2.9 Statistics: Hypothesis Testing Depth: 2-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 120 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Hypothesis Testing Toolkit: Formulae and Standard Normal Critical Values

Original material written for Revision Library, styled on Edexcel A-level Mathematics (9MA0) Statistics content.

For a binomial test of H0: p = p0, use X ~ B(n, p0) and compute the relevant cumulative probability (a tail probability, or a p-value) directly, either from cumulative binomial tables or a calculator's binomial cdf function. For a normal test of H0: mu = mu0 with known sigma, standardise using z = (xbar - mu0)/(sigma/sqrt(n)) and compare z with a critical value, or convert to a critical region for xbar directly. For a correlation test of H0: rho = 0, compare the sample product moment correlation coefficient, r, with a critical value from a table of critical values for the sample size n (such a critical value will always be given where needed). Standard normal critical z-values used throughout this pack: one-tailed 10% (or two-tailed 20%) = 1.2816; one-tailed 5% (or two-tailed 10%, i.e. 5% in each tail) = 1.6449; one-tailed 1% (or two-tailed 2%) = 2.3263; two-tailed 5% (2.5% in each tail) = 1.9600; two-tailed 1% (0.5% in each tail) = 2.5758. When finding a binomial critical region, choose the region whose cumulative probability is as close as possible to the stated significance level, unless a question specifically requires the probability to remain below that level; the actual (achieved) significance level is almost never exactly equal to the nominal one, because the binomial distribution is discrete.

1 A school claims that the probability a student passes a driving test at the first attempt is 0.15. A parent believes the probability is lower than claimed. A random sample of 20 students is checked. Let p be the true probability, and X the number in the sample, so under H0, X ~ B(20, 0.15).

(a) State suitable hypotheses for the test, in terms of p. **(2)**

(b) Using a 10% significance level, find the critical region for the test. **(3)**

(c) Write down the actual significance level of the test, and state the probability of a Type I error. **(2)**

(Total for Question 1 is 7 marks)

© Revision Library | revisionlibrary.co.uk

2 A student records 9 pairs of values for the age of a car and its resale price. The product moment correlation coefficient is r = -0.567. The critical value of r for a one-tailed test with a 5% significance level and n = 9 is 0.5822.

(a) Write down suitable hypotheses to test for negative correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 2 is 5 marks)

---

3 The proportion of a population with a certain characteristic was 0.25. A researcher believes that this proportion has decreased. In a random sample of 15 people, 0 have the characteristic.

(a) State the hypotheses for a one-tailed test. **(2)**

(b) Assuming H0 is true, find the probability of obtaining a value of X at least as extreme as 0, where X ~ B(15, 0.25). **(2)**

(c) Test the researcher's belief at the 5% level of significance, stating your conclusion in context. **(3)**

(Total for Question 3 is 7 marks)

---

4 A student records 7 pairs of values for the age of a car and its resale price. The product moment correlation coefficient is r = 0.69. The critical value of r for a one-tailed test with a 5% significance level and n = 7 is 0.6694.

(a) Write down suitable hypotheses to test for positive correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 4 is 5 marks)

---

5 A student records 6 pairs of values for the age of a car and its resale price. The product moment correlation coefficient is r = 0.384. The critical value of r for a one-tailed test with a 5% significance level and n = 6 is 0.7293.

(a) Write down suitable hypotheses to test for positive correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 5 is 5 marks)

© Revision Library | revisionlibrary.co.uk

6 The random variable X has distribution $N(\mu, 2^2)$. A random sample of 9 observations of X has mean 40.0. A test of H0: $\mu = 39$ against H1: $\mu > 39$ is carried out at the 5% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test, in context. **(2)**

(d) Find the critical value of the sample mean for this test, to 4 significant figures. **(2)**

(Total for Question 6 is 8 marks)

---

7 The proportion of a population with a certain characteristic was 0.25. A researcher believes that this proportion has decreased. In a random sample of 15 people, 3 have the characteristic.

