3.1 Measurements and their Errors: 2-hour Pack

AQA 7408 · Calculator allowed · about 120 minutes

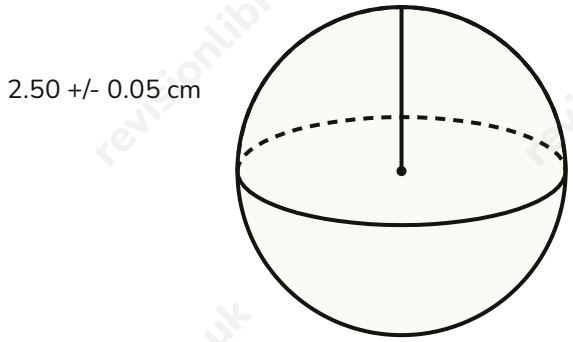
Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A small metal sphere has a measured radius r = 2.50 ± 0.05 cm.

Diagram NOT accurately drawn



(a) Calculate the percentage uncertainty in r. **(1)**

(b) The volume of the sphere is given by V = (4/3) x π x r^3. Calculate the percentage uncertainty in V. **(2)**

(c) Calculate the volume of the sphere and express your answer with its absolute uncertainty, to an appropriate number of significant figures. **(3)**

(Total for Question 1 is 6 marks)

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2 Uncertainties combine differently depending on whether quantities are added, subtracted, multiplied or divided.

(a) In a titration, a burette of resolution 0.10 cm^3 is used. The initial reading is 2.40 ± 0.05 cm^3 and the final reading is 27.65 ± 0.05 cm^3 (the uncertainty in each reading is taken as half the resolution). Calculate the titre (volume delivered) and its absolute uncertainty. **(3)**

(b) Calculate the percentage uncertainty in the titre found in part (a), giving your answer to 2 significant figures. **(2)**

(c) A thermometer of resolution 1°C is used to record the temperature of a water bath before and after heating. The initial temperature is 19.5 ± 0.5°C and the final temperature is 84.0 ± 0.5°C (the uncertainty in each reading is taken as half the resolution). Calculate the temperature rise and its absolute uncertainty. **(3)**

(Total for Question 2 is 8 marks)

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3 A student investigates the extension, x, of a spring for a range of applied forces, F, and plots a graph of F against x. A best-fit straight line is drawn through the origin, along with the steepest and least steep straight lines that still pass through all the error bars on the data points.

(a) The best-fit line has a gradient of 24.5 (in units of N per m of extension per unit, i.e. N/m). The steepest acceptable line has a gradient of 26.0 N/m, and the least steep acceptable line has a gradient of 23.0 N/m. Calculate the percentage uncertainty in the gradient. **(3)**

(b) State the gradient with its absolute uncertainty, to an appropriate number of significant figures. **(2)**

(c) Explain why error bars are included on the graph, and how they are used to find the uncertainty in the gradient. **(3)**

(d) The spring constant, k, is defined by F = k x x. Suggest two reasons why using the gradient of the graph gives a more reliable value of k than calculating k from a single pair of F and x readings. **(2)**

(Total for Question 3 is 10 marks)

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4 Two students each determine the Young modulus of the same copper wire using an applied load that produces an extension of about 2 mm. Student A uses a single wire clamped horizontally, measuring the extension directly with a metre rule of resolution 1 mm. Student B uses Searle's apparatus, with two parallel wires (one as a reference) and a vernier/micrometer scale of resolution 0.01 mm to measure the extension. Evaluate which student's method is more precise, using percentage uncertainty in the extension and any relevant features of the apparatus.

(Total for Question 4 is 6 marks)

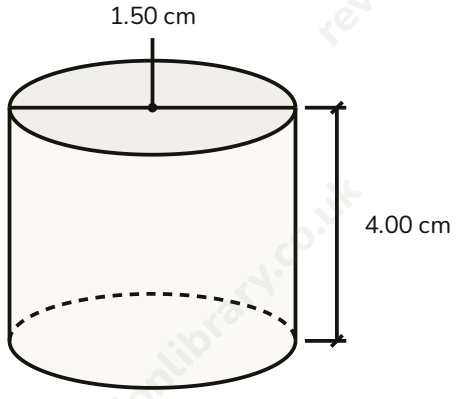
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5 Required practical: A student determines the density of a regular metal cylinder and of a small irregularly shaped stone.

