2.8 Statistics: Distributions Depth

EDEXCEL 9MA0 · Calculator allowed · about 215 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

A-Level Statistics: Distributions - Depth Practice

Original material written for Revision Library, styled on Edexcel AS/A-level Mathematics (9MA0) Statistics content.

This pack extends core statistical distributions work to full A-level depth: discrete random variables (probability distributions, cumulative distribution functions, E(X), Var(X) and linear transformations), the binomial distribution (conditions, probabilities, mean and variance, cumulative probabilities), the normal distribution (standardising, inverse normal problems, finding unknown parameters), the sum of independent normal variables, and using the normal distribution to approximate the binomial distribution with a continuity correction. All contexts and figures are original.

1 The discrete random variable X has the probability distribution shown in the table below.

x : 0 1 2 3
P(X = x) : 0.1 0.3 2k k

x	P(X = x)
0	0.1
1	0.3
2	2k
3	k

(a) Show that k = 0.2. **(2)**

(b) Find E(X). **(2)**

(c) Find $P(X \geq 2 \mid X \geq 1)$. **(2)**

(Total for Question 1 is 6 marks)

2 The random variable X is as defined in Question 1, with probability distribution

x : 0 1 2 3
P(X = x) : 0.1 0.3 0.4 0.2

Given that E(X) = 1.7.

(a) Show that $E(X^2) = 3.7$. **(2)**

(b) Hence find Var(X). **(2)**

(c) Find Var(3 - 2X). **(3)**

(Total for Question 2 is 7 marks)

3 A biased coin is such that the probability of obtaining a head on any throw is 0.3. The coin is thrown 10 times, with each throw independent of the others. Let Y be the number of heads obtained.

(a) Write down the distribution of Y, stating the value(s) of any parameter(s). **(1)**

(b) Find P(Y = 3), giving your answer to 3 significant figures. **(2)**

(c) Find P(Y ≥ 5). **(3)**

(Total for Question 3 is 6 marks)

4 The time, in minutes, taken by students at a college in Bristol to complete a mock examination is modelled by the random variable $X \sim N(60, 8^2)$.

(a) Find $P(X < 68)$. **(2)**

(b) Find $P(52 < X < 76)$. **(3)**

(Total for Question 4 is 5 marks)

5 Bags of flour produced at a mill in Norwich have masses modelled by the random variable $X \sim N(1000, 15^2)$ grams. Independently, the empty bags used to hold the flour have masses modelled by $Y \sim N(50, 4^2)$ grams. A filled bag has total mass $T = X + Y$.

(a) State the distribution of T. **(2)**

(b) Find $P(T > 1060)$. **(4)**

(c) Six filled bags are selected at random. Using the probability found in part (b), find the probability that at least 5 of the six bags have a total mass greater than 1060 g. **(3)**

(Total for Question 5 is 9 marks)

6 A machine at a factory in Swansea fills bottles of sunflower oil. The volume of oil in a randomly selected bottle, V ml, is modelled by $V \sim N(\mu, 6^2)$. The manufacturer requires that no more than 2% of bottles contain less than 500 ml.

(a) Find the minimum value of μ, to 1 decimal place, that satisfies this requirement. **(4)**

(b) Using μ = 512 ml (the value found in part (a), rounded to the nearest whole ml), find the probability that a randomly selected bottle contains more than 520 ml. **(3)**

(c) A random sample of 80 bottles is taken. Using μ = 512 and the probability found in part (b), find the probability that fewer than 5 bottles contain more than 520 ml. State the approximation used and justify why it is appropriate. **(5)**

(Total for Question 6 is 12 marks)

7 The masses, in grams, of apples from an orchard are modelled by $X \sim N(183, 6^2)$.

(a) Find $P(X > 195)$. **(3)**

(b) Find $P(183 < X < 195)$. **(3)**

(c) Find the value of a such that $P(X < a) = 0.95$, to 3 significant figures. **(3)**

(Total for Question 7 is 9 marks)

8 The random variable X has distribution B(120, 0.6).

(a) Explain why X may be approximated by a normal distribution, and state the parameters of this approximating distribution. **(2)**

(b) Use a normal approximation to find $P(X \geq 76)$, to 3 significant figures. **(3)**

(c) Calculate the exact value of $P(X \geq 76)$ and the difference from the approximation, to 3 significant figures. **(3)**

(Total for Question 8 is 8 marks)

9 A fair-looking spinner is spun 16 times and a success is a spin landing on blue. The probability of success on each trial is 0.45. Let X be the number of successes, so X ~ B(16, 0.45).

