2.5 Statistics: Hypothesis Testing

EDEXCEL 9MA0 · Calculator allowed · about 220 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A fairground stallholder claims that a coin-toss game pays out a prize with probability 0.3 on each play. A regular customer, Ben, suspects that the true probability of winning differs from 0.3. He watches a random sample of 20 plays of the game and observes that a prize is won on only 2 occasions. Let X be the number of plays, out of 20, that result in a prize.

(a) State suitable hypotheses for a two-tailed test of Ben's suspicion. **(2)**

(b) Assuming $X \sim B(20, 0.3)$, find $P(X \leq 2)$. **(3)**

(c) Using a 10% significance level, test whether there is evidence that the probability of winning a prize differs from 0.3. State your conclusion in context. **(3)**

(Total for Question 1 is 8 marks)

2 Historically, 20% of seeds of a certain plant fail to germinate. A gardener, Rhys, treats a random sample of 25 seeds with a new fertiliser, hoping it decreases the failure rate. Let X be the number of seeds, out of 25, that fail to germinate.

(a) State suitable hypotheses for Rhys's test. **(2)**

(b) Assuming X ~ B(25, 0.2), find the critical region for the test, using a 5% significance level. You should show the probabilities used to justify the critical region. **(4)**

(c) State the actual (achieved) significance level of the test. **(1)**

(d) In the sample, 1 of the 25 seeds failed to germinate. State, with a reason, the conclusion of the test in context. **(2)**

(Total for Question 2 is 9 marks)

3 The number of hours of rainfall (x) and the crop yield (y, in tonnes per hectare) are recorded for 9 randomly selected farms in a region. Priya calculates the product moment correlation coefficient for the data to be $r = 0.658$.

(a) State suitable hypotheses to test, at the 5% significance level, whether there is positive correlation between rainfall and crop yield. **(2)**

(b) Given that the critical value for a one-tail test at the 5% significance level, with $n = 9$, is 0.5822, determine whether there is significant evidence of positive correlation between rainfall and crop yield. Justify your answer. **(2)**

(c) State one assumption that is necessary for this hypothesis test to be valid. **(1)**

(d) Priya's colleague, Marcus, claims that this result proves that increased rainfall causes an increase in crop yield. Comment on whether Marcus's claim is justified. **(2)**

(Total for Question 3 is 7 marks)

4 A student records 10 pairs of values for the length of time spent revising and the test score. The product moment correlation coefficient is r = -0.539. The critical value of r for a one-tailed test with a 5% significance level and n = 10 is 0.5494.

(a) Write down suitable hypotheses to test for negative correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 4 is 5 marks)

5 The proportion of a species of tree seeds that germinate is known to be 0.5. A gardener uses a new compost and believes the proportion has decreased. She plants 12 seeds. Let p be the true probability, and X the number in the sample, so under H0, X ~ B(12, 0.5).

(a) State suitable hypotheses for the test, in terms of p. **(2)**

(b) Using a 1% significance level, find the critical region for the test. **(3)**

(c) Write down the actual significance level of the test, and state the probability of a Type I error. **(2)**

(Total for Question 5 is 7 marks)

6 A student records 10 pairs of values for the age of a car and its resale price. The product moment correlation coefficient is r = -0.681. The critical value of r for a one-tailed test with a 5% significance level and n = 10 is 0.5494.

(a) Write down suitable hypotheses to test for negative correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 6 is 5 marks)

7 The proportion of a population with a certain characteristic was 0.25. A researcher believes that this proportion has decreased. In a random sample of 25 people, 2 have the characteristic.

(a) State the hypotheses for a one-tailed test. **(2)**

(b) Assuming H0 is true, find the probability of obtaining a value of X at least as extreme as 2, where X ~ B(25, 0.25). **(2)**

(c) Test the researcher's belief at the 10% level of significance, stating your conclusion in context. **(3)**

(Total for Question 7 is 7 marks)

8 A student records 6 pairs of values for the age of a car and its resale price. The product moment correlation coefficient is r = -0.718. The critical value of r for a one-tailed test with a 5% significance level and n = 6 is 0.7293.