(a) Describe a method for determining the volume of the small irregularly shaped stone using a displacement (eureka) can and a measuring cylinder. **(3)**

(b) A metal cylinder has a diameter of 1.50 cm (measured with vernier calipers) and a height of 4.00 cm (measured with a ruler). Calculate its volume, using $V = \pi \times r^2 \times h$. **(3)**

Diagram NOT accurately drawn



(c) The cylinder has a mass of 62.3 g. Calculate the density of the cylinder in kg/m^3. **(3)**

(d) The diameter is 1.50 ± 0.02 cm and the height is 4.00 ± 0.05 cm. Calculate the percentage uncertainty in the volume of the cylinder. **(4)**

(e) For the stone, the volume found by displacement is 15.0 ± 0.5 cm^3 and the mass is 40.2 ± 0.1 g. Calculate the percentage uncertainty in the density of the stone, and state which measurement contributes the greater uncertainty. **(4)**

(Total for Question 5 is 17 marks)

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6 Synoptic question. A trolley of mass m = 0.842 ± 0.002 kg (found using a top-pan balance) is given an initial push along a horizontal, effectively frictionless air track and then passes through a single light gate fitted with an opaque interrupt card of length d = 50.0 ± 0.5 mm. The light gate's electronic timer records a time of t = 24.4 ± 0.3 ms for the card to pass through the light beam. Assume the trolley's speed does not change significantly over this short interval, so its instantaneous speed at the light gate is v = d / t.

(a) Calculate the speed, v, of the trolley at the light gate. **(2)**

(b) Calculate the percentage uncertainty in v. **(3)**

(c) The kinetic energy of the trolley is given by $E_k = 1/2 \times m \times v^2$. Calculate the percentage uncertainty in E_k, explaining how the power in the formula affects your calculation. **(2)**

(d) Calculate the kinetic energy, E_k, of the trolley and express it with its absolute uncertainty, to an appropriate number of significant figures. **(3)**

(Total for Question 6 is 10 marks)

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7 Required practical: A student determines the Young modulus of a metal wire. A mass of 1.200 ± 0.001 kg is hung from a wire of original length L = 2.000 ± 0.002 m and diameter d = 0.32 ± 0.005 mm (measured with a micrometer). The resulting extension of the wire is e = 1.40 ± 0.05 mm. Take g = 9.81 m/s^2 (treated as exact). The Young modulus is given by $E = F \times L / (A \times e)$, where F is the force applied and A is the cross-sectional area of the wire.

(a) Show that the force applied to the wire is 11.8 N. **(2)**

(b) Calculate the cross-sectional area, A, of the wire, using $A = \pi \times d^2 / 4$ and d = 0.32 mm. Give your answer in m^2. **(3)**

(c) Calculate the percentage uncertainty in F, in L, in A (using your value from part b), and in e, and hence find the percentage uncertainty in the Young modulus E. **(4)**

(d) Calculate the Young modulus, E, and express it with its absolute uncertainty, to an appropriate number of significant figures. **(3)**

(Total for Question 7 is 12 marks)

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8 Required practical: A student determines the density of a small cube cut from a sample of pine wood. The side length of the cube is measured five times, at different points and orientations, using vernier calipers of resolution 0.02 mm. The readings obtained are: 40.00 mm, 40.08 mm, 39.96 mm, 44.50 mm, 40.04 mm. The mass of the cube, measured on an electronic balance of resolution 0.1 g, is 28.3 ± 0.05 g.

Side length (mm)
40.00
40.08
39.96
44.50
40.04

(a) Identify the anomalous reading in the table of side-length measurements, and suggest one possible cause for it. **(2)**

(b) Calculate the mean side length of the cube, excluding the anomalous reading. **(2)**

(c) Estimate the absolute uncertainty in the mean side length using half the range of the four valid readings, and hence calculate the percentage uncertainty in the side length. **(2)**

(d) Calculate the volume of the cube in m^3, and the percentage uncertainty in this volume. **(3)**

(e) Calculate the density of the cube in kg/m^3, and express it with its absolute uncertainty, to an appropriate number of significant figures. **(4)**

(f) The accepted density range for this species of pine is 350-600 kg/m^3. Comment on the accuracy of the student's result, and suggest why a wooden sample might reasonably have an accepted density range rather than a single fixed value. **(2)**

