# 7.3 Spearman's Rho: Rationale, Application and Critical Values

Total Marks

AQA 7182 · about 50 minutes · Write-in edition

Name: _____ Date: ____ / ____ / _____

**Answer ALL questions.**

**1** Explain briefly how sample size affects the critical value required for significance when using Spearman's ρ and give one implication for researchers working with small samples.

**(Total for Question 1 is 2 marks)**

**2** Interpretation practice. For each Spearman's ρ value below, state briefly whether the association is positive, negative or zero, and whether it is weak, moderate or strong in magnitude. Use conventional verbal descriptors.

**(a)** ρ = 0.12 **(1)**

**(b)** ρ = -0.68 **(1)**

**(c)** ρ = 0.00 **(1)**

**(Total for Question 2 is 3 marks)**

**3** Report-writing practice. A psychologist calculates Spearman's ρ for n = 7 pairs and obtains ρ = 0.991. Write a concise sentence that reports this result for inclusion in a methods and results section, giving the test name, ρ value to three decimal places, sample size and whether the result is significant at $p < 0.05$ two-tailed using the invented table from Question 5.

**(Total for Question 3 is 3 marks)**

**4** Give one reason why researchers might prefer Spearman's ρ to Pearson's r when analysing a pair of co-variables from a small sample of ordinal data.

**(Total for Question 4 is 1 mark)**

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**5** The table below gives critical values for Spearman's ρ at the 5% significance level for an invented critical-values table. Use this table when answering Questions 2 and 8.

Critical values for Spearman's ρ at $p < 0.05$ (invented table)

| n | one-tailed critical ρ | two-tailed critical ρ |
|---|---|---|
| 5 | 0.900 | 0.950 |
| 6 | 0.829 | 0.886 |
| 7 | 0.643 | 0.786 |
| 8 | 0.571 | 0.738 |
| 9 | 0.500 | 0.700 |

State clearly which row of the table you would use for a dataset with n = 7 pairs, and give both critical values from that row.

**(Total for Question 5 is 3 marks)**

**6** Extended question: Evaluate the use of Spearman's ρ for analysing relationships in psychological research with ordinal or ranked data. In your answer, consider the rationale for the test, strengths and limitations including handling of ties and small samples, and practical implications for validity and application. Refer to real features of Spearman's ρ and give developed evaluation points.

**(Total for Question 6 is 16 marks)**

**7** Write down the Spearman's ρ formula used when there are no tied ranks, naming every symbol used.

**(Total for Question 7 is 3 marks)**

**8** Worked example. A researcher collects the following paired data from seven students on hours revised in the week before a test (X) and the test score out of 100 (Y). Convert the raw scores to ranks, handle tied X values, and use Spearman's ρ formula to calculate ρ. Finally, compare your ρ to the critical values for n = 7 from the table in Question 5 and state whether the correlation is significant at $p < 0.05$ for both a one-tailed and a two-tailed test.

Data (students in the order given):
Student 1: X = 2 hours, Y = 55
Student 2: X = 4 hours, Y = 68
Student 3: X = 4 hours, Y = 70
Student 4: X = 6 hours, Y = 85
Student 5: X = 1 hour, Y = 40
Student 6: X = 3 hours, Y = 60
Student 7: X = 5 hours, Y = 78

Show your ranked table, the d and $d^2$ column, the substitution into the formula, your working and the final decision about significance.

**(a)** Convert the X and Y raw scores to ranks, showing how you handled the tied X values, and give the ranks for each student in the order above. **(3)**

**(b)** Calculate d (rankX - rankY) and $d^2$ for each pair and give the sum of $d^2$. **(1)**

**(c)** Substitute your sum $d^2$ and n into the Spearman's ρ formula and show the arithmetic leading to ρ to three decimal places. **(3)**

**(d)** Using the critical values for n = 7 from Question 5, state whether the correlation is significant at $p < 0.05$ for a one-tailed test and for a two-tailed test, and justify each decision. **(2)**

**(Total for Question 8 is 9 marks)**

# Mark scheme · 7.3 Spearman's Rho: Rationale, Application and Critical Values

## Question 1

explains that smaller sample sizes require larger absolute ρ values to reach significance, because random variation has a bigger effect in small samples oe — **M1**

gives implication: researchers with small samples need very strong correlations to be significant or should use caution in generalising results or increase sample size oe — **A1**

**Answer: Smaller sample sizes demand larger absolute ρ values for significance, since chance variation is proportionally larger; therefore researchers with small samples need very strong correlations to achieve significance and should be cautious about generalising or try to increase sample size.**

## Question 2

(a) positive, weak association oe — **B1**

**(a) Answer: Positive, weak association.**

(b) negative, strong/moderately strong association oe — **B1**

**(b) Answer: Negative, moderately strong to strong association.**

(c) no association (zero correlation) oe — **B1**

**(c) Answer: No association, zero correlation.**

## Question 3

names the test, e.g. Spearman's ρ, and gives ρ = 0.991 to three decimal places cao — **M1**

gives the sample size n = 7 cao — **A1**

states the result is significant at $p < 0.05$ (two-tailed) and may reference the critical-value table oe — **A1**

**Answer: Spearman's ρ indicated a very strong positive association, ρ = 0.991, n = 7, which is significant at $p < 0.05$ (two-tailed) using the critical values table.**

## Question 4

states suitability for ordinal or ranked data or for non-parametric data where Pearson's assumptions (interval level and normality) are not met — **B1**

**Answer: Because Spearman's ρ is suitable for ordinal or ranked data and does not require interval-level measurement or normal distribution assumptions.**