(a) Write down suitable hypotheses to test for negative correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 8 is 5 marks)

9 The random variable X has distribution $N(\mu, 2^2)$. A random sample of 36 observations of X has mean 26.87. A test of H0: $\mu = 26$ against H1: $\mu > 26$ is carried out at the 1% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test, in context. **(2)**

(d) Find the critical value of the sample mean for this test, to 3 significant figures. **(2)**

(Total for Question 9 is 8 marks)

10 The proportion of a population with a certain characteristic was 0.2. A researcher believes that this proportion has increased. In a random sample of 15 people, 5 have the characteristic.

(a) State the hypotheses for a one-tailed test. **(2)**

(b) Assuming H0 is true, find the probability of obtaining a value of X at least as extreme as 5, where X ~ B(15, 0.2). **(2)**

(c) Test the researcher's belief at the 10% level of significance, stating your conclusion in context. **(3)**

(Total for Question 10 is 7 marks)

11 The random variable X has distribution $N(\mu, 5^2)$. A random sample of 36 observations of X has mean 74.83. A test of H0: $\mu = 73$ against H1: $\mu > 73$ is carried out at the 1% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test, in context. **(2)**

(d) Find the critical value of the sample mean for this test, to 3 significant figures. **(2)**

(Total for Question 11 is 8 marks)

12 A student records 10 pairs of values for daily temperature and the number of visitors to a park. The product moment correlation coefficient is r = 0.758. The critical value of r for a one-tailed test with a 5% significance level and n = 10 is 0.5494.

(a) Write down suitable hypotheses to test for positive correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 12 is 5 marks)

13 The proportion of a population with a certain characteristic was 0.2. A researcher believes that this proportion has increased. In a random sample of 20 people, 8 have the characteristic.

(a) State the hypotheses for a one-tailed test. **(2)**

(b) Assuming H0 is true, find the probability of obtaining a value of X at least as extreme as 8, where X ~ B(20, 0.2). **(2)**

(c) Test the researcher's belief at the 5% level of significance, stating your conclusion in context. **(3)**

(Total for Question 13 is 7 marks)

14 A student records 20 pairs of values for daily temperature and the number of visitors to a park. The product moment correlation coefficient is r = -0.435. The critical value of r for a one-tailed test with a 5% significance level and n = 20 is 0.3783.

(a) Write down suitable hypotheses to test for negative correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 14 is 5 marks)

15 The random variable X has distribution $N(\mu, 3^2)$. A random sample of 36 observations of X has mean 50.3. A test of H0: $\mu = 51$ against H1: $\mu < 51$ is carried out at the 5% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test. **(2)**

(d) Find the critical value of the sample mean for this test, to 3 significant figures. **(2)**

(Total for Question 15 is 8 marks)

16 The proportion of a population with a certain characteristic was 0.3. A researcher believes that this proportion has decreased. In a random sample of 12 people, 3 have the characteristic.

(a) State the hypotheses for a one-tailed test. **(2)**

(b) Assuming H0 is true, find the probability of obtaining a value of X at least as extreme as 3, where X ~ B(12, 0.3). **(2)**

(c) Test the researcher's belief at the 10% level of significance, stating your conclusion in context. **(3)**

(Total for Question 16 is 7 marks)

17 The proportion of a species of tree seeds that germinate is known to be 0.2. A gardener uses a new compost and believes the proportion has decreased. She plants 20 seeds. Let p be the true probability, and X the number in the sample, so under H0, X ~ B(20, 0.2).

(a) State suitable hypotheses for the test, in terms of p. **(2)**

(b) Using a 5% significance level, find the critical region for the test. **(3)**

(c) Write down the actual significance level of the test, and state the probability of a Type I error. **(2)**

(Total for Question 17 is 7 marks)

18 The proportion of a population with a certain characteristic was 0.3. A researcher believes that this proportion has increased. In a random sample of 15 people, 10 have the characteristic.