(Total for Question 8 is 15 marks)

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Mark scheme · 3.1 Measurements and their Errors: 2-hour Pack

Question 1

(a) 2.0% (cao) **B1**

(a) Answer: 2.0%

(b) for a cubed quantity, the percentage uncertainty is multiplied by 3: %unc(V) = 3 x %unc(r) **M1**

(b) 6.0% (ft from a) **A1**

(b) Answer: 6.0%

(c) $V = (4/3) \times \pi \times (2.50)^3 = 65.4\ cm^3$ **M1**

(c) absolute uncertainty = $65.4 \times 0.060 = 3.9\ cm^3$ (ft from b) **M1**

(c) $V = (65 \pm 4)\ cm^3$ (awrt, appropriate s.f.) **A1**

(c) Answer: $V = (65 \pm 4)\ cm^3$

Question 2

(a) titre = $27.65 - 2.40 = 25.25\ cm^3$ **M1**

(a) for a difference of two readings, absolute uncertainties are added: uncertainty = $0.05 + 0.05 = 0.10\ cm^3$ **M1**

(a) titre = $25.25 \pm 0.10\ cm^3$ (cao) **A1**

(a) Answer: $25.25 \pm 0.10\ cm^3$

(b) percentage uncertainty = (0.10 / 25.25) x 100 **M1**

(b) 0.40% (awrt, ft from a) **A1**

(b) Answer: 0.40%

(c) temperature rise = 84.0 - 19.5 = 64.5°C **M1**

(c) for a difference of two readings, absolute uncertainties are added: uncertainty = 0.5 + 0.5 = 1.0°C **M1**

(c) temperature rise = 64.5 ± 1.0°C (cao) **A1**

(c) Answer: 64.5 ± 1.0°C

Question 3

(a) uncertainty = half the difference between the steepest and least steep gradients = (26.0 - 23.0)/2 = 1.5 N/m **M1**

(a) percentage uncertainty = (1.5/24.5) x 100 **M1**

(a) 6.1% (awrt) **A1**

(a) Answer: 6.1%

(b) central value stated as 24.5 N/m **B1**

(b) uncertainty of 1.5 N/m quoted to the same precision (1 d.p.) as the central value: 24.5 ± 1.5 N/m **B1**

(b) Answer: 24.5 ± 1.5 N/m

(c) each error bar shows the range of uncertainty in the measured value of that data point **B1**

(c) the steepest and least steep straight lines that still pass through every error bar are drawn on the graph, and the gradient of each is calculated **B1**

(c) the uncertainty in the gradient is half the difference between these two gradients **B1**

(c) Answer: Error bars show the uncertainty range on each point; drawing the steepest and shallowest lines that still pass through all the error bars, and taking half the difference between their gradients, gives the uncertainty in the gradient.

(d) using many data points averages out the effect of random error/uncertainty in any one reading oe **B1**

(d) any anomalous point is more easily identified/its effect is reduced, giving a more representative/reliable value of k than relying on a single pair of readings **B1**

(d) Answer: The gradient uses many data points, so random errors and any single anomalous reading have much less effect, giving a more reliable value of k than a single F-x pair.

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Question 4

Level 3 A full, coherent evaluation that calculates or clearly compares the percentage uncertainty in the extension for both methods, correctly identifies extension as the measurement that dominates the overall percentage uncertainty in the Young modulus for both students, and reaches a justified conclusion (with a valid supporting reason such as the use of a reference wire) that Student B's method is more precise. **5-6**

Level 2 The answer compares the resolutions or uncertainties of the two measuring instruments and reaches a conclusion about which method is more precise, but the reasoning is incomplete, lacks a numerical comparison of percentage uncertainty, or omits reference to the small size of the extension being the limiting factor. **3-4**

Level 1 A basic statement identifying one method as better (e.g. Searle's apparatus is more precise) with little or no supporting justification. **1-2**

Level 0 No relevant content, or the answer does not relate to the question. **0**

Indicative content:

