2.5 Statistics: Hypothesis Testing: Test Pack

EDEXCEL 9MA0 · Calculator allowed · about 65 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A manufacturer states that, historically, the probability that a component produced by an automated lathe is defective is 0.1. After the lathe undergoes routine maintenance, the plant supervisor, Aisha, suspects that the probability of a component being defective has increased. She tests a random sample of 20 components and finds that 5 are defective. Let X be the number of defective components, out of 20, produced by the lathe after maintenance.

(a) State suitable null and alternative hypotheses for Aisha's test. **(2)**

(b) Assuming $X \sim B(20, 0.1)$, find $P(X \geq 5)$. **(3)**

(c) Using a 5% significance level, test whether there is evidence that the probability a component is defective has increased. State your conclusion in context. **(2)**

(Total for Question 1 is 7 marks)

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2 In a hypothesis test, a researcher uses a 5% significance level to test $H_0: p = 0.45$ against $H_1: p \neq 0.45$, where p is a population proportion.

(a) Explain what is meant by the significance level of a hypothesis test. **(1)**

(b) State what is meant by a Type I error in the context of this test. **(1)**

(c) The researcher instead decides to carry out the test using a 1% significance level. State, with a reason, the effect this would have on the width of the critical region. **(2)**

(d) A second researcher carries out the same test on the same sample data and calculates a p-value of 0.023. Explain why this p-value would lead to a different conclusion from a test at the 1% significance level, but the same conclusion as a test at the 5% significance level. **(2)**

(Total for Question 2 is 6 marks)

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3 A sports scientist records the number of hours of sleep, x, and the reaction time, y (in milliseconds), for 10 randomly selected athletes. The summary statistics are Sxx = 45.6, Syy = 812.4, Sxy = -165.3.

(a) Calculate the value of the product moment correlation coefficient, r, giving your answer to 3 significant figures. **(3)**

(b) State suitable hypotheses to test, at the 1% significance level, whether there is evidence of negative correlation between hours of sleep and reaction time. **(2)**

(c) Given that the critical value for a one-tail test at the 1% significance level, with n = 10, is -0.7155, test whether there is evidence of negative correlation. State your conclusion in context. **(3)**

(Total for Question 3 is 8 marks)

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4 It is known that, under a standard treatment, the probability that a patient recovers within a week is 0.4. A new drug is trialled on a random sample of 20 patients, and X, the number who recover within a week, is recorded. A test of H0: p = 0.4 against H1: p > 0.4 is to be carried out.

(a) Given that X ~ B(20, 0.4), show that P(X ≥ 13) = 0.0210, correct to 3 significant figures. **(3)**

(b) In the trial, 13 patients recovered within a week. State, giving a reason, the conclusion of the test at (i) the 5% significance level, (ii) the 1% significance level. **(3)**

(c) Find the set of values of the significance level, α%, for which the observation X = 13 would lead to the conclusion that the new drug increases the probability of recovery within a week. **(2)**

(Total for Question 4 is 8 marks)

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5 The time, in minutes, taken by staff to complete a task is modelled by a normal distribution with standard deviation 4.5. It is believed that the mean completion time is 30 minutes. Following a new training programme, a random sample of n staff members' completion times is recorded and the sample mean, xbar, is calculated. A test of H0: $\mu = 30$ against H1: $\mu < 30$ is to be carried out at the 1% significance level, assuming the standard deviation remains 4.5.

(a) Find, in terms of n, the critical region for the sample mean, xbar, giving your answer in the form xbar < c. **(3)**

(b) Given that a random sample of n = 50 staff members has a sample mean completion time of 28.7 minutes, test at the 1% significance level whether there is evidence that the training has decreased the mean completion time. **(4)**

(c) Find the minimum sample size, n, for which a sample mean of 28.7 minutes would indicate a significant decrease in the mean completion time at the 1% significance level. **(4)**

(Total for Question 5 is 11 marks)

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6 The random variable X has distribution $N(\mu, 4^2)$. A random sample of 36 observations of X has mean 22.4. A test of H0: $\mu = 24$ against H1: $\mu < 24$ is carried out at the 5% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test, in context. **(2)**

(d) Find the critical value of the sample mean for this test, to 3 significant figures. **(2)**

(Total for Question 6 is 8 marks)

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7 A student records 10 pairs of values for the length of time spent revising and the test score. The product moment correlation coefficient is r = -0.793. The critical value of r for a one-tailed test with a 5% significance level and n = 10 is 0.5494.

(a) Write down suitable hypotheses to test for negative correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 7 is 5 marks)

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Mark scheme · 2.5 Statistics: Hypothesis Testing: Test Pack

Question 1

(a) H0: p = 0.1, where p is the probability a component is defective **B1**

(a) H1: p > 0.1 (oe, one-tail test) **B1**

(a) Answer: H0: p = 0.1, H1: p > 0.1

(b) using X ~ B(20,0.1) and P(X≥5) = 1 - P(X≤4) **M1**

(b) P(X≤4) = 0.9568 (awrt) **A1**

(b) P(X≥5) = 0.0432 (awrt cao) **A1**

(b) Answer: P(X ≥ 5) = 0.0432 (awrt 3sf)

(c) compare 0.0432 with 0.05 and reject H0 since 0.0432 < 0.05 **M1**

(c) conclusion in context (ft): sufficient evidence, at the 5% level, that the probability a component is defective has increased **A1**

(c) Answer: 0.0432 < 0.05, so reject H0; there is significant evidence that the proportion of defective components has increased.