(a) State the hypotheses for a one-tailed test. **(2)**

(b) Assuming H0 is true, find the probability of obtaining a value of X at least as extreme as 3, where $X \sim B(15, 0.25)$. **(2)**

(c) Test the researcher's belief at the 10% level of significance, stating your conclusion in context. **(3)**

(Total for Question 7 is 7 marks)

---

8 The random variable X has distribution $N(\mu, 4^2)$. A random sample of 9 observations of X has mean 64.93. A test of H0: $\mu = 62$ against H1: $\mu > 62$ is carried out at the 1% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test, in context. **(2)**

(d) Find the critical value of the sample mean for this test, to 4 significant figures. **(2)**

(Total for Question 8 is 8 marks)

---

9 The random variable X has distribution $N(\mu, 5^2)$. A random sample of 36 observations of X has mean 61.83. A test of H0: $\mu = 63$ against H1: $\mu < 63$ is carried out at the 5% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test, in context. **(2)**

(d) Find the critical value of the sample mean for this test, to 4 significant figures. **(2)**

(Total for Question 9 is 8 marks)

© Revision Library | revisionlibrary.co.uk

10 The proportion of a population with a certain characteristic was 0.1. A researcher believes that this proportion has increased. In a random sample of 15 people, 4 have the characteristic.

(a) State the hypotheses for a one-tailed test. **(2)**

(b) Assuming H0 is true, find the probability of obtaining a value of X at least as extreme as 4, where $X \sim B(15, 0.1)$. **(2)**

(c) Test the researcher's belief at the 5% level of significance, stating your conclusion in context. **(3)**

(Total for Question 10 is 7 marks)

---

11 The random variable X has distribution $N(\mu, 3^2)$. A random sample of 49 observations of X has mean 46.97. A test of $H0: \mu = 48$ against $H1: \mu < 48$ is carried out at the 5% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test, in context. **(2)**

(d) Find the critical value of the sample mean for this test, to 4 significant figures. **(2)**

(Total for Question 11 is 8 marks)

---

12 The random variable X has distribution $N(\mu, 10^2)$. A random sample of 9 observations of X has mean 54.0. A test of $H0: \mu = 50$ against $H1: \mu > 50$ is carried out at the 5% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test, in context. **(2)**

(d) Find the critical value of the sample mean for this test, to 4 significant figures. **(2)**

(Total for Question 12 is 8 marks)

© Revision Library | revisionlibrary.co.uk

13 A manufacturer claims that 15% of its light bulbs last more than 1000 hours. A retailer believes the proportion is higher than claimed and tests a random sample of 12 bulbs. Let p be the true probability, and X the number in the sample, so under H0, X ~ B(12, 0.15).

(a) State suitable hypotheses for the test, in terms of p. **(2)**

(b) Using a 10% significance level, find the critical region for the test. **(3)**

(c) Write down the actual significance level of the test, and state the probability of a Type I error. **(2)**

(Total for Question 13 is 7 marks)

---

14 A dental practice in Swansea states that 85% of patients attend their booked check-up appointments. The practice manager, Aisha, runs a reminder-text trial and wants to test, using a random sample of 30 patients, whether the attendance rate has increased. Let X be the number of patients, out of 30, who attend, with X ~ B(30, p) under H0.

(a) Write down suitable null and alternative hypotheses. **(2)**

(b) Using a 5% significance level, find the critical region for the test, such that the probability of incorrectly rejecting H0 is as close as possible to 5% while remaining less than 5%. **(5)**

(c) State the actual significance level of the test in part (b). **(1)**

(d) After the trial, 29 of the 30 sampled patients attended their appointment. State the conclusion of the test. **(1)**

(Total for Question 14 is 9 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 2.9 Statistics: Hypothesis Testing Depth: 2-hour Pack

Question 1

(a) H0: p = 0.15 **B1**

(a) H1: p < 0.15 **B1**

(a) Answer: H0: p = 0.15, H1: p < 0.15

(b) uses the binomial distribution B(20, 0.15) to find cumulative probabilities near the tail **M1**