(a) Find P(X = 9). **(2)**

(b) Find P(X ≥ 9). **(2)**

(c) Find E(X) and Var(X), and hence the standard deviation of X to 3 significant figures. **(3)**

(d) Find P(8 ≤ X ≤ 10). **(2)**

(Total for Question 9 is 9 marks)

10 The random variable X has distribution B(80, 0.4).

(a) Explain why X may be approximated by a normal distribution, and state the parameters of this approximating distribution. **(2)**

(b) Use a normal approximation to find $P(X \geq 31)$, to 3 significant figures. **(3)**

(c) Calculate the exact value of $P(X \geq 31)$ and the difference from the approximation, to 3 significant figures. **(3)**

(Total for Question 10 is 8 marks)

11 The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $P(X < 74) = 0.8413$ and $P(X < 78) = 0.9772$.

(a) Find the z values corresponding to the two probabilities, using the inverse normal distribution function, to 2 decimal places. **(3)**

(b) Hence write two equations in μ and σ and solve them to find μ and σ, each to 1 decimal place. **(4)**

(Total for Question 11 is 7 marks)

12 The heights, in cm, of adult men in a town are modelled by $X \sim N(47, 8^2)$.

(a) Find $P(X > 67)$. **(3)**

(b) Find $P(35 < X < 67)$. **(3)**

(c) Find the value of a such that $P(X < a) = 0.8$, to 3 significant figures. **(3)**

(Total for Question 12 is 9 marks)

13 The random variable X has distribution B(50, 0.2).

(a) Explain why X may be approximated by a normal distribution, and state the parameters of this approximating distribution. **(2)**

(b) Use a normal approximation to find $P(X \geq 9)$, to 3 significant figures. **(3)**

(c) Calculate the exact value of $P(X \geq 9)$ and the difference from the approximation, to 3 significant figures. **(3)**

(Total for Question 13 is 8 marks)

14 The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $P(X < 32) = 0.0668$ and $P(X < 52) = 0.6915$.

(a) Find the z values corresponding to the two probabilities, using the inverse normal distribution function, to 2 decimal places. **(3)**

(b) Hence write two equations in μ and σ and solve them to find μ and σ, each to 1 decimal place. **(4)**

(Total for Question 14 is 7 marks)

15 The random variable X has distribution B(50, 0.2).

(a) Explain why X may be approximated by a normal distribution, and state the parameters of this approximating distribution. **(2)**

(b) Use a normal approximation to find $P(X \geq 7)$, to 3 significant figures. **(3)**

(c) Calculate the exact value of $P(X \geq 7)$ and the difference from the approximation, to 3 significant figures. **(3)**

(Total for Question 15 is 8 marks)

16 A machine produces components, and each is defective with a fixed probability, independently of the others. A sample of 12 is taken. The probability of success on each trial is 0.25. Let X be the number of successes, so X ~ B(12, 0.25).

(a) Find P(X = 2). **(2)**

(b) Find P(X ≥ 2). **(2)**

(c) Find E(X) and Var(X), and hence the standard deviation of X to 3 significant figures. **(3)**

(d) Find P(1 ≤ X ≤ 3). **(2)**

(Total for Question 16 is 9 marks)

17 The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $P(X < 50) = 0.6915$ and $P(X < 52) = 0.8413$.

(a) Find the z values corresponding to the two probabilities, using the inverse normal distribution function, to 2 decimal places. **(3)**

(b) Hence write two equations in μ and σ and solve them to find μ and σ, each to 1 decimal place. **(4)**

(Total for Question 17 is 7 marks)

18 The times, in minutes, taken by customers to be served at a bank are modelled by $X \sim N(49, 10^2)$.

(a) Find $P(X > 59)$. **(3)**

(b) Find $P(44 < X < 59)$. **(3)**

(c) Find the value of a such that $P(X < a) = 0.75$, to 3 significant figures. **(3)**

