2.8 Statistics: Distributions Depth: Test Pack

EDEXCEL 9MA0 · Calculator allowed · about 70 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

A-Level Statistics: Distributions - Depth Practice

Original material written for Revision Library, styled on Edexcel AS/A-level Mathematics (9MA0) Statistics content.

This pack extends core statistical distributions work to full A-level depth: discrete random variables (probability distributions, cumulative distribution functions, E(X), Var(X) and linear transformations), the binomial distribution (conditions, probabilities, mean and variance, cumulative probabilities), the normal distribution (standardising, inverse normal problems, finding unknown parameters), the sum of independent normal variables, and using the normal distribution to approximate the binomial distribution with a continuity correction. All contexts and figures are original.

1 A quality-control inspector at a factory in Leeds tests electronic components taken from the production line. Each component is independently defective with probability 0.08, and the outcome for each component tested is unaffected by any other. A random sample of 15 components is tested. Let X be the number of defective components found in the sample.

(a) State two conditions needed for X to be modelled by a binomial distribution. **(2)**

(b) Using the model X ~ B(15, 0.08), find P(X = 2). **(2)**

(Total for Question 1 is 4 marks)

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2 The random variable W ~ B(20, p). Given that E(W) = 5.

(a) Find the value of p. **(2)**

(b) Find Var(W). **(2)**

(c) Find P(W = 5). **(2)**

(Total for Question 2 is 6 marks)

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3 A call centre in Cardiff finds that, independently and at random, 40% of calls are resolved on first contact. A random sample of 150 calls is taken. Let X be the number of calls in the sample that are resolved on first contact.

(a) Explain why X can be approximated by a normal distribution, stating the values of any parameters used. **(3)**

(b) Using this approximation, find $P(X \leq 65)$, applying a continuity correction. **(4)**

(Total for Question 3 is 7 marks)

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4 A game uses a spinner with four sectors numbered 1, 2, 3 and 4. The spinner is biased so that the probability of it landing on sector n is proportional to n, for n = 1, 2, 3, 4. Let X be the score obtained on one spin of the spinner.

(a) Show that $P(X = n) = n/10$ for n = 1, 2, 3, 4. **(3)**

(b) Find the cumulative distribution function $F(x) = P(X \leq x)$ for x = 1, 2, 3, 4. **(3)**

(c) Find E(X) and Var(X). **(4)**

(d) Tom plays the game once. His winnings, in pounds, are W = 2X - 5. Find E(W). **(2)**

(e) Find Var(W). **(2)**

(Total for Question 4 is 14 marks)

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5 The random variable X has distribution B(50, 0.25).

(a) Explain why X may be approximated by a normal distribution, and state the parameters of this approximating distribution. **(2)**

(b) Use a normal approximation to find $P(X \geq 7)$, to 3 significant figures. **(3)**

(c) Calculate the exact value of $P(X \geq 7)$ and the difference from the approximation, to 3 significant figures. **(3)**

(Total for Question 5 is 8 marks)

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6 The daily rainfall, in mm, at a weather station in the wet season is modelled by $X \sim N(70, 12^2)$.

(a) Find $P(X > 100)$. **(3)**

(b) Find $P(64 < X < 100)$. **(3)**

(c) Find the value of a such that $P(X < a) = 0.75$, to 3 significant figures. **(3)**

(Total for Question 6 is 9 marks)

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7 The mass, in grams, of apples grown on a farm in Kent is modelled by the random variable $Y \sim N(\mu, \sigma^2)$. It is known that $P(Y < 180) = 0.2$ and $P(Y > 230) = 0.1$.

(a) Show that $\mu - 0.8416*\sigma = 180$ (values given to 4 decimal places). **(3)**

(b) Show a similar equation using $P(Y > 230) = 0.1$. **(2)**

(c) Hence find the values of μ and σ. **(4)**

(Total for Question 7 is 9 marks)

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Mark scheme · 2.8 Statistics: Distributions Depth: Test Pack

Question 1

(a) fixed number of trials, n = 15 (oe) **B1**

(a) each trial is independent with only two outcomes (defective/not defective) and a constant probability of success p = 0.08 (oe) **B1**

(a) Answer: Any two valid conditions, e.g. fixed number of trials (n = 15); independent trials with a constant probability of success (p = 0.08) and only two outcomes

