2.9 Statistics: Hypothesis Testing Depth

EDEXCEL 9MA0 · Calculator allowed · about 230 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Hypothesis Testing Toolkit: Formulae and Standard Normal Critical Values

Original material written for Revision Library, styled on Edexcel A-level Mathematics (9MA0) Statistics content.

For a binomial test of H0: p = p0, use X ~ B(n, p0) and compute the relevant cumulative probability (a tail probability, or a p-value) directly, either from cumulative binomial tables or a calculator's binomial cdf function. For a normal test of H0: mu = mu0 with known sigma, standardise using z = (xbar - mu0)/(sigma/sqrt(n)) and compare z with a critical value, or convert to a critical region for xbar directly. For a correlation test of H0: rho = 0, compare the sample product moment correlation coefficient, r, with a critical value from a table of critical values for the sample size n (such a critical value will always be given where needed). Standard normal critical z-values used throughout this pack: one-tailed 10% (or two-tailed 20%) = 1.2816; one-tailed 5% (or two-tailed 10%, i.e. 5% in each tail) = 1.6449; one-tailed 1% (or two-tailed 2%) = 2.3263; two-tailed 5% (2.5% in each tail) = 1.9600; two-tailed 1% (0.5% in each tail) = 2.5758. When finding a binomial critical region, choose the region whose cumulative probability is as close as possible to the stated significance level, unless a question specifically requires the probability to remain below that level; the actual (achieved) significance level is almost never exactly equal to the nominal one, because the binomial distribution is discrete.

1 A cafe in Leeds states that 20% of its customers choose a vegan pastry. The cafe manager, Kwame, believes this proportion has increased since a new vegan menu was introduced. In a random sample of 15 customers, 6 chose a vegan pastry. Let X be the number of customers, out of 15, choosing a vegan pastry, with X ~ B(15, p) under H0.

(a) State suitable null and alternative hypotheses for Kwame's test. **(1)**

(b) Test, at the 5% significance level, whether there is evidence that the proportion of customers choosing a vegan pastry has increased. State your conclusion in context. **(4)**

(Total for Question 1 is 5 marks)

2 A machine at a bakery in Bristol is set to produce loaves such that 25% have a well-fired crust, according to specification. After maintenance work, the quality control manager, Tomasz, samples 18 loaves at random and finds that only 1 has a well-fired crust. Let X be the number of loaves, out of 18, with a well-fired crust, with $X \sim B(18, p)$ under H0.

(a) State suitable hypotheses for a two-tailed test of whether the proportion of well-fired loaves has changed. **(2)**

(b) Assuming $X \sim B(18, 0.25)$, find $P(X \leq 1)$. **(3)**

(c) Using a 10% significance level, test whether there is evidence that the proportion of well-fired loaves has changed. State your conclusion in context. **(2)**

(Total for Question 2 is 7 marks)

3 The proportion of a population with a certain characteristic was 0.4. A researcher believes that this proportion has increased. In a random sample of 25 people, 14 have the characteristic.

(a) State the hypotheses for a one-tailed test. **(2)**

(b) Assuming H0 is true, find the probability of obtaining a value of X at least as extreme as 14, where X ~ B(25, 0.4). **(2)**

(c) Test the researcher's belief at the 5% level of significance, stating your conclusion in context. **(3)**

(Total for Question 3 is 7 marks)

4 A student records 5 pairs of values for daily temperature and the number of visitors to a park. The product moment correlation coefficient is r = 0.661. The critical value of r for a one-tailed test with a 5% significance level and n = 5 is 0.8054.

(a) Write down suitable hypotheses to test for positive correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 4 is 5 marks)

5 The random variable X has distribution $N(\mu, 3^2)$. A random sample of 36 observations of X has mean 71.8. A test of H0: $\mu = 73$ against H1: $\mu < 73$ is carried out at the 1% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test. **(2)**

(d) Find the critical value of the sample mean for this test, to 4 significant figures. **(2)**

(Total for Question 5 is 8 marks)

6 The proportion of a population with a certain characteristic was 0.3. A researcher believes that this proportion has decreased. In a random sample of 15 people, 1 have the characteristic.

