2.9 Statistics: Hypothesis Testing Depth: Test Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Hypothesis Testing Toolkit: Formulae and Standard Normal Critical Values

Original material written for Revision Library, styled on Edexcel A-level Mathematics (9MA0) Statistics content.

For a binomial test of H0: p = p0, use X ~ B(n, p0) and compute the relevant cumulative probability (a tail probability, or a p-value) directly, either from cumulative binomial tables or a calculator's binomial cdf function. For a normal test of H0: mu = mu0 with known sigma, standardise using z = (xbar - mu0)/(sigma/sqrt(n)) and compare z with a critical value, or convert to a critical region for xbar directly. For a correlation test of H0: rho = 0, compare the sample product moment correlation coefficient, r, with a critical value from a table of critical values for the sample size n (such a critical value will always be given where needed). Standard normal critical z-values used throughout this pack: one-tailed 10% (or two-tailed 20%) = 1.2816; one-tailed 5% (or two-tailed 10%, i.e. 5% in each tail) = 1.6449; one-tailed 1% (or two-tailed 2%) = 2.3263; two-tailed 5% (2.5% in each tail) = 1.9600; two-tailed 1% (0.5% in each tail) = 2.5758. When finding a binomial critical region, choose the region whose cumulative probability is as close as possible to the stated significance level, unless a question specifically requires the probability to remain below that level; the actual (achieved) significance level is almost never exactly equal to the nominal one, because the binomial distribution is discrete.

1 A garden centre in Yorkshire claims that 30% of the tulip bulbs it sells produce double blooms. A horticulturist, Priya, is investigating whether the true proportion of double-blooming bulbs is different from 30%. Let X be the number of bulbs producing double blooms in a random sample of n bulbs, and assume X ~ B(n, p) under the null hypothesis.

(a) Write down suitable null and alternative hypotheses for Priya's test. **(2)**

(b) State, with a reason, whether this is a one-tailed or two-tailed test. **(1)**

(Total for Question 1 is 3 marks)

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2 A seed company claims that 65% of its sunflower seeds germinate. Following a wet spring, an allotment holder in Norwich, Siobhan, believes the germination rate has decreased. She plants a random sample of 20 seeds, of which 8 germinate. Let X be the number of seeds, out of 20, that germinate, with $X \sim B(20, p)$ under H0: $p = 0.65$.

(a) Find the critical region for a test at the 5% significance level for whether the germination rate has decreased. The critical region should have probability as close as possible to 0.05. **(4)**

(b) State, with a reason, the conclusion of the test given that 8 of the 20 seeds germinated. **(1)**

(Total for Question 2 is 5 marks)

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3 The times taken by students at a sixth-form college in Manchester to complete an online numeracy assessment are normally distributed with mean 42 minutes and standard deviation 6 minutes, based on historical records. Following the introduction of a new revision app, the exams officer, Grace, takes a random sample of 40 students and finds a mean completion time of 40.1 minutes. Assume the standard deviation is unchanged.

(a) State suitable hypotheses for Grace's test of whether the mean completion time has decreased. **(2)**

(b) Show that the test statistic is $z = -2.00$, correct to 3 significant figures. **(2)**

(c) Using a critical value of -1.6449 for a one-tailed test at the 5% significance level, test whether there is evidence that the mean completion time has decreased. State your conclusion in context. **(3)**

(Total for Question 3 is 7 marks)

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4 A student records 8 pairs of values for the length of time spent revising and the test score. The product moment correlation coefficient is $r = 0.788$. The critical value of r for a one-tailed test with a 5% significance level and $n = 8$ is 0.6215.

(a) Write down suitable hypotheses to test for positive correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 4 is 5 marks)

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5 The proportion of a population with a certain characteristic was 0.3. A researcher believes that this proportion has increased. In a random sample of 20 people, 10 have the characteristic.