(a) State the hypotheses for a one-tailed test. **(2)**

(b) Assuming H0 is true, find the probability of obtaining a value of X at least as extreme as 10, where X ~ B(15, 0.3). **(2)**

(c) Test the researcher's belief at the 5% level of significance, stating your conclusion in context. **(3)**

(Total for Question 18 is 7 marks)

19 The random variable X has distribution $N(\mu, 6^2)$. A random sample of 49 observations of X has mean 68.8. A test of H0: $\mu = 70$ against H1: $\mu < 70$ is carried out at the 1% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test, in context. **(2)**

(d) Find the critical value of the sample mean for this test, to 3 significant figures. **(2)**

(Total for Question 19 is 8 marks)

20 A manufacturer claims that 20% of its light bulbs last more than 1000 hours. A retailer believes the proportion is different from the claim and tests a random sample of 40 bulbs. Let p be the true probability, and X the number in the sample, so under H0, X ~ B(40, 0.2).

(a) State suitable hypotheses for the test, in terms of p. **(2)**

(b) Using a 10% significance level, find the critical region for the test. **(3)**

(c) Write down the actual significance level of the test, and state the probability of a Type I error. **(2)**

(Total for Question 20 is 7 marks)

21 A telephone sales company finds that, historically, 8% of calls made result in a completed sale. Following a change to the sales script, the sales manager, Kwame, wants to test whether the probability of a call resulting in a sale has changed (either increased or decreased). A random sample of 40 calls is monitored. Let X be the number of calls, out of 40, that result in a sale.

(a) State suitable hypotheses for a two-tailed test of Kwame's question. **(2)**

(b) Assuming X ~ B(40, 0.08), find the critical region for a two-tailed test at the 10% significance level. You must show sufficient working to justify both parts of the critical region. **(5)**

(c) State the actual significance level of the test. **(1)**

(d) In the sample, 8 of the 40 calls resulted in a completed sale. State, with a reason, the conclusion of the test. **(2)**

(Total for Question 21 is 10 marks)

22 The mass, in grams, of flour in bags produced by a filling machine is modelled by a normal distribution with mean 1000 and standard deviation 8. Following a service of the machine, a random sample of 25 bags is taken and found to have a sample mean mass of 1004.2 g. The standard deviation is assumed to be unchanged. Sanjay wants to test, at the 5% significance level, whether the mean mass of flour in a bag has increased.

(a) State suitable hypotheses for Sanjay's test. **(2)**

(b) Show that the value of the test statistic is $z = 2.625$. **(3)**

(c) Using a critical value of 1.6449, test at the 5% significance level whether there is evidence that the mean mass has increased. State your conclusion in context. **(3)**

(Total for Question 22 is 8 marks)

23 A machine fills bottles with liquid such that the volume, in ml, is normally distributed with standard deviation 5. The machine is set so that the mean volume is 500 ml. During a routine quality check, a random sample of 36 bottles is taken and found to have a sample mean volume of 497.9 ml. Test, at the 1% significance level, whether there is evidence that the mean volume differs from 500 ml.

(a) State suitable hypotheses for a two-tailed test. **(2)**

(b) Calculate the value of the test statistic, giving your answer to 3 significant figures. **(3)**

(c) Given that the critical value is ±2.5758, determine whether there is significant evidence, at the 1% level, that the mean volume differs from 500 ml. State your conclusion in context. **(3)**

(Total for Question 23 is 8 marks)

24 A large company's technical support team finds that, historically, 25% of support tickets are resolved on first contact. After introducing a new troubleshooting script, the team leader, Fatima, wants to test whether the proportion of tickets resolved on first contact has increased. She takes a random sample of 30 tickets handled using the new script. Let X be the number of tickets, out of 30, resolved on first contact.

(a) State suitable hypotheses for Fatima's test. **(2)**

(b) Assuming $X \sim B(30, 0.25)$, find the critical region for the test in the form $X \geq k$, using a 5% significance level. You must show sufficient working to justify your answer. **(4)**

(c) State the actual significance level of the test. **(1)**

(d) Given that 13 of the 30 tickets sampled were resolved on first contact, state the conclusion of the test, giving a reason. **(2)**

(e) State one condition needed for X to be well modelled by a binomial distribution in this context. **(1)**

(Total for Question 24 is 10 marks)