(a) State the hypotheses for a one-tailed test. **(2)**

(b) Assuming H0 is true, find the probability of obtaining a value of X at least as extreme as 1, where X ~ B(15, 0.3). **(2)**

(c) Test the researcher's belief at the 10% level of significance, stating your conclusion in context. **(3)**

(Total for Question 6 is 7 marks)

7 A student records 9 pairs of values for the length of time spent revising and the test score. The product moment correlation coefficient is r = -0.585. The critical value of r for a one-tailed test with a 5% significance level and n = 9 is 0.5822.

(a) Write down suitable hypotheses to test for negative correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 7 is 5 marks)

8 A manufacturer claims that 15% of its light bulbs last more than 1000 hours. A retailer believes the proportion is higher than claimed and tests a random sample of 40 bulbs. Let p be the true probability, and X the number in the sample, so under H0, $X \sim B(40, 0.15)$.

(a) State suitable hypotheses for the test, in terms of p. **(2)**

(b) Using a 1% significance level, find the critical region for the test. **(3)**

(c) Write down the actual significance level of the test, and state the probability of a Type I error. **(2)**

(Total for Question 8 is 7 marks)

9 A school claims that the probability a student passes a driving test at the first attempt is 0.15. A parent believes the probability is lower than claimed. A random sample of 20 students is checked. Let p be the true probability, and X the number in the sample, so under H0, $X \sim B(20, 0.15)$.

(a) State suitable hypotheses for the test, in terms of p. **(2)**

(b) Using a 10% significance level, find the critical region for the test. **(3)**

(c) Write down the actual significance level of the test, and state the probability of a Type I error. **(2)**

(Total for Question 9 is 7 marks)

10 A student records 9 pairs of values for the age of a car and its resale price. The product moment correlation coefficient is r = -0.567. The critical value of r for a one-tailed test with a 5% significance level and n = 9 is 0.5822.

(a) Write down suitable hypotheses to test for negative correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 10 is 5 marks)

11 The proportion of a population with a certain characteristic was 0.25. A researcher believes that this proportion has decreased. In a random sample of 15 people, 0 have the characteristic.

(a) State the hypotheses for a one-tailed test. **(2)**

(b) Assuming H0 is true, find the probability of obtaining a value of X at least as extreme as 0, where X ~ B(15, 0.25). **(2)**

(c) Test the researcher's belief at the 5% level of significance, stating your conclusion in context. **(3)**

(Total for Question 11 is 7 marks)

12 A student records 7 pairs of values for the age of a car and its resale price. The product moment correlation coefficient is r = 0.69. The critical value of r for a one-tailed test with a 5% significance level and n = 7 is 0.6694.

(a) Write down suitable hypotheses to test for positive correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 12 is 5 marks)

13 A student records 6 pairs of values for the age of a car and its resale price. The product moment correlation coefficient is r = 0.384. The critical value of r for a one-tailed test with a 5% significance level and n = 6 is 0.7293.

(a) Write down suitable hypotheses to test for positive correlation. **(2)**

(b) Carry out the test and state your conclusion in context. **(3)**

(Total for Question 13 is 5 marks)

14 The random variable X has distribution $N(\mu, 2^2)$. A random sample of 9 observations of X has mean 40.0. A test of H0: $\mu = 39$ against H1: $\mu > 39$ is carried out at the 5% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test, in context. **(2)**

(d) Find the critical value of the sample mean for this test, to 4 significant figures. **(2)**

(Total for Question 14 is 8 marks)

15 The proportion of a population with a certain characteristic was 0.25. A researcher believes that this proportion has decreased. In a random sample of 15 people, 3 have the characteristic.

