2.8 Statistics: Distributions Depth: 30-minute Pack

EDEXCEL 9MA0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

A-Level Statistics: Distributions - Depth Practice

Original material written for Revision Library, styled on Edexcel AS/A-level Mathematics (9MA0) Statistics content.

This pack extends core statistical distributions work to full A-level depth: discrete random variables (probability distributions, cumulative distribution functions, E(X), Var(X) and linear transformations), the binomial distribution (conditions, probabilities, mean and variance, cumulative probabilities), the normal distribution (standardising, inverse normal problems, finding unknown parameters), the sum of independent normal variables, and using the normal distribution to approximate the binomial distribution with a continuity correction. All contexts and figures are original.

1 The time, in minutes, taken by students at a college in Bristol to complete a mock examination is modelled by the random variable $X \sim N(60, 8^2)$.

(a) Find $P(X < 68)$. **(2)**

(b) Find $P(52 < X < 76)$. **(3)**

(Total for Question 1 is 5 marks)

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2 Bags of flour produced at a mill in Norwich have masses modelled by the random variable $X \sim N(1000, 15^2)$ grams. Independently, the empty bags used to hold the flour have masses modelled by $Y \sim N(50, 4^2)$ grams. A filled bag has total mass $T = X + Y$.

(a) State the distribution of T. **(2)**

(b) Find $P(T > 1060)$. **(4)**

(c) Six filled bags are selected at random. Using the probability found in part (b), find the probability that at least 5 of the six bags have a total mass greater than 1060 g. **(3)**

(Total for Question 2 is 9 marks)

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3 A machine at a factory in Swansea fills bottles of sunflower oil. The volume of oil in a randomly selected bottle, V ml, is modelled by $V \sim N(\mu, 6^2)$. The manufacturer requires that no more than 2% of bottles contain less than 500 ml.

(a) Find the minimum value of μ, to 1 decimal place, that satisfies this requirement. **(4)**

(b) Using $\mu = 512$ ml (the value found in part (a), rounded to the nearest whole ml), find the probability that a randomly selected bottle contains more than 520 ml. **(3)**

(c) A random sample of 80 bottles is taken. Using $\mu = 512$ and the probability found in part (b), find the probability that fewer than 5 bottles contain more than 520 ml. State the approximation used and justify why it is appropriate. **(5)**

(Total for Question 3 is 12 marks)

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Mark scheme · 2.8 Statistics: Distributions Depth: 30-minute Pack

Question 1

(a) standardising: z = (68-60)/8 = 1 (oe) **M1**

(a) awrt 0.841 **A1**

(a) Answer: P(X < 68) = awrt 0.841

(b) both z-values found: z1 = (52-60)/8 = -1, z2 = (76-60)/8 = 2 (oe) **M1**

(b) correct combination Phi(2) - [1 - Phi(1)] (oe, using symmetry) **M1**

(b) awrt 0.819 **A1**

(b) Answer: P(52 < X < 76) = awrt 0.819

Question 2

(a) mean of T = 1000+50 = 1050 **B1**

(a) variance of T = 15^2+4^2 = 241 (so T ~ N(1050, 241)) **B1**

(a) Answer: T ~ N(1050, 241)

(b) use of T~N(1050,241) (ft their distribution) **M1**

(b) standardising z = (1060-1050)/$\sqrt{241}$ (oe) **M1**

(b) z = awrt 0.644 **A1**

(b) awrt 0.260 **A1**

(b) Answer: P(T > 1060) = awrt 0.260

(c) identifies N ~ B(6, 0.260) (ft their p from (b)) **M1**

(c) P(N=5)+P(N=6) computed (ft) **M1**

(c) awrt 0.00558 (ft) **A1**

(c) Answer: P(at least 5 of 6) = awrt 0.00558

Question 3

(a) boundary condition P(V<500) = 0.02 identified **M1**

(a) z = invNorm(0.02) = -2.0537 (awrt) used (or equivalently invNorm(0.98) = 2.0537) **M1**

(a) equation (500-μ)/6 = -2.0537 (oe) set up and rearranged **M1**

(a) μ = awrt 512.3 **A1**

(a) Answer: μ (minimum) = awrt 512.3 ml

(b) standardising z = (520-512)/6 (oe) **M1**

(b) use of 1 - Phi(z) (oe) **M1**

(b) awrt 0.0912 **A1**

(b) Answer: P(V > 520) = awrt 0.0912

(c) identifies M ~ B(80, 0.0912) (ft their probability from (b)) **B1**

(c) states np = awrt 7.30 > 5 and n(1-p) = awrt 72.7 > 5, justifying a normal approximation **B1**

(c) normal approximation M ~ N(7.296, 6.63) (oe, ft) with mean np and variance np(1-p) correctly computed **M1**

(c) continuity correction applied: P(M<4.5) and standardising, z = (4.5-7.296)/$\sqrt{6.63}$ (oe, ft) **M1**

(c) awrt 0.139 (ft) **A1**

(c) Answer: P(M < 5) = awrt 0.139

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