2.9 Statistics: Hypothesis Testing Depth: 30-minute Pack

EDEXCEL 9MA0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Hypothesis Testing Toolkit: Formulae and Standard Normal Critical Values

Original material written for Revision Library, styled on Edexcel A-level Mathematics (9MA0) Statistics content.

For a binomial test of H0: p = p0, use X ~ B(n, p0) and compute the relevant cumulative probability (a tail probability, or a p-value) directly, either from cumulative binomial tables or a calculator's binomial cdf function. For a normal test of H0: mu = mu0 with known sigma, standardise using z = (xbar - mu0)/(sigma/sqrt(n)) and compare z with a critical value, or convert to a critical region for xbar directly. For a correlation test of H0: rho = 0, compare the sample product moment correlation coefficient, r, with a critical value from a table of critical values for the sample size n (such a critical value will always be given where needed). Standard normal critical z-values used throughout this pack: one-tailed 10% (or two-tailed 20%) = 1.2816; one-tailed 5% (or two-tailed 10%, i.e. 5% in each tail) = 1.6449; one-tailed 1% (or two-tailed 2%) = 2.3263; two-tailed 5% (2.5% in each tail) = 1.9600; two-tailed 1% (0.5% in each tail) = 2.5758. When finding a binomial critical region, choose the region whose cumulative probability is as close as possible to the stated significance level, unless a question specifically requires the probability to remain below that level; the actual (achieved) significance level is almost never exactly equal to the nominal one, because the binomial distribution is discrete.

1 A cafe in Leeds states that 20% of its customers choose a vegan pastry. The cafe manager, Kwame, believes this proportion has increased since a new vegan menu was introduced. In a random sample of 15 customers, 6 chose a vegan pastry. Let X be the number of customers, out of 15, choosing a vegan pastry, with X ~ B(15, p) under H0.

(a) State suitable null and alternative hypotheses for Kwame's test. **(1)**

(b) Test, at the 5% significance level, whether there is evidence that the proportion of customers choosing a vegan pastry has increased. State your conclusion in context. **(4)**

(Total for Question 1 is 5 marks)

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2 The proportion of a population with a certain characteristic was 0.4. A researcher believes that this proportion has increased. In a random sample of 25 people, 14 have the characteristic.

(a) State the hypotheses for a one-tailed test. **(2)**

(b) Assuming H0 is true, find the probability of obtaining a value of X at least as extreme as 14, where X ~ B(25, 0.4). **(2)**

(c) Test the researcher's belief at the 5% level of significance, stating your conclusion in context. **(3)**

(Total for Question 2 is 7 marks)

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3 A student records 5 pairs of values for daily temperature and the number of visitors to a park. The product moment correlation coefficient is r = 0.661. The critical value of r for a one-tailed test with a 5% significance level and n = 5 is 0.8054.

(a) Write down suitable hypotheses to test for positive correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 3 is 5 marks)

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4 A call centre in Cardiff aims for 40% of calls to be resolved on first contact. The team leader, Fatima, wants to test whether this proportion has changed following a new staff training programme. She takes a random sample of 25 calls. Let X be the number of calls, out of 25, resolved on first contact, with X ~ B(25, p) under H0: p = 0.4.

(a) Find the critical region for a two-tailed test at the 10% significance level, such that the probability in each tail is as close as possible to 0.05. **(6)**

(b) Given that 5 of the 25 sampled calls were resolved on first contact, state the conclusion of the test. **(1)**

(Total for Question 4 is 7 marks)

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Mark scheme · 2.9 Statistics: Hypothesis Testing Depth: 30-minute Pack

Question 1

(a) H0: p = 0.2, H1: p > 0.2, both stated correctly **B1**

(a) Answer: H0: p = 0.2, H1: p > 0.2

(b) attempt P(X ≥ 6) using X ~ B(15, 0.2), e.g. via 1 - P(X ≤ 5) **M1**

(b) P(X ≥ 6) = 0.0611 (awrt 3sf) **A1**

(b) compare 0.0611 with 0.05 **M1**

(b) 0.0611 > 0.05, so insufficient evidence, at the 5% level, that the proportion of customers choosing a vegan pastry has increased (do not reject H0) **A1ft**

(b) Answer: P(X ≥ 6) = 0.0611 (3sf); 0.0611 > 0.05, so insufficient evidence at the 5% level that the proportion has increased.

Question 2

(a) H0: p = 0.4 **B1**

(a) H1: p > 0.4 **B1**

(a) Answer: H0: p = 0.4, H1: p > 0.4

(b) P(X ≥ 14) = 1 - P(X ≤ 13) **M1**

(b) 0.0778 **A1**

(b) Answer: 0.0778

(c) compares 0.0778 with 0.05 **M1**

(c) do not reject H0 **A1**

(c) there is insufficient evidence that the proportion has increased **B1**

(c) Answer: do not reject H0

Question 3

(a) H0: ρ = 0 **B1**

(a) H1: ρ > 0 **B1**

(a) Answer: H0: ρ = 0, H1: ρ > 0

(b) compares |r| = 0.661 with the critical value 0.8054 in the correct tail **M1**

(b) do not reject H0 **A1**

(b) there is insufficient evidence of positive correlation **B1**

(b) Answer: do not reject H0

Question 4

(a) attempt cumulative probabilities for X ~ B(25, 0.4) in the lower tail **M1**

(a) P(X ≤ 5) = 0.0294 and P(X ≤ 6) = 0.0736 (awrt 3sf) **A1**

(a) lower critical region X ≤ 5 (0.0294 is closer to 0.05 than 0.0736: |0.05-0.0294| = 0.0206 < |0.0736-0.05| = 0.0236) **B1**

(a) attempt cumulative probabilities for X ~ B(25, 0.4) in the upper tail **M1**

(a) P(X ≥ 15) = 0.0344 and P(X ≥ 14) = 0.0778 (awrt 3sf) **A1**

(a) upper critical region X ≥ 15 (0.0344 is closer to 0.05 than 0.0778: |0.05-0.0344| = 0.0156 < |0.0778-0.05| = 0.0278) **B1**

(a) Answer: Critical region: X ≤ 5 or X ≥ 15; combined actual significance level 0.0294 + 0.0344 = 0.0638.

(b) X = 5 lies in the critical region (X ≤ 5), so reject H0; evidence, at the 10% level, that the proportion of calls resolved on first contact has changed **B1ft**

(b) Answer: X = 5 is in the critical region, so reject H0: evidence the proportion has changed.

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