(a) State the hypotheses for a one-tailed test. **(2)**

(b) Assuming H0 is true, find the probability of obtaining a value of X at least as extreme as 3, where X ~ B(15, 0.25). **(2)**

(c) Test the researcher's belief at the 10% level of significance, stating your conclusion in context. **(3)**

(Total for Question 15 is 7 marks)

16 The random variable X has distribution $N(\mu, 4^2)$. A random sample of 9 observations of X has mean 64.93. A test of H0: $\mu = 62$ against H1: $\mu > 62$ is carried out at the 1% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test, in context. **(2)**

(d) Find the critical value of the sample mean for this test, to 4 significant figures. **(2)**

(Total for Question 16 is 8 marks)

17 The random variable X has distribution $N(\mu, 5^2)$. A random sample of 36 observations of X has mean 61.83. A test of H0: $\mu = 63$ against H1: $\mu < 63$ is carried out at the 5% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test, in context. **(2)**

(d) Find the critical value of the sample mean for this test, to 4 significant figures. **(2)**

(Total for Question 17 is 8 marks)

18 The proportion of a population with a certain characteristic was 0.1. A researcher believes that this proportion has increased. In a random sample of 15 people, 4 have the characteristic.

(a) State the hypotheses for a one-tailed test. **(2)**

(b) Assuming H0 is true, find the probability of obtaining a value of X at least as extreme as 4, where X ~ B(15, 0.1). **(2)**

(c) Test the researcher's belief at the 5% level of significance, stating your conclusion in context. **(3)**

(Total for Question 18 is 7 marks)

19 The random variable X has distribution $N(\mu, 3^2)$. A random sample of 49 observations of X has mean 46.97. A test of H0: $\mu = 48$ against H1: $\mu < 48$ is carried out at the 5% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test, in context. **(2)**

(d) Find the critical value of the sample mean for this test, to 4 significant figures. **(2)**

(Total for Question 19 is 8 marks)

20 The random variable X has distribution $N(\mu, 10^2)$. A random sample of 9 observations of X has mean 54.0. A test of H0: $\mu = 50$ against H1: $\mu > 50$ is carried out at the 5% significance level.

(a) State the distribution of the sample mean under H0. **(2)**

(b) Calculate the p-value for the test, to 3 significant figures. **(2)**

(c) State the conclusion of the test, in context. **(2)**

(d) Find the critical value of the sample mean for this test, to 4 significant figures. **(2)**

(Total for Question 20 is 8 marks)

21 A manufacturer claims that 15% of its light bulbs last more than 1000 hours. A retailer believes the proportion is higher than claimed and tests a random sample of 12 bulbs. Let p be the true probability, and X the number in the sample, so under H0, X ~ B(12, 0.15).

(a) State suitable hypotheses for the test, in terms of p. **(2)**

(b) Using a 10% significance level, find the critical region for the test. **(3)**

(c) Write down the actual significance level of the test, and state the probability of a Type I error. **(2)**

(Total for Question 21 is 7 marks)

22 A call centre in Cardiff aims for 40% of calls to be resolved on first contact. The team leader, Fatima, wants to test whether this proportion has changed following a new staff training programme. She takes a random sample of 25 calls. Let X be the number of calls, out of 25, resolved on first contact, with $X \sim B(25, p)$ under H0: $p = 0.4$.

(a) Find the critical region for a two-tailed test at the 10% significance level, such that the probability in each tail is as close as possible to 0.05. **(6)**

(b) Given that 5 of the 25 sampled calls were resolved on first contact, state the conclusion of the test. **(1)**

(Total for Question 22 is 7 marks)

23 The mass of bags of flour produced at a factory in Preston is modelled as X ~ N(1000, 64) grams (mean 1000 g, standard deviation 8 g), as stated on the packaging. Following a change of flour supplier, quality inspector Olusegun wants to test whether the mean mass has changed, using a random sample of 25 bags.

(a) State suitable hypotheses, and find, in terms of xbar (the sample mean mass), the critical region for a two-tailed test at the 1% significance level. **(5)**

(b) Given that the mean mass of the sample of 25 bags is found to be 1003.2 g, comment on whether there is evidence that the mean mass has changed. **(2)**

(Total for Question 23 is 7 marks)

24 A researcher, Chen, investigates whether there is a linear relationship between the number of hours of daylight (x) and the number of visitors (y) to a National Trust garden in Kent, using data from a random sample of 12 days spread throughout the year. The product moment correlation coefficient for the sample is calculated as r = 0.62.

