) z = 0.50 **A1**

(a) Answer: z = -1.00 and 0.50

(b) $(93 - \mu)/\sigma = -1.00$ and $(108 - \mu)/\sigma = 0.50$ **M1**

(b) subtracts to eliminate μ **M1**

(b) $\sigma = 10.0$ **A1**

(b) $\mu = 103.0$ **A1**

(b) Answer: $\mu = 103.0$, $\sigma = 10.0$

Question 11

(a) inverse normal of 0.1587 **M1**

(a) z = -1.00 **A1**

(a) z = 1.00 **A1**

(a) Answer: z = -1.00 and 1.00

(b) $(47 - \mu)/\sigma = -1.00$ and $(63 - \mu)/\sigma = 1.00$ **M1**

(b) subtracts to eliminate μ **M1**

(b) $\sigma = 8.0$ **A1**

(b) $\mu = 55.0$ **A1**

(b) Answer: $\mu = 55.0$, $\sigma = 8.0$

Question 12

(a) standardises: $z = (166 - 164)/2 = 1$ **M1**

(a) $P(Z > 1) = 1 - 0.8413$ **A1**

(a) awrt 0.159 **A1**

(a) Answer: awrt 0.159

(b) finds both z values: -1.5 and 1 **M1**

(b) P(Z < upper) - P(Z < lower) **M1**

(b) awrt 0.775 **A1**

(b) Answer: awrt 0.775

(c) z = inverse normal of 0.95 = 1.6449 **M1**

(c) $a = 164 + z \times 2$ **M1**

(c) a = 167 **A1**

(c) Answer: a = 167

Question 13

(a) n is large and p is close enough to 0.5 (np = 30 and n(1 - p) = 20 both large) **B1**

(a) mean 30, variance 12.00 **B1**

(a) Answer: N(30, 12.00)

(b) continuity correction: P(Y > 24.5) **M1**

(b) $z = (24.5 - 30)/3.4641 = -1.5877$ **M1**

(b) 0.944 **A1**

(b) Answer: 0.944

(c) $1 - P(X \leq 24)$ with the binomial distribution **M1**

(c) 0.943 **A1**

(c) difference 0.0012 **A1**

(c) Answer: exact 0.943, difference 0.0012

Question 14

(a) inverse normal of 0.3085 **M1**

(a) z = -0.50 **A1**

(a) z = 1.00 **A1**

(a) Answer: z = -0.50 and 1.00

(b) $(40 - \mu)/\sigma = -0.50$ and $(46 - \mu)/\sigma = 1.00$ **M1**

(b) subtracts to eliminate μ **M1**

(b) $\sigma = 4.0$ **A1**

(b) $\mu = 42.0$ **A1**

(b) Answer: $\mu = 42.0$, $\sigma = 4.0$

Question 15

(a) inverse normal of 0.0668 **M1**

(a) z = -1.50 **A1**

(a) z = -0.50 **A1**

(a) Answer: z = -1.50 and -0.50

(b) (60 - μ)/σ = -1.50 and (66 - μ)/σ = -0.50 **M1**

(b) subtracts to eliminate μ **M1**

(b) σ = 6.0 **A1**

(b) μ = 69.0 **A1**

(b) Answer: μ = 69.0, σ = 6.0

Question 16

(a) standardises: z = (91 - 81)/5 = 2 **M1**

(a) P(Z > 2) = 1 - 0.9772 **A1**

(a) awrt 0.0228 **A1**

(a) Answer: awrt 0.0228

(b) finds both z values: -0.5 and 2 **M1**

(b) P(Z < upper) - P(Z < lower) **M1**

(b) awrt 0.669 **A1**

(b) Answer: awrt 0.669

(c) z = inverse normal of 0.8 = 0.8416 **M1**

(c) a = 81 + z x 5 **M1**

(c) a = 85.2 **A1**

(c) Answer: a = 85.2

Question 17

(a) n is large and p is close enough to 0.5 (np = 24 and n(1 - p) = 56 both large) **B1**

(a) mean 24, variance 16.80 **B1**

(a) Answer: N(24, 16.80)

