3.1 Measurements and their Errors

AQA 7408 · Calculator allowed · about 240 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 SI units, prefixes and estimation are the foundation of all physical measurements.

(a) State the SI base unit of: (i) mass, (ii) time, (iii) electric current. **(3)**

(b) A component in a circuit is 250 nm thick. Write this thickness in metres, using standard form. **(1)**

(c) A capacitor is labelled as having a capacitance of 4.7 nF. Express this value in picofarads (pF). **(2)**

(d) Estimate, to one significant figure, the diameter of a single human hair in metres. **(2)**
State the order of magnitude of your estimate.

(Total for Question 1 is 8 marks)

2 Significant figures indicate the precision of a measured or calculated quantity.

(a) A student measures the diameter of a wire as 0.42 mm using a micrometer of resolution 0.01 mm. State the number of significant figures in this measurement. **(1)**

(b) Calculate the cross-sectional area of the wire in part (a), using $A = \pi \times d^2 / 4$, giving your answer in m^2 to an appropriate number of significant figures. **(3)**

(c) A calculation gives a result of 12.3456 N. Round this value to 3 significant figures. **(1)**

(d) Explain why it is not appropriate to quote the final answer to a calculation to more significant figures than the least precise measurement used in that calculation. **(2)**

(Total for Question 2 is 7 marks)

3 Precision, accuracy and resolution describe different aspects of the quality of a measurement.

(a) Define the term 'resolution' of a measuring instrument. **(1)**

(b) Define the term 'accuracy' of a measurement. **(1)**

(c) Two students each measure the length of the same object several times. Student A's repeated readings are tightly clustered together but consistently 3 mm below the true length. Student B's readings are centred on the true length but are widely spread out. Describe what this tells you about the precision and accuracy of each student's set of results. **(2)**

(d) A metre ruler has a resolution of 1 mm. State the absolute uncertainty in a single length measurement made with this ruler. **(1)**

(Total for Question 3 is 5 marks)

4 Significant figures indicate how precisely a value is known; recognising them correctly, and rounding calculated results appropriately, is an essential exam skill.

(a) State the number of significant figures in the measurement 0.070060 m. **(1)**

(b) A calculation gives a result of 143.7568 N, using two measured quantities that were each quoted to 3 significant figures. State this result to an appropriate number of significant figures. **(1)**

(c) A student measures a potential difference V = 6.0 V (2 significant figures) across a resistor and a current I = 0.24 A (2 significant figures) through it. Calculate the resistance R = V / I, giving your answer to an appropriate number of significant figures. **(2)**

(d) Explain the difference between the number of decimal places and the number of significant figures in a measured value, using 0.070060 m as your example. **(2)**

(Total for Question 4 is 6 marks)

5 Measurements are always subject to systematic and/or random error.

(a) A top-pan balance is not zeroed before use and reads 0.05 g with nothing on the pan. Every subsequent mass measured with this balance is too high by the same 0.05 g. Identify the type of error this illustrates, and state how the student should correct the affected readings. **(2)**

(b) A student reads the position of a pointer on an analogue ammeter scale from an angle, rather than looking directly along the pointer from directly above the scale. Name the type of error this introduces, and state one way to reduce it. **(2)**

(c) The student re-zeros the balance from part (a) correctly, then measures the same mass 6 times, obtaining slightly different readings each time because of air currents disturbing the balance. Explain whether this variation is a systematic or a random error, and describe how the student could reduce its effect on the reported mean value. **(3)**

(Total for Question 5 is 7 marks)

6 Reading an instrument scale correctly is a fundamental practical skill.

(a) A micrometer screw gauge has a resolution of 0.01 mm. For a particular reading, the main scale shows 5.5 mm and the rotating thimble scale, on which each division represents 0.01 mm, is aligned with the line labelled 23. Calculate the diameter reading shown, and state the absolute uncertainty in this single reading. **(3)**

(b) A vernier caliper has a resolution of 0.02 mm. For a particular reading, the main scale shows 12 mm and the 4th line on the vernier scale is the one that aligns exactly with a main scale line. Calculate the diameter reading shown, and state the absolute uncertainty in this single reading. **(3)**

(Total for Question 6 is 6 marks)

7 A student measures the length of a metal rod using vernier calipers of resolution 0.1 mm, obtaining a reading of 45.0 mm.