Mark scheme · 2.5 Statistics: Hypothesis Testing

Question 1

(a) H0: p = 0.3, where p is the probability of winning a prize **B1**

(a) H1: p ≠ 0.3 (oe, two-tail test) **B1**

(a) Answer: H0: p = 0.3, H1: p ≠ 0.3

(b) using X ~ B(20,0.3) and summing P(X=0)+P(X=1)+P(X=2) **M1**

(b) two of P(X=0)=0.000798, P(X=1)=0.00684, P(X=2)=0.02785 correct (awrt) **A1**

(b) P(X≤2) = 0.0355 (awrt cao) **A1**

(b) Answer: P(X ≤ 2) = 0.0355 (awrt 3sf)

(c) compares 0.0355 with 0.05 (half of the 10% two-tail level) **B1**

(c) 0.0355 < 0.05 so reject H0 **M1**

(c) conclusion in context (ft): significant evidence, at the 10% level, that the probability of winning a prize differs from 0.3 (appears lower than claimed) **A1**

(c) Answer: 0.0355 < 0.05, so reject H0; there is significant evidence that the true probability of winning differs from 0.3.

Question 2

(a) H0: p = 0.2, where p is the probability a seed fails to germinate **B1**

(a) H1: p < 0.2 (oe, one-tail test) **B1**

(a) Answer: H0: p = 0.2, H1: p < 0.2

(b) attempt to find P(X≤1) or P(X≤2) using B(25,0.2) **M1**

(b) P(X≤1) = 0.0274 (awrt) **A1**

(b) P(X≤2) = 0.0982 (awrt), showing 0.0274 ≤ 0.05 < 0.0982 **A1**

(b) correct critical region: X ≤ 1 (cao) **A1**

(b) Answer: Critical region: X ≤ 1

(c) actual significance level = 0.0274 (2.74%) (ft) **B1**

(c) Answer: 2.74%

(d) 1 lies in the critical region {0,1} (ft) **M1**

(d) reject H0; evidence, at the 5% level, that the fertiliser has decreased the probability a seed fails to germinate **A1**

(d) Answer: Since X = 1 is in the critical region, reject H0; there is evidence the fertiliser has decreased the failure probability.

Question 3

(a) H0: ρ = 0, where ρ is the population product moment correlation coefficient **B1**

(a) H1: ρ > 0 (oe, one-tail test) **B1**

(a) Answer: H0: ρ = 0, H1: ρ > 0

(b) compare r = 0.658 with the critical value 0.5822 **M1**

(b) 0.658 > 0.5822 so reject H0; significant evidence of positive correlation between rainfall and crop yield **A1**

(b) Answer: 0.658 > 0.5822, so reject H0; significant evidence of positive correlation.

(c) the pairs (rainfall, crop yield) come from a population that follows a bivariate normal distribution (oe) **B1**

(c) Answer: The population of (rainfall, crop yield) pairs is bivariate normally distributed.

(d) correlation does not imply causation **B1**

(d) there may be other factors (e.g. soil quality, temperature, farming practice) that affect crop yield, so causation cannot be concluded from correlation alone (oe) **B1**

(d) Answer: No, Marcus's claim is not justified; correlation does not prove causation, as other factors could explain the relationship.

Question 4

(a) H0: ρ = 0 **B1**

(a) H1: ρ < 0 **B1**

(a) Answer: H0: ρ = 0, H1: ρ < 0

(b) compares |r| = 0.539 with the critical value 0.5494 in the correct tail **M1**

(b) do not reject H0 **A1**

(b) there is insufficient evidence of negative correlation **B1**

(b) Answer: do not reject H0

Question 5

(a) H0: p = 0.5 **B1**

(a) H1: p < 0.5 **B1**

(a) Answer: H0: p = 0.5, H1: p < 0.5

(b) uses the binomial distribution B(12, 0.5) to find cumulative probabilities near the tail **M1**

(b) P(X ≤ 1) = 0.003174 ≤ 0.01 **A1**

(b) critical region: X ≤ 1 **A1**

(b) Answer: critical region X ≤ 1

(c) actual significance level = 0.00317 **B1**

(c) this is the probability of a Type I error (rejecting H0 when it is true) **B1**