(a) State suitable hypotheses to test whether there is positive linear correlation between hours of daylight and number of visitors. **(2)**

(b) Given that the critical value for a one-tailed test at the 5% significance level, for a sample of size 12, is 0.4973, determine whether there is significant evidence of positive correlation. **(3)**

(c) Chen argues that the value of r shows that longer daylight hours directly cause more visitors. Comment on this claim. **(2)**

(Total for Question 24 is 7 marks)

25 A dental practice in Swansea states that 85% of patients attend their booked check-up appointments. The practice manager, Aisha, runs a reminder-text trial and wants to test, using a random sample of 30 patients, whether the attendance rate has increased. Let X be the number of patients, out of 30, who attend, with $X \sim B(30, p)$ under H0.

(a) Write down suitable null and alternative hypotheses. **(2)**

(b) Using a 5% significance level, find the critical region for the test, such that the probability of incorrectly rejecting H0 is as close as possible to 5% while remaining less than 5%. **(5)**

(c) State the actual significance level of the test in part (b). **(1)**

(d) After the trial, 29 of the 30 sampled patients attended their appointment. State the conclusion of the test. **(1)**

(Total for Question 25 is 9 marks)

26 An orchard in Herefordshire has historically produced apples with weights normally distributed with mean 180 g and standard deviation 22 g. After introducing a new irrigation system, the orchard manager, Liam, samples 50 apples at random and records a sample mean weight of 186.4 g. Assume the standard deviation is unchanged.

(a) State suitable hypotheses to test whether the irrigation system has changed the mean weight of the apples. **(2)**

(b) Show that the test statistic is $z = 2.06$ (awrt), and hence carry out the test at the 5% significance level, stating your conclusion clearly. **(5)**

(c) Liam decides to repeat the test using a 1% significance level instead. State, with a reason, whether his conclusion would change. **(2)**

(d) Explain what is meant by a Type I error in this context, and state the probability of a Type I error for the test in part (c). **(2)**

(Total for Question 26 is 11 marks)

Mark scheme · 2.9 Statistics: Hypothesis Testing Depth

Question 1

(a) H0: p = 0.2, H1: p > 0.2, both stated correctly **B1**

(a) Answer: H0: p = 0.2, H1: p > 0.2

(b) attempt P(X ≥ 6) using X ~ B(15, 0.2), e.g. via 1 - P(X ≤ 5) **M1**

(b) P(X ≥ 6) = 0.0611 (awrt 3sf) **A1**

(b) compare 0.0611 with 0.05 **M1**

(b) 0.0611 > 0.05, so insufficient evidence, at the 5% level, that the proportion of customers choosing a vegan pastry has increased (do not reject H0) **A1ft**

(b) Answer: P(X ≥ 6) = 0.0611 (3sf); 0.0611 > 0.05, so insufficient evidence at the 5% level that the proportion has increased.

Question 2

(a) H0: p = 0.25, where p is the population proportion of well-fired loaves **B1**

(a) H1: p ≠ 0.25 (oe) **B1**

(a) Answer: H0: p = 0.25, H1: p ≠ 0.25

(b) attempt P(X=0) + P(X=1) using X ~ B(18, 0.25) **M1**

(b) P(X = 0) = 0.75^{18} = 0.00564 (awrt) **A1**

(b) P(X ≤ 1) = 0.0395 (awrt 3sf) **A1**

(b) Answer: P(X ≤ 1) = 0.0395 (3sf)

(c) compare 0.0395 with 0.05 (half of the 10% two-tailed significance level) **M1**

(c) 0.0395 < 0.05, so reject H0; evidence, at the 10% level, that the proportion of well-fired loaves has changed **A1ft**

(c) Answer: 0.0395 < 0.05, so reject H0: evidence the proportion of well-fired loaves has changed.