Question 2

(a) the probability of incorrectly rejecting H0 when H0 is actually true (oe) **B1**

(a) Answer: The probability of rejecting H0 when H0 is true.

(b) rejecting H0 (concluding p ≠ 0.45) when in fact p = 0.45 (oe) **B1**

(b) Answer: Concluding that p ≠ 0.45 when in fact p = 0.45.

(c) the critical region becomes smaller/narrower **B1**

(c) reason: a lower significance level requires stronger evidence against H0 before it is rejected (oe) **B1**

(c) Answer: The critical region becomes narrower, because a lower significance level requires more extreme evidence before H0 is rejected.

(d) 0.023 < 0.05 so H0 would be rejected at the 5% level **M1**

(d) 0.023 > 0.01 so H0 would not be rejected at the 1% level, hence the differing conclusions are correctly explained **A1**

(d) Answer: 0.023 < 0.05 (reject at 5%) but 0.023 > 0.01 (do not reject at 1%), so the two significance levels give different conclusions.

Question 3

(a) use of r = Sxy/$\sqrt{Sxx*Syy}$ **M1**

(a) $\sqrt{45.6*812.4}$ = $\sqrt{37045.44}$ = 192 (awrt, unsimplified value acceptable) **A1**

(a) r = -0.859 (awrt 3sf) **A1**

(a) Answer: r = -0.859

(b) H0: ρ = 0, where ρ is the population product moment correlation coefficient **B1**

(b) H1: ρ < 0 (oe, one-tail test) **B1**

(b) Answer: H0: ρ = 0, H1: ρ < 0

(c) compare r = -0.859 with the critical value -0.7155 **M1**

(c) -0.859 < -0.7155 (more extreme) so reject H0 **A1**

(c) conclusion in context: significant evidence, at the 1% level, of negative correlation between hours of sleep and reaction time (more sleep is associated with faster reaction times) **A1**

(c) Answer: -0.859 < -0.7155, so reject H0; significant evidence of negative correlation between sleep and reaction time.

Question 4

(a) attempt P(X≤12) or an equivalent complement approach using B(20,0.4) **M1**

(a) P(X≤12) = 0.9790 (awrt) **A1**

(a) P(X≥13) = 1 - 0.9790 = 0.0210 (cso) **A1**

(a) Answer: P(X ≥ 13) = 0.0210

(b) (i) 0.0210 < 0.05 so reject H0 at the 5% level **B1**

(b) (ii) 0.0210 > 0.01 so do not reject H0 at the 1% level **B1**

(b) both comparisons stated correctly with correct reasoning **B1**

(b) Answer: (i) Reject H0 at 5% (0.0210 < 0.05). (ii) Do not reject H0 at 1% (0.0210 > 0.01).

(c) recognise H0 is rejected whenever α ≥ p-value **M1**

(c) α ≥ 2.10% (accept awrt 2.1%) **A1**

(c) Answer: α ≥ 2.10% (awrt 2.1%)

Question 5

(a) identify the one-tail 1% critical z-value as -2.3263 **M1**

(a) set up xbar = μ - z*σ/$\sqrt{n}$, or equivalent standardising expression **M1**

(a) xbar < 30 - 10.5/$\sqrt{n}$ (awrt 10.5; accept exact form 30 - 2.3263*4.5/$\sqrt{n}$) **A1**

(a) Answer: xbar < 30 - 10.5/$\sqrt{n}$

(b) standardise using n=50: z = (28.7-30)/(4.5/$\sqrt{50}$) **M1**

(b) z = -2.04 (awrt 3sf) **A1**

(b) compare -2.04 with the critical value -2.3263 **M1**

(b) -2.04 > -2.3263 so do not reject H0; insufficient evidence, at the 1% level, that the mean completion time has decreased **A1**

(b) Answer: z = -2.04; do not reject H0; insufficient evidence of a decrease in mean completion time.

(c) set up inequality (28.7-30)/(4.5/$\sqrt{n}$) ≤ -2.3263 **M1**

(c) $\sqrt{n}$ ≥ 8.05 (awrt) **A1**

(c) square both sides: n ≥ 64.8 (awrt) **M1**

(c) minimum integer n = 65, with justification that n must be rounded up **A1**

(c) Answer: n = 65

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Question 6

(a) X-bar ~ N(24, $4^2/36$) **B1**

(a) standard error $4/\sqrt{36} = 0.6667$ **B1**

(a) Answer: N(24, $4^2/36$)

(b) z = (22.4 - 24)/0.6667 = -2.400 **M1**

(b) p-value = 0.00820 **A1**

(b) Answer: 0.00820

(c) compares p-value 0.00820 with 0.05 **M1**

(c) reject H0: there is sufficient evidence that the mean has decreased **A1**

(c) Answer: reject H0

(d) z critical = -1.6449 **M1**

(d) critical value = 22.9 **A1**

(d) Answer: 22.9

Question 7

(a) H0: $\rho = 0$ **B1**

(a) H1: $\rho < 0$ **B1**

(a) Answer: H0: $\rho = 0$, H1: $\rho < 0$

(b) compares $|r| = 0.793$ with the critical value 0.5494 in the correct tail **M1**

(b) reject H0 **A1**

(b) there is evidence of negative correlation **B1**

(b) Answer: reject H0

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