2.7 Statistics: Probability Depth: Test Pack

EDEXCEL 9MA0 · Calculator allowed · about 65 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

A-Level Statistics: Probability Depth Practice

Original material written for Revision Library, styled on Edexcel AS/A-level Mathematics (9MA0) Statistics content.

This pack extends core probability skills to full A-level depth: Venn diagrams and set notation for two and three events, tree diagrams for dependent and independent trials, conditional probability, testing and proving independence algebraically, and probability models involving an unknown to be found from a given equation, including quadratics arising from probability statements. All contexts and figures are original.

1 At a sixth-form college in Leeds, A is the event that a randomly chosen student studies History and B is the event that a randomly chosen student studies Psychology. It is known that P(A) = 0.55, P(B) = 0.35 and P(A and B) = 0.20.

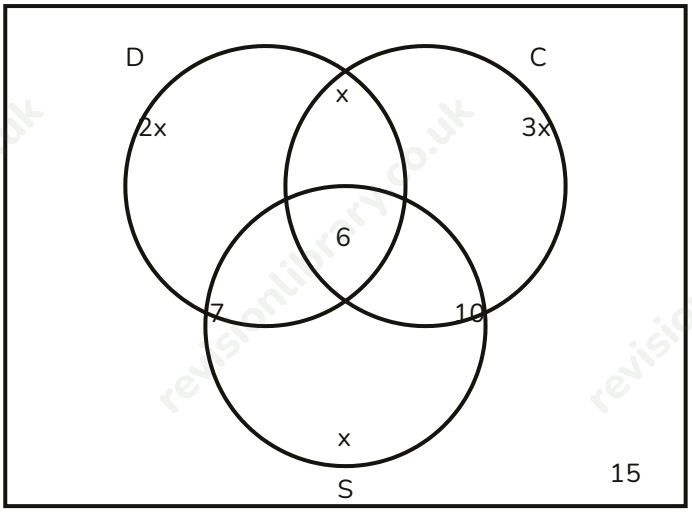
(a) Find P(A or B). **(2)**

(b) Find P(not A and not B). **(1)**

(Total for Question 1 is 3 marks)

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2 In a survey of 94 students at a sixth-form college in Bristol, D is the event that a student takes part in Drama, C is the event that a student takes part in Choir, and S is the event that a student takes part in Sport. Using a Venn diagram of the survey results: the number of students in D only is 2x, in C only is 3x, in S only is x, in D and C only (not S) is x, in D and S only (not C) is 7, in C and S only (not D) is 10, in all three of D, C and S is 6, and in none of D, C or S is 15.



(a) Form an equation in x and hence find the value of x. **(3)**

(b) Using x = 8, find P(D). **(2)**

(c) Find P(S | C). **(2)**

(d) Determine, showing your working, whether D and S are independent. **(2)**

(Total for Question 2 is 9 marks)

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3 A bag contains n red counters and 8 blue counters, where n is a positive integer. Two counters are drawn at random from the bag, one after another, without replacement.

(a) Show that P(both counters drawn are red) = n(n-1) / [(n+8)(n+7)]. **(2)**

(b) Given that P(both counters drawn are red) = 5/17, show that $3n^2 - 23n - 70 = 0$. **(4)**

(c) Hence find the value of n, explaining why the other solution of the equation is rejected. **(2)**

(d) Using n = 10, given that the first counter drawn is blue, find the probability that the second counter drawn is red. **(1)**

(e) Using n = 10, find the probability that exactly one of the two counters drawn is red. **(3)**

(Total for Question 3 is 12 marks)

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4 A bag contains 7 red counters and 6 blue counters. Two counters are taken at random from the bag, one after the other, without replacement.

(a) Draw a tree diagram, labelling each branch with its probability, and state the probability that both counters are red. **(4)**

(b) Find the probability that the two counters are the same colour. **(2)**

(c) Given that the two counters are different colours, find the probability that the first counter was red. **(2)**

(Total for Question 4 is 8 marks)

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5 Events J and K are mutually exclusive, with P(J) = 1/3 and P(K) = 1/4.

(a) Write down P(J and K). **(1)**

(b) Find P(J or K). **(1)**

(c) L is the event that neither J nor K occurs. Find P(L). **(2)**

(d) Explain, with reference to your answer to part (a), why J and K cannot be independent. **(3)**

(Total for Question 5 is 7 marks)

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6 Events M and N are independent, with P(M) = x and P(N) = x + 0.2, where $0 < x < 0.8$. It is given that P(M or N) = 0.6.