(c) Answer: 0.00317

Question 6

(a) H0: ρ = 0 **B1**

(a) H1: ρ < 0 **B1**

(a) Answer: H0: ρ = 0, H1: ρ < 0

(b) compares |r| = 0.681 with the critical value 0.5494 in the correct tail **M1**

(b) reject H0 **A1**

(b) there is evidence of negative correlation **B1**

(b) Answer: reject H0

Question 7

(a) H0: p = 0.25 **B1**

(a) H1: p < 0.25 **B1**

(a) Answer: H0: p = 0.25, H1: p < 0.25

(b) P(X ≤ 2) **M1**

(b) 0.0321 **A1**

(b) Answer: 0.0321

(c) compares 0.0321 with 0.1 **M1**

(c) reject H0 **A1**

(c) there is sufficient evidence that the proportion has decreased **B1**

(c) Answer: reject H0

Question 8

(a) H0: ρ = 0 **B1**

(a) H1: ρ < 0 **B1**

(a) Answer: H0: ρ = 0, H1: ρ < 0

(b) compares |r| = 0.718 with the critical value 0.7293 in the correct tail **M1**

(b) do not reject H0 **A1**

(b) there is insufficient evidence of negative correlation **B1**

(b) Answer: do not reject H0

Question 9

(a) X-bar ~ N(26, 2^2/36) **B1**

(a) standard error $2/\sqrt{36}$ = 0.3333 **B1**

(a) Answer: N(26, 2^2/36)

(b) z = (26.87 - 26)/0.3333 = 2.610 **M1**

(b) p-value = 0.00453 **A1**

(b) Answer: 0.00453

(c) compares p-value 0.00453 with 0.01 **M1**

(c) reject H0: there is sufficient evidence that the mean has increased **A1**

(c) Answer: reject H0

(d) z critical = 2.3263 **M1**

(d) critical value = 26.8 **A1**

(d) Answer: 26.8

Question 10

(a) H0: p = 0.2 **B1**

(a) H1: p > 0.2 **B1**

(a) Answer: H0: p = 0.2, H1: p > 0.2

(b) P(X ≥ 5) = 1 - P(X ≤ 4) **M1**

(b) 0.164 **A1**

(b) Answer: 0.164

(c) compares 0.164 with 0.1 **M1**

(c) do not reject H0 **A1**

(c) there is insufficient evidence that the proportion has increased **B1**

(c) Answer: do not reject H0

Question 11

(a) X-bar ~ N(73, 5^2/36) **B1**

(a) standard error $5/\sqrt{36}$ = 0.8333 **B1**

(a) Answer: N(73, 5^2/36)

(b) z = (74.83 - 73)/0.8333 = 2.196 **M1**

(b) p-value = 0.0140 **A1**

(b) Answer: 0.0140

(c) compares p-value 0.0140 with 0.01 **M1**

(c) do not reject H0: insufficient evidence that the mean has increased **A1**

(c) Answer: do not reject H0

(d) z critical = 2.3263 **M1**

(d) critical value = 74.9 **A1**

(d) Answer: 74.9

Question 12

(a) H0: ρ = 0 **B1**

(a) H1: ρ > 0 **B1**

(a) Answer: H0: ρ = 0, H1: ρ > 0

(b) compares |r| = 0.758 with the critical value 0.5494 in the correct tail **M1**

(b) reject H0 **A1**

(b) there is evidence of positive correlation **B1**

(b) Answer: reject H0

Question 13

(a) H0: p = 0.2 **B1**

(a) H1: p > 0.2 **B1**

(a) Answer: H0: p = 0.2, H1: p > 0.2

(b) P(X ≥ 8) = 1 - P(X ≤ 7) **M1**

(b) 0.0321 **A1**

(b) Answer: 0.0321

(c) compares 0.0321 with 0.05 **M1**

(c) reject H0 **A1**

(c) there is sufficient evidence that the proportion has increased **B1**

(c) Answer: reject H0

Question 14

(a) H0: ρ = 0 **B1**

(a) H1: ρ < 0 **B1**

(a) Answer: H0: ρ = 0, H1: ρ < 0

(b) compares |r| = 0.435 with the critical value 0.3783 in the correct tail **M1**