(a) Calculate the percentage uncertainty in the single reading of 45.0 mm. **(2)**

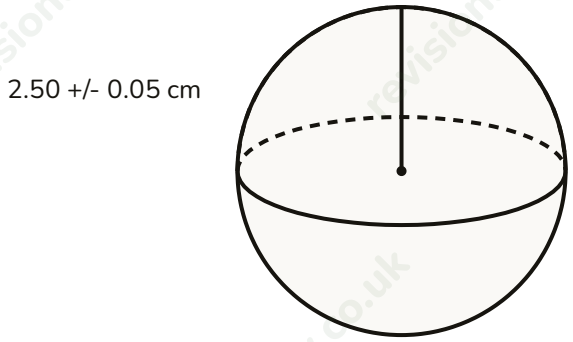
(b) The student instead repeats the measurement 5 times, obtaining: 45.0 mm, 45.2 mm, 44.8 mm, 45.1 mm, 44.9 mm. Calculate the mean length and the absolute uncertainty using the range method (half the range of the readings). **(3)**

(c) Explain why the uncertainty found in part (b) is greater than the uncertainty found in part (a). **(2)**

(Total for Question 7 is 7 marks)

8 A small metal sphere has a measured radius r = 2.50 ± 0.05 cm.

Diagram NOT accurately drawn



(a) Calculate the percentage uncertainty in r. **(1)**

(b) The volume of the sphere is given by $V = (4/3) \times \pi \times r^3$. Calculate the percentage uncertainty in V. **(2)**

(c) Calculate the volume of the sphere and express your answer with its absolute uncertainty, to an appropriate number of significant figures. **(3)**

(Total for Question 8 is 6 marks)

9 Errors in measurement can be systematic or random.

(a) State one difference between a systematic error and a random error. **(2)**

(b) A newton-meter (spring balance) reads 0.20 N when nothing is hung from it. Identify the type of error this illustrates and explain how the student should account for it. **(2)**

(c) A student times 20 complete oscillations of a pendulum, rather than timing a single oscillation, to determine its period. Explain how timing 20 oscillations reduces the percentage uncertainty in the period, compared with timing a single oscillation. **(3)**

(Total for Question 9 is 7 marks)

10 Uncertainties combine differently depending on whether quantities are added, subtracted, multiplied or divided.

(a) In a titration, a burette of resolution 0.10 cm^3 is used. The initial reading is 2.40 ± 0.05 cm^3 and the final reading is 27.65 ± 0.05 cm^3 (the uncertainty in each reading is taken as half the resolution). Calculate the titre (volume delivered) and its absolute uncertainty. **(3)**

(b) Calculate the percentage uncertainty in the titre found in part (a), giving your answer to 2 significant figures. **(2)**

(c) A thermometer of resolution 1°C is used to record the temperature of a water bath before and after heating. The initial temperature is 19.5 ± 0.5°C and the final temperature is 84.0 ± 0.5°C (the uncertainty in each reading is taken as half the resolution). Calculate the temperature rise and its absolute uncertainty. **(3)**

(Total for Question 10 is 8 marks)

11 The percentage uncertainty in a calculated quantity depends on how that quantity is combined mathematically from its measured inputs: percentage uncertainties are added for a product or quotient, and multiplied by the power for a term raised to a power (including a square root, which multiplies the percentage uncertainty by 1/2).

(a) A heating element has a potential difference V = 6.0 ± 0.1 V across it and a resistance R = 15.0 ± 0.3 ohm. Calculate the percentage uncertainty in V and the percentage uncertainty in R. **(2)**

(b) The power dissipated is given by $P = V^2 / R$. Calculate the percentage uncertainty in P, and hence state the power P with its absolute uncertainty, to an appropriate number of significant figures. **(3)**

(c) A mass m = 0.250 ± 0.005 kg is attached to a spring of spring constant k = 18.0 ± 0.5 N/m. Calculate the percentage uncertainty in m and the percentage uncertainty in k. **(2)**

(d) The period of oscillation is given by $T = 2 \times \pi \times \sqrt{m/k}$. Calculate the percentage uncertainty in T, and hence state the period T with its absolute uncertainty, to an appropriate number of significant figures. **(3)**

(Total for Question 11 is 10 marks)

12 A student investigates the extension, x, of a spring for a range of applied forces, F, and plots a graph of F against x. A best-fit straight line is drawn through the origin, along with the steepest and least steep straight lines that still pass through all the error bars on the data points.