Question 3

(a) H0: p = 0.4 **B1**

(a) H1: p > 0.4 **B1**

(a) Answer: H0: p = 0.4, H1: p > 0.4

(b) P(X ≥ 14) = 1 - P(X ≤ 13) **M1**

(b) 0.0778 **A1**

(b) Answer: 0.0778

(c) compares 0.0778 with 0.05 **M1**

(c) do not reject H0 **A1**

(c) there is insufficient evidence that the proportion has increased **B1**

(c) Answer: do not reject H0

Question 4

(a) H0: ρ = 0 **B1**

(a) H1: ρ > 0 **B1**

(a) Answer: H0: ρ = 0, H1: ρ > 0

(b) compares |r| = 0.661 with the critical value 0.8054 in the correct tail **M1**

(b) do not reject H0 **A1**

(b) there is insufficient evidence of positive correlation **B1**

(b) Answer: do not reject H0

Question 5

(a) X-bar ~ N(73, 3^2/36) **B1**

(a) standard error $3/\sqrt{36}$ = 0.5000 **B1**

(a) Answer: N(73, 3^2/36)

(b) z = (71.8 - 73)/0.5000 = -2.400 **M1**

(b) p-value = 0.00820 **A1**

(b) Answer: 0.00820

(c) compares p-value 0.00820 with 0.01 **M1**

(c) reject H0: there is sufficient evidence that the mean has decreased **A1**

(c) Answer: reject H0

(d) z critical = -2.3263 **M1**

(d) critical value = 71.84 **A1**

(d) Answer: 71.84

Question 6

(a) H0: p = 0.3 **B1**

(a) H1: p < 0.3 **B1**

(a) Answer: H0: p = 0.3, H1: p < 0.3

(b) P(X ≤ 1) **M1**

(b) 0.0353 **A1**

(b) Answer: 0.0353

(c) compares 0.0353 with 0.1 **M1**

(c) reject H0 **A1**

(c) there is sufficient evidence that the proportion has decreased **B1**

(c) Answer: reject H0

Question 7

(a) H0: ρ = 0 **B1**

(a) H1: ρ < 0 **B1**

(a) Answer: H0: ρ = 0, H1: ρ < 0

(b) compares |r| = 0.585 with the critical value 0.5822 in the correct tail **M1**

(b) reject H0 **A1**

(b) there is evidence of negative correlation **B1**

(b) Answer: reject H0

Question 8

(a) H0: p = 0.15 **B1**

(a) H1: p > 0.15 **B1**

(a) Answer: H0: p = 0.15, H1: p > 0.15

(b) uses the binomial distribution B(40, 0.15) to find cumulative probabilities near the tail **M1**

(b) P(X ≥ 13) = 0.004313 ≤ 0.01 **A1**

(b) critical region: X ≥ 13 **A1**

(b) Answer: critical region X ≥ 13

(c) actual significance level = 0.00431 **B1**

(c) this is the probability of a Type I error (rejecting H0 when it is true) **B1**

(c) Answer: 0.00431

Question 9

(a) H0: $p = 0.15$ **B1**

(a) H1: $p < 0.15$ **B1**

(a) Answer: H0: $p = 0.15$, H1: $p < 0.15$

(b) uses the binomial distribution B(20, 0.15) to find cumulative probabilities near the tail **M1**

(b) $P(X \leq 0) = 0.03876 \leq 0.1$ **A1**

(b) critical region: $X \leq 0$ **A1**

(b) Answer: critical region $X \leq 0$

(c) actual significance level = 0.0388 **B1**

(c) this is the probability of a Type I error (rejecting H0 when it is true) **B1**

(c) Answer: 0.0388

Question 10

(a) H0: $\rho = 0$ **B1**

(a) H1: $\rho < 0$ **B1**

(a) Answer: H0: $\rho = 0$, H1: $\rho < 0$

(b) compares $|r| = 0.567$ with the critical value 0.5822 in the correct tail **M1**

