2.8 Statistics: Distributions Depth: 1-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

A-Level Statistics: Distributions - Depth Practice

Original material written for Revision Library, styled on Edexcel AS/A-level Mathematics (9MA0) Statistics content.

This pack extends core statistical distributions work to full A-level depth: discrete random variables (probability distributions, cumulative distribution functions, E(X), Var(X) and linear transformations), the binomial distribution (conditions, probabilities, mean and variance, cumulative probabilities), the normal distribution (standardising, inverse normal problems, finding unknown parameters), the sum of independent normal variables, and using the normal distribution to approximate the binomial distribution with a continuity correction. All contexts and figures are original.

1 The masses, in grams, of apples from an orchard are modelled by $X \sim N(183, 6^2)$.

(a) Find $P(X > 195)$. **(3)**

(b) Find $P(183 < X < 195)$. **(3)**

(c) Find the value of a such that $P(X < a) = 0.95$, to 3 significant figures. **(3)**

(Total for Question 1 is 9 marks)

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2 The random variable X has distribution B(120, 0.6).

(a) Explain why X may be approximated by a normal distribution, and state the parameters of this approximating distribution. **(2)**

(b) Use a normal approximation to find $P(X \geq 76)$, to 3 significant figures. **(3)**

(c) Calculate the exact value of $P(X \geq 76)$ and the difference from the approximation, to 3 significant figures. **(3)**

(Total for Question 2 is 8 marks)

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3 A fair-looking spinner is spun 16 times and a success is a spin landing on blue. The probability of success on each trial is 0.45. Let X be the number of successes, so X ~ B(16, 0.45).

(a) Find P(X = 9). **(2)**

(b) Find P(X ≥ 9). **(2)**

(c) Find E(X) and Var(X), and hence the standard deviation of X to 3 significant figures. **(3)**

(d) Find P(8 ≤ X ≤ 10). **(2)**

(Total for Question 3 is 9 marks)

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4 The random variable X has distribution B(80, 0.4).

(a) Explain why X may be approximated by a normal distribution, and state the parameters of this approximating distribution. **(2)**

(b) Use a normal approximation to find P(X ≥ 31), to 3 significant figures. **(3)**

(c) Calculate the exact value of P(X ≥ 31) and the difference from the approximation, to 3 significant figures. **(3)**

(Total for Question 4 is 8 marks)

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5 A large multiple-choice test has questions with 5 possible answers each, of which only one is correct. Priya has not revised, and guesses the answer to every question independently at random. She attempts 12 questions. Let R be the number of questions Priya answers correctly.

(a) State the distribution of R. **(1)**

(b) Find P(R = 4). **(2)**

(c) Find P(2 ≤ R ≤ 5). **(3)**

(d) The pass mark for the test is 6 correct answers out of 12. Find the probability that Priya passes the test by guessing. **(2)**

(Total for Question 5 is 8 marks)

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6 The heights, in cm, of adult sunflowers grown by a gardener in Chester are modelled by the random variable $H \sim N(150, 15^2)$.

(a) Find the height h such that P(H < h) = 0.90. **(3)**

(b) Find the interquartile range of H. **(3)**

(Total for Question 6 is 6 marks)

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Mark scheme · 2.8 Statistics: Distributions Depth: 1-hour Pack

Question 1

(a) standardises: z = (195 - 183)/6 = 2 **M1**

(a) P(Z > 2) = 1 - 0.9772 **A1**

(a) awrt 0.0228 **A1**

(a) Answer: awrt 0.0228

(b) finds both z values: 0 and 2 **M1**

(b) P(Z < upper) - P(Z < lower) **M1**

(b) awrt 0.477 **A1**

(b) Answer: awrt 0.477

(c) z = inverse normal of 0.95 = 1.6449 **M1**

(c) a = 183 + z x 6 **M1**

(c) a = 193 **A1**

(c) Answer: a = 193

Question 2

(a) n is large and p is close enough to 0.5 (np = 72 and n(1 - p) = 48 both large) **B1**

(a) mean 72, variance 28.80 **B1**

(a) Answer: N(72, 28.80)

(b) continuity correction: P(Y > 75.5) **M1**

(b) z = (75.5 - 72)/5.3666 = 0.6522 **M1**

(b) 0.257 **A1**

(b) Answer: 0.257

(c) 1 - P(X ≤ 75) with the binomial distribution **M1**

(c) 0.258 **A1**

(c) difference 0.00125 **A1**

(c) Answer: exact 0.258, difference 0.00125

Question 3

(a) $16C9 \times 0.45^9 \times 0.55^7$ **M1**

(a) awrt 0.132 **A1**

(a) Answer: awrt 0.132

(b) 1 - P(X ≤ 8) **M1**

(b) awrt 0.256 **A1**

(b) Answer: awrt 0.256

(c) E(X) = 16 x 0.45 = 7.2 **B1**

(c) Var(X) = 16 x 0.45 x 0.55 = 3.960 **B1**

(c) standard deviation = 1.99 **B1**

(c) Answer: E(X) = 7.2, Var(X) = 3.960, sd 1.99

(d) P(X ≤ 10) - P(X ≤ 7) **M1**

(d) awrt 0.388 **A1**

(d) Answer: awrt 0.388

Question 4

(a) n is large and p is close enough to 0.5 (np = 32 and n(1 - p) = 48 both large) **B1**

(a) mean 32, variance 19.20 **B1**

(a) Answer: N(32, 19.20)

(b) continuity correction: P(Y > 30.5) **M1**

(b) z = (30.5 - 32)/4.3818 = -0.3423 **M1**

(b) 0.634 **A1**

(b) Answer: 0.634

(c) 1 - P(X ≤ 30) with the binomial distribution **M1**

(c) 0.631 **A1**

(c) difference 0.00267 **A1**

(c) Answer: exact 0.631, difference 0.00267

Question 5

(a) R ~ B(12, 0.2) **B1**

(a) Answer: R ~ B(12, 0.2)

(b) correct expression $C(12,4) * 0.2^4 * 0.8^8$ (oe) **M1**

(b) awrt 0.133 **A1**

(b) Answer: P(R = 4) = awrt 0.133

(c) identifies P(2≤R≤5) = P(R=2)+P(R=3)+P(R=4)+P(R=5) (oe, or cdf subtraction) **M1**

(c) at least two of P(R=2)=awrt 0.283, P(R=3)=awrt 0.236, P(R=5)=awrt 0.0532 correct (ft) **M1**

(c) awrt 0.706 **A1**

(c) Answer: P(2 ≤ R ≤ 5) = awrt 0.706

(d) P(R≥6) = 1 - P(R≤5) (oe) **M1**

(d) awrt 0.0194 **A1**

(d) Answer: P(pass) = awrt 0.0194

Question 6

(a) correct inverse normal z-value used, z = 1.2816 (awrt) **M1**

(a) h = 150 + 1.2816*15 (oe, ft their z) **M1**

(a) awrt 169 **A1**

(a) Answer: h = awrt 169 cm

(b) z = 0.6745 (awrt) identified for both quartiles by symmetry **M1**

(b) IQR = 2*0.6745*15 (oe) **M1**

(b) awrt 20.2 **A1**

(b) Answer: IQR = awrt 20.2 cm

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