(a) The best-fit line has a gradient of 24.5 (in units of N per m of extension per unit, i.e. N/m). The steepest acceptable line has a gradient of 26.0 N/m, and the least steep acceptable line has a gradient of 23.0 N/m. Calculate the percentage uncertainty in the gradient. **(3)**

(b) State the gradient with its absolute uncertainty, to an appropriate number of significant figures. **(2)**

(c) Explain why error bars are included on the graph, and how they are used to find the uncertainty in the gradient. **(3)**

(d) The spring constant, k, is defined by F = k x x. Suggest two reasons why using the gradient of the graph gives a more reliable value of k than calculating k from a single pair of F and x readings. **(2)**

(Total for Question 12 is 10 marks)

13 A student uses a simple pendulum to determine the acceleration due to gravity, g, using $T = 2 \times \pi \times \sqrt{l/g}$, which rearranges to $g = 4 \times \pi^2 \times l / T^2$. The pendulum length is $l = 0.850 \pm 0.002$ m. The time for 20 complete oscillations is measured as 37.0 ± 0.4 s.

(a) Show that the period, T, of the pendulum is 1.85 s. **(2)**

(b) Calculate the percentage uncertainty in the time for 20 oscillations, and hence state the percentage uncertainty in the period, T. **(3)**

(c) Calculate the percentage uncertainty in T^2. **(1)**

(d) The percentage uncertainty in l is 0.24%. Calculate the percentage uncertainty in g. **(2)**

(e) Calculate the value of g and its absolute uncertainty, to an appropriate number of significant figures. **(4)**

(f) The accepted value of g is 9.81 m/s^2. Comment on whether the student's result in part (e) is consistent with the accepted value. **(2)**

(Total for Question 13 is 14 marks)

14 Two students each determine the Young modulus of the same copper wire using an applied load that produces an extension of about 2 mm. Student A uses a single wire clamped horizontally, measuring the extension directly with a metre rule of resolution 1 mm. Student B uses Searle's apparatus, with two parallel wires (one as a reference) and a vernier/micrometer scale of resolution 0.01 mm to measure the extension. Evaluate which student's method is more precise, using percentage uncertainty in the extension and any relevant features of the apparatus.

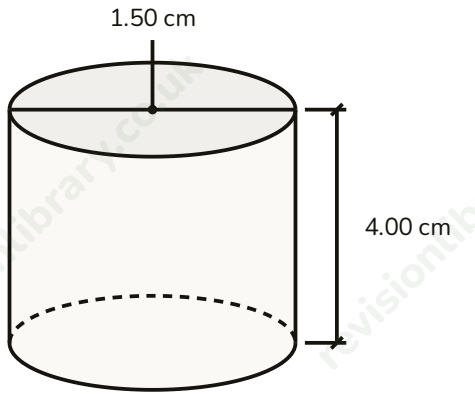
(Total for Question 14 is 6 marks)

15 Required practical: A student determines the density of a regular metal cylinder and of a small irregularly shaped stone.

(a) Describe a method for determining the volume of the small irregularly shaped stone using a displacement (eureka) can and a measuring cylinder. **(3)**

(b) A metal cylinder has a diameter of 1.50 cm (measured with vernier calipers) and a height of 4.00 cm (measured with a ruler). Calculate its volume, using $V = \pi \times r^2 \times h$. **(3)**

Diagram NOT accurately drawn



(c) The cylinder has a mass of 62.3 g. Calculate the density of the cylinder in kg/m^3. **(3)**