(Total for Question 18 is 9 marks)

19 The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $P(X < 86) = 0.9332$ and $P(X < 90) = 0.9772$.

(a) Find the z values corresponding to the two probabilities, using the inverse normal distribution function, to 2 decimal places. **(3)**

(b) Hence write two equations in μ and σ and solve them to find μ and σ, each to 1 decimal place. **(4)**

(Total for Question 19 is 7 marks)

20 The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $P(X < 106) = 0.3085$ and $P(X < 122) = 0.9332$.

(a) Find the z values corresponding to the two probabilities, using the inverse normal distribution function, to 2 decimal places. **(3)**

(b) Hence write two equations in μ and σ and solve them to find μ and σ, each to 1 decimal place. **(4)**

(Total for Question 20 is 7 marks)

21 A large multiple-choice test has questions with 5 possible answers each, of which only one is correct. Priya has not revised, and guesses the answer to every question independently at random. She attempts 12 questions. Let R be the number of questions Priya answers correctly.

(a) State the distribution of R. **(1)**

(b) Find $P(R = 4)$. **(2)**

(c) Find $P(2 \leq R \leq 5)$. **(3)**

(d) The pass mark for the test is 6 correct answers out of 12. Find the probability that Priya passes the test by guessing. **(2)**

(Total for Question 21 is 8 marks)

22 The heights, in cm, of adult sunflowers grown by a gardener in Chester are modelled by the random variable $H \sim N(150, 15^2)$.

(a) Find the height h such that $P(H < h) = 0.90$. **(3)**

(b) Find the interquartile range of H. **(3)**

(Total for Question 22 is 6 marks)

Mark scheme · 2.8 Statistics: Distributions Depth

Question 1

(a) sum of probabilities = 1 used, i.e. 0.1 + 0.3 + 2k + k = 1 (oe)	**M1**
(a) k = 0.2 cso	**A1**
(a) Answer: k = 0.2	
(b) E(X) = 0(0.1)+1(0.3)+2(0.4)+3(0.2) (oe, ft their k)	**M1**
(b) 1.7 cao	**A1**
(b) Answer: E(X) = 1.7	
(c) P(X≥2)/P(X≥1) = (0.4+0.2)/(0.3+0.4+0.2) (oe)	**M1**
(c) 2/3 oe (awrt 0.667)	**A1**
(c) Answer: P(X ≥ 2 | X ≥ 1) = 2/3 (awrt 0.667)	

Question 2

(a) $E(X^2) = 0^2(0.1)+1^2(0.3)+2^2(0.4)+3^2(0.2)$ (oe)	**M1**
(a) 3.7 cso	**A1**
(a) Answer: $E(X^2) = 3.7$	
(b) $Var(X) = E(X^2) - [E(X)]^2 = 3.7 - 1.7^2$ (oe)	**M1**
(b) 0.81 cao	**A1**
(b) Answer: Var(X) = 0.81	
(c) use of Var(aX+b) = a^2 * Var(X)	**B1**
(c) $(-2)^2$ * 0.81 (ft their Var(X))	**M1**
(c) 3.24 cao	**A1**
(c) Answer: Var(3 - 2X) = 3.24	

Question 3

(a) Y ~ B(10, 0.3) stated	**B1**
(a) Answer: Y ~ B(10, 0.3)	
(b) correct expression C(10,3) * 0.3^3 * 0.7^7 (oe)	**M1**
(b) awrt 0.267	**A1**
(b) Answer: P(Y = 3) = awrt 0.267	
(c) use of P(Y≥5) = 1 - P(Y≤4) (oe)	**M1**
(c) correct computation of P(Y≤4) (sum of P(0) to P(4), or calculator cdf), e.g. awrt 0.850	**M1**
(c) awrt 0.150	**A1**
(c) Answer: P(Y ≥ 5) = awrt 0.150	