- The extension itself is small (about 2 mm) in both experiments, so the absolute uncertainty in measuring the extension is the critical factor in the overall percentage uncertainty of the Young modulus.
- Student A's metre rule has a resolution of 1 mm, giving an absolute uncertainty of about 0.5 mm on an extension of about 2 mm, a percentage uncertainty of roughly 25%.
- Student B's vernier/micrometer scale has a resolution of 0.01 mm, giving an absolute uncertainty of about 0.005-0.01 mm on a similar extension, a percentage uncertainty of only around 0.5%.
- Because the extension has by far the largest percentage uncertainty of any quantity in the Young modulus calculation, it dominates the overall percentage uncertainty in E for both students, but far more severely for Student A.
- Searle's apparatus also uses a reference wire under the same conditions (temperature, sag) as the test wire, reducing systematic errors that would otherwise arise from thermal expansion or the wire settling under load.
- Conclusion: Student B's method (Searle's apparatus) gives a more precise value of the Young modulus, because its much finer resolution greatly reduces the percentage uncertainty in the extension, the limiting measurement in this experiment.

Question 5

(a) fill the displacement can with water until it overflows from the side spout, and allow the dripping to stop before starting **B1**

(a) place an empty measuring cylinder under the spout, then lower the stone gently into the can on a thread until fully submerged, without touching the sides or base **B1**

(a) the volume of water collected in the measuring cylinder equals the volume of the stone **B1**

(a) Answer: Fill the eureka can to the spout and let it stop dripping; place a measuring cylinder under the spout; lower the stone in on a thread until fully submerged; the volume of water collected equals the stone's volume.

(b) radius = diameter/2 = 0.75 cm **M1**

(b) correct substitution and evaluation: $V = \pi \times (0.75)^2 \times 4.00$ **M1**

(b) 7.07 cm^3 (awrt, cao) **A1**

(b) Answer: V = 7.07 cm^3

(c) density = mass/volume = 62.3/7.07 = 8.81 g/cm^3 (ft from b) **M1**

(c) converts g/cm^3 to kg/m^3 by multiplying by 1000 **M1**

(c) 8810 kg/m^3 (awrt, accept 8800-8820 kg/m^3) **A1**

(c) Answer: 8810 kg/m^3

(d) percentage uncertainty in diameter = (0.02/1.50) x 100 = 1.33% **M1**

(d) percentage uncertainty in height = (0.05/4.00) x 100 = 1.25% **M1**

(d) since V depends on d^2 x h, percentage uncertainty is doubled for the diameter term: %unc(V) = (2 x 1.33%) + 1.25% **M1**

(d) 3.9% (awrt) **A1**

(d) Answer: 3.9%

(e) percentage uncertainty in mass = (0.1/40.2) x 100 = 0.25% **M1**

(e) percentage uncertainty in volume = (0.5/15.0) x 100 = 3.3% **M1**

(e) percentage uncertainty in density = 0.25% + 3.3% = 3.6% (ft) **A1**

(e) the volume measurement contributes the greater uncertainty, since 3.3% is much larger than 0.25% **B1**

(e) Answer: Percentage uncertainty in density = 3.6%; the volume (displacement) measurement contributes the greater uncertainty.

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Question 6

(a) converts to SI units and substitutes: v = 0.0500/0.0244 **M1**

(a) v = 2.05 m/s (awrt, cao) **A1**

(a) Answer: v = 2.05 m/s

(b) percentage uncertainty in d = (0.5/50.0) x 100 = 1.0% **M1**

(b) percentage uncertainty in t = (0.3/24.4) x 100 = 1.2% (awrt) **M1**

(b) for a quotient, percentage uncertainties add: %unc(v) = 1.0 + 1.2 = 2.2% (ft) **A1**

(b) Answer: 2.2%

(c) percentage uncertainty in m = (0.002/0.842) x 100 = 0.24%; since v is squared, its percentage uncertainty doubles to 2 x 2.23% = 4.5% (ft from b) **M1**

(c) for a product, percentage uncertainties add: %unc(Ek) = 0.24 + 4.5 = 4.7% (ft) **A1**

(c) Answer: 4.7%

(d) Ek = 0.5 x 0.842 x $(2.05)^2$ **M1**

(d) Ek = 1.77 J (awrt, ft from a) **M1**

(d) absolute uncertainty = 1.77 x 0.047 = 0.083, so Ek = (1.77 ± 0.08) J (ft, awrt) **A1**