(b) correct expression $C(15,2) * 0.08^2 * 0.92^{13}$ (oe) **M1**

(b) awrt 0.227 **A1**

(b) Answer: P(X = 2) = awrt 0.227

Question 2

(a) 20p = 5 (oe) **M1**

(a) p = 0.25 cao **A1**

(a) Answer: p = 0.25

(b) Var(W) = 20 * 0.25 * 0.75 (ft their p) **M1**

(b) 3.75 cao **A1**

(b) Answer: Var(W) = 3.75

(c) correct expression $C(20,5) * 0.25^5 * 0.75^{15}$ (ft their p) **M1**

(c) awrt 0.202 **A1**

(c) Answer: P(W = 5) = awrt 0.202

Question 3

(a) np = 150*0.4 = 60 > 5 stated **B1**

(a) n(1-p) = 150*0.6 = 90 > 5 stated (justifying the approximation is valid) **B1**

(a) X ~ N(60, 36) stated (mean 60, variance 36) **B1**

(a) Answer: X ~ N(60, 36) approximately, since np = 60 > 5 and n(1-p) = 90 > 5

(b) continuity correction applied: P(X<65.5) (oe) **M1**

(b) standardising z = (65.5-60)/6 (ft their mean/sd) **M1**

(b) z = awrt 0.917 **A1**

(b) awrt 0.820 **A1**

(b) Answer: P(X ≤ 65) = awrt 0.820

Question 4

(a) P(X=n) = k*n set up for some constant k **M1**

(a) sum to 1: k(1+2+3+4) = 1, i.e. 10k = 1 (oe) **M1**

(a) k = 1/10, so P(X=n) = n/10 cso **A1**

(a) Answer: P(X = n) = n/10

(b) cumulative summation method used, e.g. F(2) = P(X=1)+P(X=2) (oe) **M1**

(b) at least two of F(1)=0.1, F(2)=0.3, F(3)=0.6 correct **A1**

(b) all four correct: F(1)=0.1, F(2)=0.3, F(3)=0.6, F(4)=1 **A1**

(b) Answer: F(1)=0.1, F(2)=0.3, F(3)=0.6, F(4)=1

(c) E(X) = 1(0.1)+2(0.2)+3(0.3)+4(0.4) (oe) **M1**

(c) E(X) = 3 **A1**

(c) $E(X^2) = 1(0.1)+4(0.2)+9(0.3)+16(0.4)$ used with $Var(X) = E(X^2) - [E(X)]^2$ (oe) **M1**

(c) Var(X) = 1 **A1**

(c) Answer: E(X) = 3, Var(X) = 1

(d) E(W) = 2*E(X) - 5 (ft their E(X)) **M1**

(d) E(W) = 1 (i.e. £1) **A1**

(d) Answer: E(W) = 1 (£1)

(e) $Var(W) = 2^2 * Var(X)$ (ft their Var(X)) **M1**

(e) Var(W) = 4 **A1**

(e) Answer: Var(W) = 4

Question 5

(a) n is large and p is close enough to 0.5 (np = 12.5 and n(1 - p) = 37.5 both large) **B1**

(a) mean 12.5, variance 9.375 **B1**

(a) Answer: N(12.5, 9.375)

(b) continuity correction: P(Y > 6.5) **M1**

(b) z = (6.5 - 12.5)/3.0619 = -1.9596 **M1**

(b) 0.975 **A1**

(b) Answer: 0.975

(c) 1 - P(X ≤ 6) with the binomial distribution **M1**

(c) 0.981 **A1**

(c) difference 0.00563 **A1**

(c) Answer: exact 0.981, difference 0.00563

Question 6

(a) standardises: z = (100 - 70)/12 = 2.5 **M1**

(a) P(Z > 2.5) = 1 - 0.9938 **A1**

(a) awrt 0.00621 **A1**

(a) Answer: awrt 0.00621

(b) finds both z values: -0.5 and 2.5 **M1**

(b) P(Z < upper) - P(Z < lower) **M1**

(b) awrt 0.685 **A1**

(b) Answer: awrt 0.685

(c) z = inverse normal of 0.75 = 0.6745 **M1**

(c) a = 70 + z x 12 **M1**

(c) a = 78.1 **A1**

(c) Answer: a = 78.1

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Question 7

(a) standardising P(Y<180)=0.2, i.e. (180-μ)/σ = z (oe)	**M1**
(a) z = invNorm(0.2) = -0.8416 (awrt) used	**M1**
(a) μ - 0.8416*σ = 180 cso	**A1**

(a) Answer: μ - 0.8416*σ = 180

(b) standardising with z = invNorm(0.9) = 1.2816 (awrt), i.e. (230-μ)/σ = 1.2816 (oe)	**M1**
(b) μ + 1.2816*σ = 230 cso	**A1**

(b) Answer: μ + 1.2816*σ = 230

(c) solving the two equations simultaneously (subtracting to eliminate μ)	**M1**
(c) σ = awrt 23.5	**A1**
(c) substituting their σ back to find μ	**M1**
(c) μ = awrt 199.8 (or awrt 200)	**A1**

(c) Answer: σ = awrt 23.5 (g), μ = awrt 199.8 (g)

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