Question 4

(a) standardising: z = (68-60)/8 = 1 (oe)	**M1**
(a) awrt 0.841	**A1**
(a) Answer: P(X < 68) = awrt 0.841	
(b) both z-values found: z1 = (52-60)/8 = -1, z2 = (76-60)/8 = 2 (oe)	**M1**
(b) correct combination Phi(2) - [1 - Phi(1)] (oe, using symmetry)	**M1**
(b) awrt 0.819	**A1**
(b) Answer: P(52 < X < 76) = awrt 0.819	

Question 5

(a) mean of T = 1000+50 = 1050	**B1**
(a) variance of T = 15^2+4^2 = 241 (so T ~ N(1050, 241))	**B1**
(a) Answer: T ~ N(1050, 241)	
(b) use of T~N(1050,241) (ft their distribution)	**M1**
(b) standardising z = (1060-1050)/$\sqrt{241}$ (oe)	**M1**
(b) z = awrt 0.644	**A1**
(b) awrt 0.260	**A1**
(b) Answer: P(T > 1060) = awrt 0.260	
(c) identifies N ~ B(6, 0.260) (ft their p from (b))	**M1**
(c) P(N=5)+P(N=6) computed (ft)	**M1**
(c) awrt 0.00558 (ft)	**A1**
(c) Answer: P(at least 5 of 6) = awrt 0.00558	

Question 6

(a) boundary condition P(V<500) = 0.02 identified	**M1**
(a) z = invNorm(0.02) = -2.0537 (awrt) used (or equivalently invNorm(0.98) = 2.0537)	**M1**
(a) equation (500-μ)/6 = -2.0537 (oe) set up and rearranged	**M1**
(a) μ = awrt 512.3	**A1**
(a) Answer: μ (minimum) = awrt 512.3 ml	
(b) standardising z = (520-512)/6 (oe)	**M1**
(b) use of 1 - Phi(z) (oe)	**M1**
(b) awrt 0.0912	**A1**
(b) Answer: P(V > 520) = awrt 0.0912	
(c) identifies M ~ B(80, 0.0912) (ft their probability from (b))	**B1**
(c) states np = awrt 7.30 > 5 and n(1-p) = awrt 72.7 > 5, justifying a normal approximation	**B1**
(c) normal approximation M ~ N(7.296, 6.63) (oe, ft) with mean np and variance np(1-p) correctly computed	**M1**
(c) continuity correction applied: P(M<4.5) and standardising, z = (4.5-7.296)/$\sqrt{6.63}$ (oe, ft)	**M1**
(c) awrt 0.139 (ft)	**A1**
(c) Answer: P(M < 5) = awrt 0.139	

Question 7

(a) standardises: z = (195 - 183)/6 = 2	**M1**
(a) P(Z > 2) = 1 - 0.9772	**A1**
(a) awrt 0.0228	**A1**
(a) Answer: awrt 0.0228	
(b) finds both z values: 0 and 2	**M1**
(b) P(Z < upper) - P(Z < lower)	**M1**
(b) awrt 0.477	**A1**
(b) Answer: awrt 0.477	
(c) z = inverse normal of 0.95 = 1.6449	**M1**
(c) a = 183 + z x 6	**M1**
(c) a = 193	**A1**
(c) Answer: a = 193	

Question 8

(a) n is large and p is close enough to 0.5 (np = 72 and n(1 - p) = 48 both large) **B1**

(a) mean 72, variance 28.80 **B1**

(a) Answer: N(72, 28.80)

(b) continuity correction: P(Y > 75.5) **M1**

(b) z = (75.5 - 72)/5.3666 = 0.6522 **M1**

(b) 0.257 **A1**

(b) Answer: 0.257

(c) 1 - P(X ≤ 75) with the binomial distribution **M1**

(c) 0.258 **A1**

(c) difference 0.00125 **A1**

(c) Answer: exact 0.258, difference 0.00125

Question 9

(a) $16C9 \times 0.45^9 \times 0.55^7$ **M1**

(a) awrt 0.132 **A1**

(a) Answer: awrt 0.132

(b) 1 - P(X ≤ 8) **M1**

(b) awrt 0.256 **A1**

(b) Answer: awrt 0.256

(c) E(X) = 16 x 0.45 = 7.2 **B1**

(c) Var(X) = 16 x 0.45 x 0.55 = 3.960 **B1**

(c) standard deviation = 1.99 **B1**

(c) Answer: E(X) = 7.2, Var(X) = 3.960, sd 1.99

(d) P(X ≤ 10) - P(X ≤ 7) **M1**

(d) awrt 0.388 **A1**

(d) Answer: awrt 0.388

Question 10

(a) n is large and p is close enough to 0.5 (np = 32 and n(1 - p) = 48 both large) **B1**