(d) Answer: Ek = (1.77 ± 0.08) J

Question 7

(a) F = mg = 1.200 x 9.81 **M1**

(a) F = 11.772 N = 11.8 N (3 s.f.) (cso, answer given) **A1**

(a) Answer: F = 11.8 N

(b) converts diameter to metres: d = 3.2 x 10^{-4} m **M1**

(b) correct substitution and evaluation: A = π x $(3.2 \times 10^{-4})^2$ / 4 = 8.04 x 10^{-8} m^2 **M1**

(b) 8.0 x 10^{-8} m^2 (awrt, to 2 s.f., matching the precision of d) **A1**

(b) Answer: A = 8.0 x 10^{-8} m^2

(c) percentage uncertainty in F = (0.001/1.200) x 100 = 0.083%, and in L = (0.002/2.000) x 100 = 0.10% **M1**

(c) percentage uncertainty in d = (0.005/0.32) x 100 = 1.56%, so (since A depends on d^2) percentage uncertainty in A = 2 x 1.56% = 3.1% **M1**

(c) percentage uncertainty in e = (0.05/1.40) x 100 = 3.6% **M1**

(c) for a product/quotient of several quantities percentage uncertainties add: %unc(E) = 0.083 + 0.10 + 3.1 + 3.6 = 6.9% (ft) **A1**

(c) Answer: 6.9%

(d) correct substitution: E = (11.772 x 2.000) / (8.04 x 10^{-8} x 1.40 x 10^{-3}) **M1**

(d) E = 2.09 x 10^{11} Pa (209 GPa) (awrt, ft from b) **M1**

(d) absolute uncertainty = 2.09 x 10^{11} x 0.069 = 1.4 x 10^{10} Pa, so E = (2.09 ± 0.14) x 10^{11} Pa (ft, awrt) **A1**

(d) Answer: E = (2.09 ± 0.14) x 10^{11} Pa

Question 8

(a) 44.50 mm is anomalous, since it is much larger than (and inconsistent with) the other four readings, which cluster closely around 40.0 mm **B1**

(a) a possible cause, e.g. the calipers were misread (a slipped decimal/scale misread), the jaws were not closed correctly on the cube, or the measurement was taken across a corner/diagonal rather than a face of the cube oe **B1**

(a) Answer: 44.50 mm is the anomalous reading; a likely cause is a misread scale or the calipers being incorrectly positioned across the cube (e.g. along a diagonal rather than a face).

(b) mean = (40.00 + 40.08 + 39.96 + 40.04) / 4 **M1**

(b) 40.02 mm (cao) **A1**

(b) Answer: 40.02 mm

(c) uncertainty = half the range = (40.08 - 39.96)/2 = 0.06 mm **M1**

(c) percentage uncertainty = (0.06/40.02) x 100 = 0.15% (awrt, ft from b) **A1**

(c) Answer: 0.15%

(d) converts the mean side length to metres and cubes it: V = $(4.002 \times 10^{-2})^3$ **M1**

(d) V = 6.41 x 10^{-5} m^3 (awrt, ft from b) **M1**

(d) since V depends on (side length)^3, percentage uncertainty in V = 3 x 0.15% = 0.45% (ft from c) **A1**

(d) Answer: V = 6.41 x 10^{-5} m^3, percentage uncertainty 0.45%

(e) percentage uncertainty in mass = (0.05/28.3) x 100 = 0.18% **M1**

(e) density = mass/volume = 0.0283/(6.41 x 10^{-5}) = 442 kg/m^3 (awrt, ft from d) **M1**

(e) for a quotient, percentage uncertainties add: percentage uncertainty in density = 0.18 + 0.45 = 0.63% (ft) **M1**

(e) absolute uncertainty = 442 x 0.0063 = 2.8, so density = (442 ± 3) kg/m^3 (ft, awrt) **A1**

(e) Answer: density = (442 ± 3) kg/m^3

(f) 442 kg/m^3 lies within the stated accepted range of 350-600 kg/m^3, so the student's result is accurate/consistent with the accepted value oe **B1**

(f) wood is a natural material, so its density varies between (and within) samples owing to factors such as moisture content, growth rate/ring spacing and the presence of knots or voids, which is why a range rather than a single value is quoted oe **B1**

(f) Answer: The result (442 kg/m^3) lies within the accepted range of 350-600 kg/m^3, so it is accurate; wood density naturally varies between samples (moisture content, growth conditions, knots), which is why a range rather than a single fixed value is quoted.

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