(a) Show that $5x^2 - 9x + 2 = 0$. **(4)**

(b) Hence find the value of x, giving a reason why the other root of the equation should be rejected. **(3)**

(c) Find P(M and N). **(2)**

(d) Determine whether M and not N are independent, justifying your answer. **(2)**

(Total for Question 6 is 11 marks)

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Mark scheme · 2.7 Statistics: Probability Depth: Test Pack

Question 1

(a) P(A) + P(B) - P(A and B) **M1**

(a) cao 0.70 **A1**

(a) Answer: P(A or B) = 0.70

(b) 1 - P(A or B), following through from part (a); cao 0.30 **B1ft**

(b) Answer: P(not A and not B) = 0.30

Question 2

(a) sum all 8 regions and set equal to 94 **M1**

(a) correct equation 7x + 38 = 94, oe **A1**

(a) cao x = 8 **A1**

(a) Answer: x = 8

(b) n(D) = 16 + 8 + 7 + 6, using their x **M1ft**

(b) cao 37/94 (awrt 0.394) **A1**

(b) Answer: P(D) = 37/94 (awrt 0.394)

(c) n(C and S)/n(C) = (10+6)/(24+8+10+6), using their x **M1ft**

(c) cao 1/3 (awrt 0.333) **A1**

(c) Answer: P(S | C) = 1/3 (awrt 0.333)

(d) compute P(D) x P(S) = (37/94) x (31/94), using their x **M1ft**

(d) cso: P(D) x P(S) = awrt 0.130, which does not equal P(D and S) = 13/94 (awrt 0.138), so D and S are not independent **A1**

(d) Answer: Not independent, since P(D)P(S) = awrt 0.130 not equal to P(D and S) = 13/94 (awrt 0.138)

Question 3

(a) P(first red) = n/(n+8) and P(second red | first red) = (n-1)/(n+7) **M1**

(a) cso: product simplifies to n(n-1) / [(n+8)(n+7)], as required **A1**

(a) Answer: P(both red) = n(n-1) / [(n+8)(n+7)] (shown)

(b) set n(n-1) / [(n+8)(n+7)] = 5/17, using the result from part (a) **M1**

(b) cross-multiply: 17n(n-1) = 5(n+8)(n+7) **M1**

(b) expand both sides and collect all terms on one side, set to zero **dM1**

(b) cso: $3n^2 - 23n - 70 = 0$, fully shown **A1**

(b) Answer: $3n^2 - 23n - 70 = 0$ (shown)

(c) solve $3n^2 - 23n - 70 = 0$ by factorisation, e.g. (n - 10)(3n + 7) = 0, or the quadratic formula **M1**

(c) cso: n = 10 (n = -7/3 rejected, since n must be a positive integer number of counters) **A1**

(c) Answer: n = 10 (n = -7/3 rejected)

(d) cao 10/17, from 10 red and 7 blue remaining out of 17 **B1**

(d) Answer: P(second red | first blue) = 10/17 (awrt 0.588)

(e) (10/18) x (8/17), oe, for one order of colours **M1**

(e) recognise the second order (8/18) x (10/17) gives the same value, and double it (or add both products) **M1**

(e) cao 80/153 (awrt 0.523) **A1**

(e) Answer: P(exactly one red) = 80/153 (awrt 0.523)

Question 4

(a) first branches 7/13 (red) and 6/13 (blue) **B1**

(a) second branches 1/2, 1/2 after red; 7/12, 5/12 after blue **B1**

(a) P(RR) = 7/13 x 1/2 **M1**

(a) 7/26 **A1**

(a) Answer: 7/26

(b) P(RR) + P(BB) **M1**

(b) 6/13 **A1**

(b) Answer: 6/13

(c) conditional probability: P(RB)/P(different) **M1**

(c) 1/2 **A1**

(c) Answer: 1/2

Question 5

(a) cao 0 **B1**

(a) Answer: P(J and K) = 0

(b) cao 7/12, using P(J or K) = P(J) + P(K) since P(J and K) = 0 (ft from a) **B1ft**

(b) Answer: P(J or K) = 7/12

(c) 1 - P(J or K), following through from part (b) **M1ft**

(c) cao 5/12 **A1**

(c) Answer: P(L) = 5/12

(d) states the condition for independence: P(J and K) = P(J) x P(K), oe **B1**

(d) calculates P(J) x P(K) = (1/3)(1/4) = 1/12 **B1**

(d) correct conclusion: since P(J and K) = 0 (from part a) and 0 does not equal 1/12, and P(J), P(K) are both nonzero, J and K cannot be independent **B1**

(d) Answer: Since P(J and K) = 0 but P(J) x P(K) = 1/12, and these are not equal, J and K cannot be independent

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Question 6

(a) use P(M or N) = P(M) + P(N) - P(M)P(N), using independence **M1**

(a) correct substitution: $x + (x + 0.2) - x(x + 0.2) = 0.6$ **M1**

(a) expand and rearrange into a three-term quadratic set to zero **dM1**

(a) cso: $5x^2 - 9x + 2 = 0$, fully shown **A1**

(a) Answer: $5x^2 - 9x + 2 = 0$ (shown)

(b) solve $5x^2 - 9x + 2 = 0$ using the quadratic formula or a calculator **M1**

(b) awrt $x = 0.260$ (other root awrt 1.540) **A1**

(b) reason: the other root (awrt 1.540) is rejected as it does not satisfy $0 < x < 0.8$ (and exceeds 1, which is impossible for a probability), oe **B1**

(b) Answer: $x = 0.260$ (3 s.f.); other root $x = 1.540$ rejected as it lies outside $0 < x < 0.8$

(c) $x(x + 0.2)$, using their value of x from part (b) **M1ft**

(c) awrt 0.119 **A1**

(c) Answer: P(M and N) = awrt 0.119

(d) states the result that if two events are independent, then one event and the complement of the other event are also independent, oe **B1**

(d) correct conclusion: since M and N are independent, M and not N are also independent **B1ft**

(d) Answer: M and not N are independent, since M and N are independent

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