## Question 5

identifies the row for n = 7 — **B1**

gives the one-tailed critical ρ for n = 7 as 0.643 — **B1**

gives the two-tailed critical ρ for n = 7 as 0.786 — **B1**

**Answer: Use the row n = 7. One-tailed critical ρ = 0.643. Two-tailed critical ρ = 0.786.**

## Question 6

**Level 4** Knowledge of Spearman's rho and its rationale is accurate and detailed. Evaluation is thorough and balanced, covering several well-developed strengths and limitations, including handling of tied ranks, sample size effects and meaning of significance, with clear implications for validity and application. Responses integrate evidence and reasoned argument, consider alternatives or adjustments, and are well organised with appropriate technical language. — **13-16**

**Level 3** Knowledge is accurate with reasonable detail. Evaluation includes several relevant strengths and limitations, but development may be less thorough or slightly unbalanced. Some consideration of ties, sample size or interpretation of significance is present. Overall clear and mostly well organised. — **9-12**

**Level 2** Knowledge is present but limited. Evaluation tends to be descriptive or brief, with only one or two underdeveloped points about strengths or weaknesses. Little consideration of ties or sample-size effects. Organisation may be patchy and technical language used inconsistently. — **5-8**

**Level 1** Very limited knowledge and evaluation. Responses are simplistic, list-like, or largely inaccurate. Little or no application to practical implications. Very poor organisation and minimal use of technical terms. — **1-4**

**Level 0** No creditable content. — **0**

**Indicative content:**

- AO1: Rationale: Spearman's rho assesses the strength and direction of a monotonic association between two variables when data are ordinal or can be meaningfully ranked; it uses ranked differences and a simple formula based on sum of squared differences.
- AO1: Procedure: raw scores converted to ranks, ties assigned average ranks, compute differences d between paired ranks, sum d^2 and substitute in rho formula, compare to critical values for given n and tail.
- AO3 Strength: appropriate for ordinal data and non-parametric situations where Pearson's assumptions (interval level, linearity, normality) are violated, increasing the validity of inferences when data are ranks.
- AO3 Strength: simple to calculate by hand for small samples and interpretable; ranking reduces effect of outliers compared with raw-score correlations, providing robustness.
- AO3 Limitation: tied ranks complicate calculation and reduce the maximum attainable magnitude of rho; many ties can reduce sensitivity and may require tie-correction formulas or exact permutation tests.
- AO3 Limitation: small sample sizes require very large rho to reach significance and critical-value tables are coarse, so statistical power is low and Type II error risk is high; importance of reporting effect size and confidence remains.
- AO3 Limitation: Spearman's rho tests monotonic relationships, not necessarily linear ones, so interpretation must state monotonic association rather than causation; correlational design still cannot establish causality and is vulnerable to confounding variables.
- AO3 Practical implications: researchers should choose Spearman's rho when measurement level or distribution justifies it, report how ties were handled and whether exact or approximate p values were used, and consider increasing sample size or using permutation tests for small samples.
- AO3 Alternative approaches: when many tied ranks or larger samples with interval data, consider Kendall's tau or Pearson's r with appropriate transformations; discuss trade-offs in power and interpretability.
- AO3 Ethical and reporting considerations: be transparent about tail selection, pre-registered directional hypotheses if using one-tailed tests, and avoid overstating significance when sample size is small.

## Question 7

states formula: $\rho = 1 - (6 * \text{sum } d^2) / (n * (n^2 - 1))$ or equivalent **M1**

defines d as the difference between paired ranks and n as the number of pairs **A1**

notes that sum $d^2$ means the sum of squared rank differences across all pairs **A1**

**Answer: $\rho = 1 - (6 * \text{sum } d^2) / (n * (n^2 - 1))$, where d = difference between ranks for each pair, n = number of pairs, and sum $d^2$ is the total of squared differences.**

## Question 8

(a) correct X ranks across the seven students: 2, 4.5, 4.5, 7, 1, 3, 6 (in that order) oe **B1**

(a) correct Y ranks across the seven students: 2, 4, 5, 7, 1, 3, 6 (in that order) oe **B1**

(a) explicitly states tied X values were assigned the average rank 4.5 (the mean of ranks 4 and 5) oe **B1**

**(a) Answer: X ranks: 2, 4.5, 4.5, 7, 1, 3, 6. Y ranks: 2, 4, 5, 7, 1, 3, 6. Tied X values (the two 4-hour entries) each receive rank 4.5 as the average of ranks 4 and 5.**

(b) lists d values and $d^2$ correctly and gives sum $d^2 = 0.5$ oe **M1**

**(b) Answer: d and $d^2$ by student: (0,0), (0.5,0.25), (-0.5,0.25), (0,0), (0,0), (0,0), (0,0). Sum $d^2 = 0.5$.**

(c) substitutes correctly: $\rho = 1 - (6 * 0.5) / (7 * (7^2 - 1))$ oe **M1**

(c) shows correct intermediate arithmetic: $6 * 0.5 = 3$ and $7 * (49 - 1) = 7 * 48 = 336$ oe **M1**

(c) gives $\rho = 1 - (3 / 336) = 0.991$ (to three decimal places) cao **A1**

**(c) Answer: $\rho = 1 - (6 * 0.5) / (7 * (49 - 1)) = 1 - 3 / 336 = 1 - 0.00892857 = 0.991$ (3 dp).**

(d) states that $\rho = 0.991$ exceeds the one-tailed critical value 0.643, so it is significant one-tailed at $p < 0.05$ oe **B1**

(d) states that $\rho = 0.991$ also exceeds the two-tailed critical value 0.786, so it is significant two-tailed at $p < 0.05$ oe **B1**

**(d) Answer: $\rho = 0.991$ exceeds both critical values for n = 7 (one-tailed 0.643 and two-tailed 0.786), so the correlation is significant at $p < 0.05$ for both one-tailed and two-tailed tests.**