(a) mean 32, variance 19.20 **B1**

(a) Answer: N(32, 19.20)

(b) continuity correction: P(Y > 30.5) **M1**

(b) z = (30.5 - 32)/4.3818 = -0.3423 **M1**

(b) 0.634 **A1**

(b) Answer: 0.634

(c) 1 - P(X ≤ 30) with the binomial distribution **M1**

(c) 0.631 **A1**

(c) difference 0.00267 **A1**

(c) Answer: exact 0.631, difference 0.00267

Question 11

(a) inverse normal of 0.8413 **M1**

(a) z = 1.00 **A1**

(a) z = 2.00 **A1**

(a) Answer: z = 1.00 and 2.00

(b) (74 - μ)/σ = 1.00 and (78 - μ)/σ = 2.00 **M1**

(b) subtracts to eliminate μ **M1**

(b) σ = 4.0 **A1**

(b) μ = 70.0 **A1**

(b) Answer: μ = 70.0, σ = 4.0

Question 12

(a) standardises: z = (67 - 47)/8 = 2.5 **M1**

(a) P(Z > 2.5) = 1 - 0.9938 **A1**

(a) awrt 0.00621 **A1**

(a) Answer: awrt 0.00621

(b) finds both z values: -1.5 and 2.5 **M1**

(b) P(Z < upper) - P(Z < lower) **M1**

(b) awrt 0.927 **A1**

(b) Answer: awrt 0.927

(c) z = inverse normal of 0.8 = 0.8416 **M1**

(c) a = 47 + z x 8 **M1**

(c) a = 53.7 **A1**

(c) Answer: a = 53.7

Question 13

(a) n is large and p is close enough to 0.5 (np = 10 and n(1 - p) = 40 both large) **B1**

(a) mean 10, variance 8.000 **B1**

(a) Answer: N(10, 8.000)

(b) continuity correction: P(Y > 8.5) **M1**

(b) z = (8.5 - 10)/2.8284 = -0.5303 **M1**

(b) 0.702 **A1**

(b) Answer: 0.702

(c) 1 - P(X ≤ 8) with the binomial distribution **M1**

(c) 0.693 **A1**

(c) difference 0.00939 **A1**

(c) Answer: exact 0.693, difference 0.00939

Question 14

(a) inverse normal of 0.0668 **M1**

(a) z = -1.50 **A1**

(a) z = 0.50 **A1**

(a) Answer: z = -1.50 and 0.50

(b) (32 - μ)/σ = -1.50 and (52 - μ)/σ = 0.50 **M1**

(b) subtracts to eliminate μ **M1**

(b) σ = 10.0 **A1**

(b) μ = 47.0 **A1**

(b) Answer: μ = 47.0, σ = 10.0

Question 15

(a) n is large and p is close enough to 0.5 (np = 10 and n(1 - p) = 40 both large) **B1**

(a) mean 10, variance 8.000 **B1**

(a) Answer: N(10, 8.000)