(b) $P(X \leq 0) = 0.03876 \leq 0.1$ **A1**

(b) critical region: $X \leq 0$ **A1**

(b) Answer: critical region $X \leq 0$

(c) actual significance level = 0.0388 **B1**

(c) this is the probability of a Type I error (rejecting H0 when it is true) **B1**

(c) Answer: 0.0388

Question 2

(a) H0: ρ = 0 **B1**

(a) H1: ρ < 0 **B1**

(a) Answer: H0: ρ = 0, H1: ρ < 0

(b) compares |r| = 0.567 with the critical value 0.5822 in the correct tail **M1**

(b) do not reject H0 **A1**

(b) there is insufficient evidence of negative correlation **B1**

(b) Answer: do not reject H0

Question 3

(a) H0: p = 0.25 **B1**

(a) H1: p < 0.25 **B1**

(a) Answer: H0: p = 0.25, H1: p < 0.25

(b) P(X ≤ 0) **M1**

(b) 0.0134 **A1**

(b) Answer: 0.0134

(c) compares 0.0134 with 0.05 **M1**

(c) reject H0 **A1**

(c) there is sufficient evidence that the proportion has decreased **B1**

(c) Answer: reject H0

Question 4

(a) H0: ρ = 0 **B1**

(a) H1: ρ > 0 **B1**

(a) Answer: H0: ρ = 0, H1: ρ > 0

(b) compares |r| = 0.69 with the critical value 0.6694 in the correct tail **M1**

(b) reject H0 **A1**

(b) there is evidence of positive correlation **B1**

(b) Answer: reject H0

Question 5

(a) H0: ρ = 0 **B1**

(a) H1: ρ > 0 **B1**

(a) Answer: H0: ρ = 0, H1: ρ > 0

(b) compares |r| = 0.384 with the critical value 0.7293 in the correct tail **M1**

(b) do not reject H0 **A1**

(b) there is insufficient evidence of positive correlation **B1**

(b) Answer: do not reject H0

Question 6

(a) X-bar ~ $N(39, 2^2/9)$ **B1**

(a) standard error $2/\sqrt{9} = 0.6667$ **B1**

(a) Answer: $N(39, 2^2/9)$

(b) $z = (40.0 - 39)/0.6667 = 1.500$ **M1**

(b) p-value = 0.0668 **A1**

(b) Answer: 0.0668

(c) compares p-value 0.0668 with 0.05 **M1**

(c) do not reject H0: insufficient evidence that the mean has increased **A1**

(c) Answer: do not reject H0

(d) z critical = 1.6449 **M1**

(d) critical value = 40.10 **A1**

(d) Answer: 40.10

Question 7

(a) H0: p = 0.25 **B1**

(a) H1: p < 0.25 **B1**

(a) Answer: H0: p = 0.25, H1: p < 0.25

(b) P(X ≤ 3) **M1**

(b) 0.461 **A1**

(b) Answer: 0.461

(c) compares 0.461 with 0.1 **M1**

(c) do not reject H0 **A1**

(c) there is insufficient evidence that the proportion has decreased **B1**

(c) Answer: do not reject H0

Question 8

(a) X-bar ~ $N(62, 4^2/9)$ **B1**

(a) standard error $4/\sqrt{9} = 1.333$ **B1**

(a) Answer: $N(62, 4^2/9)$

(b) $z = (64.93 - 62)/1.333 = 2.198$ **M1**

(b) p-value = 0.0140 **A1**

(b) Answer: 0.0140

(c) compares p-value 0.0140 with 0.01 **M1**

(c) do not reject H0: insufficient evidence that the mean has increased **A1**

(c) Answer: do not reject H0

(d) z critical = 2.3263 **M1**

(d) critical value = 65.10 **A1**

(d) Answer: 65.10

© Revision Library | revisionlibrary.co.uk

Question 9

(a) X-bar ~ N(63, 5^2/36) **B1**

(a) standard error $5/\sqrt{36}$ = 0.8333 **B1**

(a) Answer: N(63, 5^2/36)