(b) reject H0 **A1**

(b) there is evidence of negative correlation **B1**

(b) Answer: reject H0

Question 15

(a) X-bar ~ N(51, 3^2/36) **B1**

(a) standard error $3/\sqrt{36}$ = 0.5000 **B1**

(a) Answer: N(51, 3^2/36)

(b) z = (50.3 - 51)/0.5000 = -1.400 **M1**

(b) p-value = 0.0808 **A1**

(b) Answer: 0.0808

(c) compares p-value 0.0808 with 0.05 **M1**

(c) do not reject H0: insufficient evidence at the 5% level that $\mu < 51$ **A1**

(c) Answer: do not reject H0

(d) z critical = -1.6449 **M1**

(d) critical value = 50.2 **A1**

(d) Answer: 50.2

Question 16

(a) H0: p = 0.3 **B1**

(a) H1: p < 0.3 **B1**

(a) Answer: H0: p = 0.3, H1: p < 0.3

(b) P(X ≤ 3) **M1**

(b) 0.493 **A1**

(b) Answer: 0.493

(c) compares 0.493 with 0.1 **M1**

(c) do not reject H0 **A1**

(c) there is insufficient evidence that the proportion has decreased **B1**

(c) Answer: do not reject H0

Question 17

(a) H0: p = 0.2 **B1**

(a) H1: p < 0.2 **B1**

(a) Answer: H0: p = 0.2, H1: p < 0.2

(b) uses the binomial distribution B(20, 0.2) to find cumulative probabilities near the tail **M1**

(b) P(X ≤ 0) = 0.01153 ≤ 0.05 **A1**

(b) critical region: X ≤ 0 **A1**

(b) Answer: critical region X ≤ 0

(c) actual significance level = 0.0115 **B1**

(c) this is the probability of a Type I error (rejecting H0 when it is true) **B1**

(c) Answer: 0.0115

Question 18

(a) H0: p = 0.3 **B1**

(a) H1: p > 0.3 **B1**

(a) Answer: H0: p = 0.3, H1: p > 0.3

(b) P(X ≥ 10) = 1 - P(X ≤ 9) **M1**

(b) 0.00365 **A1**

(b) Answer: 0.00365

(c) compares 0.00365 with 0.05 **M1**

(c) reject H0 **A1**

(c) there is sufficient evidence that the proportion has increased **B1**

(c) Answer: reject H0

Question 19

(a) X-bar ~ N(70, 6^2/49) **B1**

(a) standard error $6/\sqrt{49}$ = 0.8571 **B1**

(a) Answer: N(70, 6^2/49)

(b) z = (68.8 - 70)/0.8571 = -1.400 **M1**

(b) p-value = 0.0808 **A1**

(b) Answer: 0.0808

(c) compares p-value 0.0808 with 0.01 **M1**

(c) do not reject H0: insufficient evidence that the mean has decreased **A1**

(c) Answer: do not reject H0

(d) z critical = -2.3263 **M1**

(d) critical value = 68.0 **A1**

(d) Answer: 68.0

Question 20

(a) H0: p = 0.2 **B1**

(a) H1: p ≠ 0.2 **B1**

(a) Answer: H0: p = 0.2, H1: p ≠ 0.2

(b) uses the binomial distribution B(40, 0.2) to find cumulative probabilities near the tail (each tail uses 0.05) **M1**

(b) P(X ≤ 3) = 0.02846, P(X ≥ 13) = 0.04324 **A1**

(b) critical region: X ≤ 3 or X ≥ 13 **A1**

(b) Answer: critical region X ≤ 3 or X ≥ 13

(c) actual significance level = 0.0717 **B1**

(c) this is the probability of a Type I error (rejecting H0 when it is true) **B1**

(c) Answer: 0.0717

Question 21

(a) H0: p = 0.08, where p is the probability a call results in a sale **B1**

(a) H1: p ≠ 0.08 (oe, two-tail test) **B1**

(a) Answer: H0: p = 0.08, H1: p ≠ 0.08

(b) attempt lower tail: find P(X=0) and P(X≤1) using B(40,0.08) **M1**

(b) P(X=0) = 0.0356 (awrt) and P(X≤1) = 0.159 (awrt), showing the lower critical region is X = 0 **A1**