(b) continuity correction: P(Y > 6.5) **M1**

(b) z = (6.5 - 10)/2.8284 = -1.2374 **M1**

(b) 0.892 **A1**

(b) Answer: 0.892

(c) 1 - P(X ≤ 6) with the binomial distribution **M1**

(c) 0.897 **A1**

(c) difference 0.00456 **A1**

(c) Answer: exact 0.897, difference 0.00456

Question 16

(a) $12C2 \times 0.25^2 \times 0.75^{10}$ **M1**

(a) awrt 0.232 **A1**

(a) Answer: awrt 0.232

(b) $1 - P(X \leq 1)$ **M1**

(b) awrt 0.842 **A1**

(b) Answer: awrt 0.842

(c) $E(X) = 12 \times 0.25 = 3$ **B1**

(c) $Var(X) = 12 \times 0.25 \times 0.75 = 2.250$ **B1**

(c) standard deviation = 1.50 **B1**

(c) Answer: E(X) = 3, Var(X) = 2.250, sd 1.50

(d) $P(X \leq 3) - P(X \leq 0)$ **M1**

(d) awrt 0.617 **A1**

(d) Answer: awrt 0.617

Question 17

(a) inverse normal of 0.6915 **M1**

(a) z = 0.50 **A1**

(a) z = 1.00 **A1**

(a) Answer: z = 0.50 and 1.00

(b) $(50 - \mu)/\sigma = 0.50$ and $(52 - \mu)/\sigma = 1.00$ **M1**

(b) subtracts to eliminate μ **M1**

(b) $\sigma = 4.0$ **A1**

(b) $\mu = 48.0$ **A1**

(b) Answer: $\mu = 48.0$, $\sigma = 4.0$

Question 18

(a) standardises: $z = (59 - 49)/10 = 1$ **M1**

(a) $P(Z > 1) = 1 - 0.8413$ **A1**

(a) awrt 0.159 **A1**

(a) Answer: awrt 0.159

(b) finds both z values: -0.5 and 1 **M1**

(b) P(Z < upper) - P(Z < lower) **M1**

(b) awrt 0.533 **A1**

(b) Answer: awrt 0.533

(c) z = inverse normal of 0.75 = 0.6745 **M1**

(c) a = 49 + z x 10 **M1**

(c) a = 55.7 **A1**

(c) Answer: a = 55.7

Question 19

(a) inverse normal of 0.9332 **M1**

(a) z = 1.50 **A1**

(a) z = 2.00 **A1**

(a) Answer: z = 1.50 and 2.00

(b) $(86 - \mu)/\sigma = 1.50$ and $(90 - \mu)/\sigma = 2.00$ **M1**

(b) subtracts to eliminate μ **M1**

(b) $\sigma = 8.0$ **A1**

(b) $\mu = 74.0$ **A1**

(b) Answer: $\mu = 74.0$, $\sigma = 8.0$

Question 20

(a) inverse normal of 0.3085 **M1**

(a) z = -0.50 **A1**

(a) z = 1.50 **A1**

(a) Answer: z = -0.50 and 1.50

(b) $(106 - \mu)/\sigma = -0.50$ and $(122 - \mu)/\sigma = 1.50$ **M1**

(b) subtracts to eliminate μ **M1**

(b) $\sigma = 8.0$ **A1**

(b) $\mu = 110.0$ **A1**

(b) Answer: $\mu = 110.0$, $\sigma = 8.0$

Question 21

(a) $R \sim B(12, 0.2)$ **B1**

(a) Answer: $R \sim B(12, 0.2)$

(b) correct expression $C(12,4) * 0.2^4 * 0.8^8$ (oe) **M1**

(b) awrt 0.133 **A1**

(b) Answer: P(R = 4) = awrt 0.133

(c) identifies $P(2 \leq R \leq 5) = P(R=2)+P(R=3)+P(R=4)+P(R=5)$ (oe, or cdf subtraction) **M1**

(c) at least two of P(R=2)=awrt 0.283, P(R=3)=awrt 0.236, P(R=5)=awrt 0.0532 correct (ft) **M1**

(c) awrt 0.706 **A1**

(c) Answer: $P(2 \leq R \leq 5)$ = awrt 0.706

(d) $P(R \geq 6) = 1 - P(R \leq 5)$ (oe) **M1**

(d) awrt 0.0194 **A1**

(d) Answer: P(pass) = awrt 0.0194

Question 22

(a) correct inverse normal z-value used, z = 1.2816 (awrt) **M1**

(a) h = 150 + 1.2816*15 (oe, ft their z) **M1**

(a) awrt 169 **A1**

(a) Answer: h = awrt 169 cm

(b) z = 0.6745 (awrt) identified for both quartiles by symmetry **M1**

(b) IQR = 2*0.6745*15 (oe) **M1**

(b) awrt 20.2 **A1**

(b) Answer: IQR = awrt 20.2 cm