(b) z = (61.83 - 63)/0.8333 = -1.404 **M1**

(b) p-value = 0.0802 **A1**

(b) Answer: 0.0802

(c) compares p-value 0.0802 with 0.05 **M1**

(c) do not reject H0: insufficient evidence that the mean has decreased **A1**

(c) Answer: do not reject H0

(d) z critical = -1.6449 **M1**

(d) critical value = 61.63 **A1**

(d) Answer: 61.63

Question 10

(a) H0: p = 0.1 **B1**

(a) H1: p > 0.1 **B1**

(a) Answer: H0: p = 0.1, H1: p > 0.1

(b) P(X ≥ 4) = 1 - P(X ≤ 3) **M1**

(b) 0.0556 **A1**

(b) Answer: 0.0556

(c) compares 0.0556 with 0.05 **M1**

(c) do not reject H0 **A1**

(c) there is insufficient evidence that the proportion has increased **B1**

(c) Answer: do not reject H0

Question 11

(a) X-bar ~ N(48, 3^2/49) **B1**

(a) standard error $3/\sqrt{49}$ = 0.4286 **B1**

(a) Answer: N(48, 3^2/49)

(b) z = (46.97 - 48)/0.4286 = -2.403 **M1**

(b) p-value = 0.00812 **A1**

(b) Answer: 0.00812

(c) compares p-value 0.00812 with 0.05 **M1**

(c) reject H0: there is sufficient evidence that the mean has decreased **A1**

(c) Answer: reject H0

(d) z critical = -1.6449 **M1**

(d) critical value = 47.30 **A1**

(d) Answer: 47.30

Question 12

(a) X-bar ~ N(50, 10^2/9) **B1**

(a) standard error $10/\sqrt{9}$ = 3.333 **B1**

(a) Answer: N(50, 10^2/9)

(b) z = (54.0 - 50)/3.333 = 1.200 **M1**

(b) p-value = 0.115 **A1**

(b) Answer: 0.115

(c) compares p-value 0.115 with 0.05 **M1**

(c) do not reject H0: insufficient evidence that the mean has increased **A1**

(c) Answer: do not reject H0

(d) z critical = 1.6449 **M1**

(d) critical value = 55.48 **A1**

(d) Answer: 55.48

Question 13

(a) H0: p = 0.15 **B1**

(a) H1: p > 0.15 **B1**

(a) Answer: H0: p = 0.15, H1: p > 0.15

(b) uses the binomial distribution B(12, 0.15) to find cumulative probabilities near the tail **M1**

(b) P(X ≥ 4) = 0.09221 ≤ 0.1 **A1**

(b) critical region: X ≥ 4 **A1**

(b) Answer: critical region X ≥ 4

(c) actual significance level = 0.0922 **B1**

(c) this is the probability of a Type I error (rejecting H0 when it is true) **B1**

(c) Answer: 0.0922

Question 14

(a) H0: p = 0.85, where p is the population proportion of patients who attend **B1**

(a) H1: p > 0.85 (oe) **B1**

(a) Answer: H0: p = 0.85, H1: p > 0.85

(b) attempt to find upper-tail cumulative probabilities for X ~ B(30, 0.85), e.g. via Y = 30 - X ~ B(30, 0.15) **M1**

(b) P(X ≥ 28) = 0.151 (awrt 3sf) - too large, so rejected **A1**

(b) P(X ≥ 29) = 0.0480 (awrt 3sf) - valid, since less than 5% **A1**

(b) correctly identifies P(X ≥ 29) = 0.0480 as the required boundary, since P(X ≥ 28) exceeds 5% **A1**

(b) critical region X ≥ 29 (accept {29, 30}) **A1**

(b) Answer: Critical region: X ≥ 29 (i.e. {29, 30}).

(c) 0.0480 (4.80%, awrt) (ft) **B1**

(c) Answer: 0.0480 (4.80%)

(d) 29 lies in the critical region (X ≥ 29), so reject H0; evidence, at the 5% level, that the attendance rate has increased **B1ft**

(d) Answer: 29 is in the critical region, so reject H0: evidence the attendance rate has increased.

© Revision Library | revisionlibrary.co.uk