(b) do not reject H0 **A1**

(b) there is insufficient evidence of negative correlation **B1**

(b) Answer: do not reject H0

Question 11

(a) H0: $p = 0.25$ **B1**

(a) H1: $p < 0.25$ **B1**

(a) Answer: H0: $p = 0.25$, H1: $p < 0.25$

(b) $P(X \leq 0)$ **M1**

(b) 0.0134 **A1**

(b) Answer: 0.0134

(c) compares 0.0134 with 0.05 **M1**

(c) reject H0 **A1**

(c) there is sufficient evidence that the proportion has decreased **B1**

(c) Answer: reject H0

Question 12

(a) H0: $\rho = 0$ **B1**

(a) H1: $\rho > 0$ **B1**

(a) Answer: H0: $\rho = 0$, H1: $\rho > 0$

(b) compares $|r| = 0.69$ with the critical value 0.6694 in the correct tail **M1**

(b) reject H0 **A1**

(b) there is evidence of positive correlation **B1**

(b) Answer: reject H0

Question 13

(a) H0: $\rho = 0$ **B1**

(a) H1: $\rho > 0$ **B1**

(a) Answer: H0: $\rho = 0$, H1: $\rho > 0$

(b) compares $|r| = 0.384$ with the critical value 0.7293 in the correct tail **M1**

(b) do not reject H0 **A1**

(b) there is insufficient evidence of positive correlation **B1**

(b) Answer: do not reject H0

Question 14

(a) X-bar ~ $N(39, 2^2/9)$ **B1**

(a) standard error $2/\sqrt{9} = 0.6667$ **B1**

(a) Answer: $N(39, 2^2/9)$

(b) $z = (40.0 - 39)/0.6667 = 1.500$ **M1**

(b) p-value = 0.0668 **A1**

(b) Answer: 0.0668

(c) compares p-value 0.0668 with 0.05 **M1**

(c) do not reject H0: insufficient evidence that the mean has increased **A1**

(c) Answer: do not reject H0

(d) z critical = 1.6449 **M1**

(d) critical value = 40.10 **A1**

(d) Answer: 40.10

Question 15

(a) H0: $p = 0.25$ **B1**

(a) H1: $p < 0.25$ **B1**

(a) Answer: H0: $p = 0.25$, H1: $p < 0.25$

(b) $P(X \leq 3)$ **M1**

(b) 0.461 **A1**

(b) Answer: 0.461

(c) compares 0.461 with 0.1 **M1**

(c) do not reject H0 **A1**

(c) there is insufficient evidence that the proportion has decreased **B1**

(c) Answer: do not reject H0

Question 16

(a) X-bar ~ $N(62, 4^2/9)$ **B1**

(a) standard error $4/\sqrt{9} = 1.333$ **B1**

(a) Answer: $N(62, 4^2/9)$

(b) $z = (64.93 - 62)/1.333 = 2.198$ **M1**

(b) p-value = 0.0140 **A1**

(b) Answer: 0.0140

(c) compares p-value 0.0140 with 0.01 **M1**

(c) do not reject H0: insufficient evidence that the mean has increased **A1**

(c) Answer: do not reject H0

(d) z critical = 2.3263 **M1**

(d) critical value = 65.10 **A1**

(d) Answer: 65.10

Question 17

(a) X-bar ~ N(63, 5^2/36) **B1**

(a) standard error $5/\sqrt{36}$ = 0.8333 **B1**

(a) Answer: N(63, 5^2/36)

(b) z = (61.83 - 63)/0.8333 = -1.404 **M1**

(b) p-value = 0.0802 **A1**

(b) Answer: 0.0802

(c) compares p-value 0.0802 with 0.05 **M1**

(c) do not reject H0: insufficient evidence that the mean has decreased **A1**

(c) Answer: do not reject H0

(d) z critical = -1.6449 **M1**

(d) critical value = 61.63 **A1**

(d) Answer: 61.63

Question 18

(a) H0: p = 0.1 **B1**

(a) H1: p > 0.1 **B1**

(a) Answer: H0: p = 0.1, H1: p > 0.1

(b) P(X ≥ 4) = 1 - P(X ≤ 3) **M1**

(b) 0.0556 **A1**

(b) Answer: 0.0556

(c) compares 0.0556 with 0.05 **M1**

(c) do not reject H0 **A1**

(c) there is insufficient evidence that the proportion has increased **B1**

(c) Answer: do not reject H0

Question 19

(a) X-bar ~ N(48, 3^2/49) **B1**

(a) standard error $3/\sqrt{49}$ = 0.4286 **B1**

(a) Answer: N(48, 3^2/49)