(b) attempt upper tail: find P(X≥6) and P(X≥7) **M1**

(b) P(X≥7) = 0.0376 (awrt) and P(X≥6) = 0.0967 (awrt), showing the upper critical region is X ≥ 7 **A1**

(b) correct critical region stated: X = 0 or X ≥ 7 (cao) **A1**

(b) Answer: Critical region: X = 0 or X ≥ 7

(c) actual significance level = 0.0356 + 0.0376 = 0.0732 (7.32%) (ft) **B1**

(c) Answer: 7.32%

(d) X = 8 lies in the critical region {7,8,...,40} (ft) **M1**

(d) reject H0; evidence, at the 10% level, that the probability of a call resulting in a sale has changed **A1**

(d) Answer: X = 8 is in the critical region, so reject H0; evidence the probability has changed.

Question 22

(a) H0: $\mu = 1000$, where μ is the population mean mass of flour **B1**

(a) H1: $\mu > 1000$ (oe, one-tail test) **B1**

(a) Answer: H0: $\mu = 1000$, H1: $\mu > 1000$

(b) use of $z = (\text{xbar} - \mu)/(\sigma/\sqrt{n})$ **M1**

(b) correct substitution: $z = (1004.2 - 1000)/(8/\sqrt{25})$ **A1**

(b) $z = 2.625$ (cso) **A1**

(b) Answer: z = 2.625

(c) compare 2.625 with 1.6449 **M1**

(c) 2.625 > 1.6449 so reject H0 **A1**

(c) conclusion in context: significant evidence, at the 5% level, that the mean mass of flour in a bag has increased **A1**

(c) Answer: 2.625 > 1.6449, so reject H0; evidence the mean mass has increased.

Question 23

(a) H0: $\mu = 500$, where μ is the population mean volume **B1**

(a) H1: $\mu \neq 500$ (oe, two-tail test) **B1**

(a) Answer: H0: $\mu = 500$, H1: $\mu \neq 500$

(b) use of $z = (\text{xbar} - \mu)/(\sigma/\sqrt{n})$ **M1**

(b) correct substitution: $z = (497.9 - 500)/(5/\sqrt{36})$ **A1**

(b) $z = -2.52$ (awrt 3sf) **A1**

(b) Answer: z = -2.52

(c) compare $|z| = 2.52$ with 2.5758 **M1**

(c) 2.52 < 2.5758 so do not reject H0 **A1**

(c) conclusion in context: insufficient evidence, at the 1% level, that the mean volume differs from 500 ml **A1**

(c) Answer: 2.52 < 2.5758, so do not reject H0; no significant evidence the mean volume has changed.

Question 24

(a) H0: $p = 0.25$, where p is the probability a ticket is resolved on first contact **B1**

(a) H1: $p > 0.25$ (oe, one-tail test) **B1**

(a) Answer: H0: p = 0.25, H1: p > 0.25

(b) attempt cumulative binomial probability, e.g. $P(X \geq 12)$ or $P(X \leq 11)$, using B(30,0.25) **M1**

(b) $P(X \geq 12) = 0.0507$ (awrt) and $P(X \geq 13) = 0.0216$ (awrt) (or equivalent cumulative values) **A1**

(b) correct comparison: $0.0507 > 0.05$ (fails) but $0.0216 \leq 0.05$ (satisfies) **A1**

(b) correct critical region: $X \geq 13$ (cao) **A1**

(b) Answer: Critical region: X ≥ 13

(c) actual significance level = 0.0216 (2.16%) (ft) **B1**

(c) Answer: 2.16%

(d) X = 13 lies in the critical region {13,14,...,30} (ft) **M1**

(d) reject H0; evidence, at the 5% level, that the proportion of tickets resolved on first contact has increased **A1**

(d) Answer: X = 13 is in the critical region, so reject H0; evidence the proportion resolved on first contact has increased.

(e) each ticket is resolved independently of the others / the probability a ticket is resolved on first contact is constant for every ticket (oe) **B1**

(e) Answer: Each ticket is resolved independently, with a constant probability of resolution on first contact.