(b) continuity correction: P(Y > 25.5) **M1**

(b) z = (25.5 - 24)/4.0988 = 0.3660 **M1**

(b) 0.357 **A1**

(b) Answer: 0.357

(c) 1 - P(X ≤ 25) with the binomial distribution **M1**

(c) 0.352 **A1**

(c) difference 0.0051 **A1**

(c) Answer: exact 0.352, difference 0.0051

Question 18

(a) $15C5 \times 0.25^5 \times 0.75^{10}$ **M1**

(a) awrt 0.165 **A1**

(a) Answer: awrt 0.165

(b) 1 - P(X ≤ 4) **M1**

(b) awrt 0.314 **A1**

(b) Answer: awrt 0.314

(c) E(X) = 15 x 0.25 = 3.75 **B1**

(c) Var(X) = 15 x 0.25 x 0.75 = 2.8125 **B1**

(c) standard deviation = 1.68 **B1**

(c) Answer: E(X) = 3.75, Var(X) = 2.8125, sd 1.68

(d) P(X ≤ 6) - P(X ≤ 3) **M1**

(d) awrt 0.482 **A1**

(d) Answer: awrt 0.482

Question 19

(a) standardises: z = (60 - 56)/8 = 0.5 **M1**

(a) P(Z > 0.5) = 1 - 0.6915 **A1**

(a) awrt 0.309 **A1**

(a) Answer: awrt 0.309

(b) finds both z values: -0.5 and 0.5 **M1**

(b) P(Z < upper) - P(Z < lower) **M1**

(b) awrt 0.383 **A1**

(b) Answer: awrt 0.383

(c) z = inverse normal of 0.05 = -1.6449 **M1**

(c) a = 56 + z x 8 **M1**

(c) a = 42.8 **A1**

(c) Answer: a = 42.8

Question 20

(a) standardises: z = (129 - 117)/8 = 1.5 **M1**

(a) P(Z > 1.5) = 1 - 0.9332 **A1**

(a) awrt 0.0668 **A1**

(a) Answer: awrt 0.0668

(b) finds both z values: -1.5 and 1.5 **M1**

(b) P(Z < upper) - P(Z < lower) **M1**

(b) awrt 0.866 **A1**

(b) Answer: awrt 0.866

(c) z = inverse normal of 0.9 = 1.2816 **M1**

(c) a = 117 + z x 8 **M1**

(c) a = 127 **A1**

(c) Answer: a = 127

Question 21

(a) H0: p = 0.3 **B1**

(a) H1: p > 0.3 **B1**

(a) Answer: H0: p=0.3, H1: p>0.3

(b) under H0, X ~ B(20, 0.3), and attempts P(X≥k) for values of k near the upper tail **M1**

(b) P(X≥10) = awrt 0.0479 (< 0.05) **A1**

(b) P(X≥9) = awrt 0.1133 (> 0.05) **A1**

(b) critical region is X ≥ 10, since this is the smallest region with probability ≤ 0.05, cso **A1**

(b) Answer: Critical region: X ≥ 10

(c) X=10 lies in the critical region (X≥10), so H0 is rejected **B1**

(c) there is sufficient evidence, at the 5% level, that the proportion of customers making a purchase has increased since the new campaign, cao **B1**

(c) Answer: Reject H0; evidence that the purchase proportion has increased

(d) actual significance level = P(X≥10) = awrt 0.0479 (4.79%) **B1**

(d) Answer: 0.0479 (4.79%)

Question 22

(a) X ~ B(16, 0.85) **B1**

(a) Answer: X ~ B(16, 0.85)

(b) uses $P(X=16) = 0.85^{16}$ **M1**

(b) P(X=16) = awrt 0.0743 **A1**

(b) Answer: P(X=16) = awrt 0.0743

(c) identifies $P(X \geq 14) = P(X=14)+P(X=15)+P(X=16)$ **M1**

(c) computes each term correctly using the binomial formula **M1**

(c) $P(X \geq 14)$ = awrt 0.561 **A1**

(c) Answer: P(X≥14) = awrt 0.561

(d) standardises: z = (10.5-9.2)/1.1 **M1**

(d) z = 1.18 **A1**

(d) P(L>10.5) = awrt 0.119 **A1**

(d) Answer: P(L>10.5) = awrt 0.119

(e) assumes pod length is symmetric about the mean and can theoretically take any value (including implausibly large or negative lengths), which may not be realistic, oe **B1**

(e) Answer: Pod length cannot realistically be negative or unbounded, but the normal model allows this