(b) z = (46.97 - 48)/0.4286 = -2.403 **M1**

(b) p-value = 0.00812 **A1**

(b) Answer: 0.00812

(c) compares p-value 0.00812 with 0.05 **M1**

(c) reject H0: there is sufficient evidence that the mean has decreased **A1**

(c) Answer: reject H0

(d) z critical = -1.6449 **M1**

(d) critical value = 47.30 **A1**

(d) Answer: 47.30

Question 20

(a) X-bar ~ N(50, 10^2/9) **B1**

(a) standard error $10/\sqrt{9}$ = 3.333 **B1**

(a) Answer: N(50, 10^2/9)

(b) z = (54.0 - 50)/3.333 = 1.200 **M1**

(b) p-value = 0.115 **A1**

(b) Answer: 0.115

(c) compares p-value 0.115 with 0.05 **M1**

(c) do not reject H0: insufficient evidence that the mean has increased **A1**

(c) Answer: do not reject H0

(d) z critical = 1.6449 **M1**

(d) critical value = 55.48 **A1**

(d) Answer: 55.48

Question 21

(a) H0: p = 0.15 **B1**

(a) H1: p > 0.15 **B1**

(a) Answer: H0: p = 0.15, H1: p > 0.15

(b) uses the binomial distribution B(12, 0.15) to find cumulative probabilities near the tail **M1**

(b) P(X ≥ 4) = 0.09221 ≤ 0.1 **A1**

(b) critical region: X ≥ 4 **A1**

(b) Answer: critical region X ≥ 4

(c) actual significance level = 0.0922 **B1**

(c) this is the probability of a Type I error (rejecting H0 when it is true) **B1**

(c) Answer: 0.0922

Question 22

(a) attempt cumulative probabilities for X ~ B(25, 0.4) in the lower tail **M1**

(a) P(X ≤ 5) = 0.0294 and P(X ≤ 6) = 0.0736 (awrt 3sf) **A1**

(a) lower critical region X ≤ 5 (0.0294 is closer to 0.05 than 0.0736: |0.05-0.0294| = 0.0206 < |0.0736-0.05| = 0.0236) **B1**

(a) attempt cumulative probabilities for X ~ B(25, 0.4) in the upper tail **M1**

(a) P(X ≥ 15) = 0.0344 and P(X ≥ 14) = 0.0778 (awrt 3sf) **A1**

(a) upper critical region X ≥ 15 (0.0344 is closer to 0.05 than 0.0778: |0.05-0.0344| = 0.0156 < |0.0778-0.05| = 0.0278) **B1**

(a) Answer: Critical region: X ≤ 5 or X ≥ 15; combined actual significance level 0.0294 + 0.0344 = 0.0638.

(b) X = 5 lies in the critical region (X ≤ 5), so reject H0; evidence, at the 10% level, that the proportion of calls resolved on first contact has changed **B1ft**

(b) Answer: X = 5 is in the critical region, so reject H0: evidence the proportion has changed.

Question 23

(a) H0: $\mu = 1000$, H1: $\mu \neq 1000$ **B1**

(a) identify the two-tailed 1% critical value $z = 2.5758$ **M1**

(a) use $\text{xbar} = \mu \pm z^*\sigma/\sqrt{n}$ **M1**

(a) lower boundary = 995.9 (awrt) **A1**

(a) upper boundary = 1004.1 (awrt) **A1**

(a) Answer: Critical region: xbar < 995.9 or xbar > 1004.1 (g).