(a) State the hypotheses for a one-tailed test. **(2)**

(b) Assuming H0 is true, find the probability of obtaining a value of X at least as extreme as 10, where X ~ B(20, 0.3). **(2)**

(c) Test the researcher's belief at the 5% level of significance, stating your conclusion in context. **(3)**

(Total for Question 5 is 7 marks)

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6 A pharmaceutical trial investigates a new insomnia treatment. Without treatment, the time taken by adults to fall asleep is normally distributed with mean 34 minutes and standard deviation 9 minutes, based on a large historical dataset. Researcher Amara records a mean time of 30.5 minutes to fall asleep in a random sample of 36 patients using the new treatment.

(a) Using a suitable hypothesis test at the 1% significance level, determine whether there is evidence that the treatment reduces the mean time taken to fall asleep. State your hypotheses clearly. **(6)**

(b) Interpret, in context, what is meant by the test in part (a) being 'significant at the 1% level'. **(2)**

(Total for Question 6 is 8 marks)

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7 A students' union at a university in Nottingham believes that 55% of first-year students use the on-campus gym at least once a week. Following a new fitness marketing campaign, the sabbatical officer, Nadia, wants to investigate whether this proportion has changed. She takes a random sample of 22 first-year students. Let X be the number, out of the 22 sampled, who use the gym at least once a week, with $X \sim B(22, p)$ under H0: $p = 0.55$.

(a) Nadia initially wants to test only whether the proportion has increased. State appropriate hypotheses for this one-tailed test. **(2)**

(b) Find the critical region for this one-tailed test at the 5% significance level, such that the probability of rejecting H0 is as close as possible to 0.05. **(5)**

(c) State the actual significance level of the test in part (b). **(1)**

(d) Nadia then reconsiders, and decides she should instead test whether the proportion has simply changed (in either direction), rather than only increased. Explain how this changes the hypotheses, and find the new critical region for a two-tailed test at the 10% significance level (5% in each tail), such that each tail's probability is as close as possible to 0.05. You are given that, under H0, $P(X \leq 7) = 0.0243$ and $P(X \leq 8) = 0.0617$. **(3)**

(e) A survey afterwards finds that 16 of the 22 students sampled use the gym at least once a week. Using your critical region from part (d), state the conclusion of the two-tailed test, and briefly explain, in context, one advantage of using a two-tailed test rather than a one-tailed test in this situation. **(2)**

(Total for Question 7 is 13 marks)

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Mark scheme · 2.9 Statistics: Hypothesis Testing Depth: Test Pack

Question 1

(a) H0: p = 0.3, where p is the population proportion of bulbs producing double blooms **B1**

(a) H1: p ≠ 0.3 (oe) **B1**

(a) Answer: H0: p = 0.3, H1: p ≠ 0.3

(b) two-tailed, since H1 tests for a difference in either direction (p ≠ 0.3) rather than a specific increase or decrease **B1**

(b) Answer: Two-tailed test.

Question 2

(a) attempt cumulative probabilities P(X ≤ c) for X ~ B(20, 0.65) for small c **M1**

(a) P(X ≤ 8) = 0.0196 (awrt 3sf) **A1**

(a) P(X ≤ 9) = 0.0532 (awrt 3sf) **A1**

(a) critical region X ≤ 9 (since |0.0532-0.05| = 0.0032 is smaller than |0.0196-0.05| = 0.0304) **A1**

(a) Answer: Critical region: X ≤ 9 (i.e. {0,1,...,9}); actual significance level 0.0532.

(b) 8 lies within the critical region {0,...,9}, so reject H0; evidence, at the 5% level, that the germination rate has decreased **B1ft**

(b) Answer: 8 is in the critical region, so reject H0: evidence the germination rate has decreased.

Question 3

(a) H0: μ = 42, where μ is the population mean completion time in minutes **B1**

(a) H1: μ < 42 (oe) **B1**

(a) Answer: H0: μ = 42, H1: μ < 42

(b) use of $z = (\text{xbar} - \mu)/(\sigma/\sqrt{n})$ **M1**

(b) z = -2.00 cso **A1**

(b) Answer: z = -2.00 (-2.003 to 3dp)

(c) compare -2.00 with -1.6449 **M1**

(c) -2.00 < -1.6449, so reject H0 **A1**

(c) conclusion in context: evidence, at the 5% level, that the mean completion time has decreased **A1ft**

(c) Answer: -2.00 < -1.6449, so reject H0: evidence the mean completion time has decreased.