(b) 1003.2 does not lie in the critical region, since 1003.2 < 1004.1 **B1**

(b) insufficient evidence, at the 1% level, that the mean mass of the bags has changed (do not reject H0) **B1ft**

(b) Answer: 1003.2 is not in the critical region, so insufficient evidence the mean mass has changed.

Question 24

(a) H0: $\rho = 0$, where ρ is the population product moment correlation coefficient **B1**

(a) H1: $\rho > 0$ (oe) **B1**

(a) Answer: H0: $\rho = 0$, H1: $\rho > 0$

(b) compares $r = 0.62$ with the critical value 0.4973 **B1**

(b) 0.62 > 0.4973, so reject H0 **B1**

(b) conclusion in context: significant evidence, at the 5% level, of positive correlation between hours of daylight and the number of visitors **B1**

(b) Answer: 0.62 > 0.4973, so reject H0: significant evidence of positive correlation.

(c) correlation does not imply causation **B1**

(c) valid alternative explanation, e.g. a confounding/lurking variable such as warmer weather or school holidays could independently affect both daylight hours (via season) and visitor numbers, so a direct causal link cannot be concluded from correlation alone **B1**

(c) Answer: Correlation does not imply causation; a confounding variable (e.g. season or weather) may explain the association without daylight hours directly causing higher visitor numbers.

Question 25

(a) H0: $p = 0.85$, where p is the population proportion of patients who attend **B1**

(a) H1: $p > 0.85$ (oe) **B1**

(a) Answer: H0: $p = 0.85$, H1: $p > 0.85$

(b) attempt to find upper-tail cumulative probabilities for $X \sim B(30, 0.85)$, e.g. via $Y = 30 - X \sim B(30, 0.15)$ **M1**

(b) $P(X \geq 28) = 0.151$ (awrt 3sf) - too large, so rejected **A1**

(b) $P(X \geq 29) = 0.0480$ (awrt 3sf) - valid, since less than 5% **A1**

(b) correctly identifies $P(X \geq 29) = 0.0480$ as the required boundary, since $P(X \geq 28)$ exceeds 5% **A1**

(b) critical region $X \geq 29$ (accept {29, 30}) **A1**

(b) Answer: Critical region: $X \geq 29$ (i.e. {29, 30}).

(c) 0.0480 (4.80%, awrt) (ft) **B1**

(c) Answer: 0.0480 (4.80%)

(d) 29 lies in the critical region ($X \geq 29$), so reject H0; evidence, at the 5% level, that the attendance rate has increased **B1ft**

(d) Answer: 29 is in the critical region, so reject H0: evidence the attendance rate has increased.

Question 26

(a) H0: $\mu = 180$, where μ is the population mean weight in grams **B1**

(a) H1: $\mu \neq 180$ (oe) **B1**

(a) Answer: H0: $\mu = 180$, H1: $\mu \neq 180$

(b) standardise, $z = (\text{xbar} - \mu)/(\sigma/\sqrt{n})$ **M1**

(b) $z = 2.06$ (2.057 to 3dp) cso **A1**

(b) critical value ± 1.96 (two-tailed, 5%) stated for comparison **B1**

(b) valid comparison, $|z| > 1.96$ (2.057 > 1.96) **M1**

(b) reject H0; evidence, at the 5% level, that the mean weight of the apples has changed **A1ft**

(b) Answer: z = 2.06 (2.057, 3dp); 2.057 > 1.96, so reject H0: evidence the mean weight has changed.

(c) new critical value ± 2.5758 (two-tailed, 1%) identified, and $2.057 < 2.5758$ **B1**

(c) conclusion changes: insufficient evidence, at the 1% level, that the mean weight of the apples has changed (do not reject H0) **B1**

(c) Answer: 2.057 < 2.5758, so at the 1% level there is insufficient evidence of a change (the conclusion changes).

(d) a Type I error is rejecting H0 (concluding the mean weight has changed) when H0 is actually true (the mean weight of the apples is still 180 g) **B1**

(d) P(Type I error) = 0.01 **B1**

(d) Answer: A Type I error is rejecting H0 when it is true; P(Type I error) = 0.01 for the 1% level test.