Question 4

(a) H0: ρ = 0 **B1**

(a) H1: ρ > 0 **B1**

(a) Answer: H0: ρ = 0, H1: ρ > 0

(b) compares |r| = 0.788 with the critical value 0.6215 in the correct tail **M1**

(b) reject H0 **A1**

(b) there is evidence of positive correlation **B1**

(b) Answer: reject H0

Question 5

(a) H0: p = 0.3 **B1**

(a) H1: p > 0.3 **B1**

(a) Answer: H0: p = 0.3, H1: p > 0.3

(b) P(X ≥ 10) = 1 - P(X ≤ 9) **M1**

(b) 0.0480 **A1**

(b) Answer: 0.0480

(c) compares 0.0480 with 0.05 **M1**

(c) reject H0 **A1**

(c) there is sufficient evidence that the proportion has increased **B1**

(c) Answer: reject H0

Question 6

(a) H0: μ = 34, H1: μ < 34 **B1**

(a) standardise, $z = (\text{xbar} - \mu)/(\sigma/\sqrt{n})$ **M1**

(a) z = -2.33 (awrt, -2.333 to 3dp) **A1**

(a) critical value -2.3263 (one-tailed, 1%) correctly stated for comparison **B1**

(a) valid comparison (z < -2.3263, or equivalently a p-value less than 0.01) **M1**

(a) reject H0; evidence, at the 1% level, that the treatment reduces the mean time taken to fall asleep **A1ft**

(a) Answer: z = -2.33 (3sf); -2.33 < -2.3263, so reject H0: evidence the treatment reduces the mean time to fall asleep.

(b) refers to the probability of obtaining a sample mean as low as (or lower than) 30.5 minutes, if the treatment genuinely had no effect (i.e. under H0, μ = 34) **B1**

(b) this probability is less than 0.01 (1%), so a result this extreme is unlikely to have occurred by chance alone, giving evidence against H0 **B1**

(b) Answer: It means that, if the treatment had no effect, a sample mean as low as 30.5 minutes (or lower) would occur with probability less than 0.01, so the result is unlikely to be due to chance.

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Question 7

(a) H0: p = 0.55 **B1**

(a) H1: p > 0.55 (oe) **B1**

(a) Answer: H0: p = 0.55, H1: p > 0.55

(b) attempt upper-tail cumulative probabilities for X ~ B(22, 0.55) **M1**

(b) P(X ≥ 17) = 0.0271 (awrt 3sf) **A1**

(b) P(X ≥ 16) = 0.0705 (awrt 3sf) **A1**

(b) correct comparison identifying X ≥ 16 as closer to 0.05 (|0.0705-0.05| = 0.0205 < |0.0271-0.05| = 0.0229) **B1**

(b) critical region X ≥ 16 (accept {16,17,...,22}) **A1**

(b) Answer: Critical region: X ≥ 16.

(c) 0.0705 (7.05%, awrt) (ft) **B1**

(c) Answer: 0.0705 (7.05%)

(d) revised hypotheses H0: p = 0.55, H1: p ≠ 0.55 (two-tailed) **B1**

(d) lower critical region X ≤ 8, since P(X ≤ 8) = 0.0617 is closer to 0.05 than P(X ≤ 7) = 0.0243 (|0.0617-0.05| = 0.0117 < |0.0243-0.05| = 0.0257) **B1**

(d) overall critical region X ≤ 8 or X ≥ 16 (reusing the upper region found in part (b)) **B1**

(d) Answer: H1: p ≠ 0.55; critical region: X ≤ 8 or X ≥ 16.

(e) X = 16 lies in the critical region (X ≥ 16), so reject H0; evidence, at the 10% level, that the proportion of first-year students using the gym has changed **B1ft**

(e) valid advantage of the two-tailed test in context, e.g. it detects a change in either direction without assuming in advance that the campaign could only increase (rather than decrease) gym use, appropriate since Nadia was not certain of the direction of any change before collecting the data **B1**

(e) Answer: 16 is in the critical region, so reject H0: evidence the proportion has changed. Advantage: a two-tailed test does not require assuming the direction of change in advance.

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