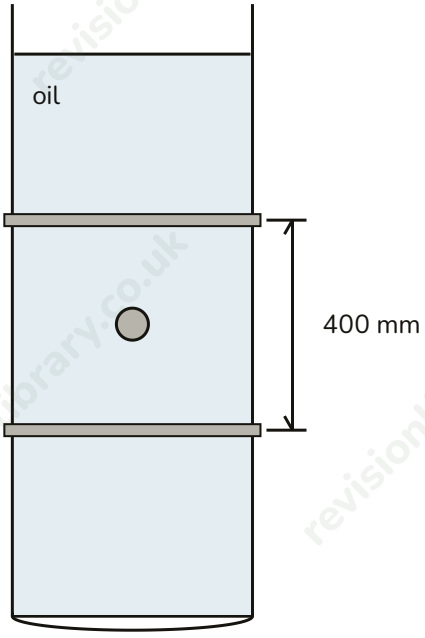
(d) The diameter is 1.50 ± 0.02 cm and the height is 4.00 ± 0.05 cm. Calculate the percentage uncertainty in the volume of the cylinder. **(4)**

(e) For the stone, the volume found by displacement is 15.0 ± 0.5 cm^3 and the mass is 40.2 ± 0.1 g. Calculate the percentage uncertainty in the density of the stone, and state which measurement contributes the greater uncertainty. **(4)**

(Total for Question 15 is 17 marks)

16 A student determines the terminal velocity of steel ball bearings falling through a viscous oil held in a tall, wide measuring cylinder. Two rubber bands are placed around the outside of the cylinder to mark two points a measured 400 mm apart, positioned well below the point at which each ball reaches terminal velocity. For each of five different ball bearing diameters, the student drops one ball bearing into the oil near the centre of the cylinder, starts a hand-held stopwatch by eye as the ball passes the upper band, and stops it as the ball passes the lower band. Each ball is timed once only. The terminal velocity for each ball is then calculated as 400 mm divided by the measured time. The student's five calculated terminal velocities are all slightly, but consistently, higher than a set of accepted reference values for oil of this viscosity. Evaluate the procedure described for a systematic weakness that could explain why the calculated terminal velocities are consistently too high, propose an improved procedure, and justify how your improvement would reduce this systematic uncertainty.

Diagram NOT accurately drawn



(Total for Question 16 is 6 marks)

17 Synoptic question. A trolley of mass m = 0.842 ± 0.002 kg (found using a top-pan balance) is given an initial push along a horizontal, effectively frictionless air track and then passes through a single light gate fitted with an opaque interrupt card of length d = 50.0 ± 0.5 mm. The light gate's electronic timer records a time of t = 24.4 ± 0.3 ms for the card to pass through the light beam. Assume the trolley's speed does not change significantly over this short interval, so its instantaneous speed at the light gate is v = d / t.

(a) Calculate the speed, v, of the trolley at the light gate. **(2)**

(b) Calculate the percentage uncertainty in v. **(3)**

(c) The kinetic energy of the trolley is given by Ek = 1/2 x m x v^2. Calculate the percentage uncertainty in Ek, explaining how the power in the formula affects your calculation. **(2)**

(d) Calculate the kinetic energy, Ek, of the trolley and express it with its absolute uncertainty, to an appropriate number of significant figures. **(3)**

(Total for Question 17 is 10 marks)

18 Required practical: A student determines the Young modulus of a metal wire. A mass of 1.200 ± 0.001 kg is hung from a wire of original length L = 2.000 ± 0.002 m and diameter d = 0.32 ± 0.005 mm (measured with a micrometer). The resulting extension of the wire is e = 1.40 ± 0.05 mm. Take g = 9.81 m/s^2 (treated as exact). The Young modulus is given by E = F x L / (A x e), where F is the force applied and A is the cross-sectional area of the wire.

(a) Show that the force applied to the wire is 11.8 N. **(2)**

(b) Calculate the cross-sectional area, A, of the wire, using A = π x d^2 / 4 and d = 0.32 mm. Give your answer in m^2. **(3)**

(c) Calculate the percentage uncertainty in F, in L, in A (using your value from part b), and in e, and hence find the percentage uncertainty in the Young modulus E. **(4)**

(d) Calculate the Young modulus, E, and express it with its absolute uncertainty, to an appropriate number of significant figures. **(3)**

(Total for Question 18 is 12 marks)

19 Required practical: A student determines the density of a small cube cut from a sample of pine wood. The side length of the cube is measured five times, at different points and orientations, using vernier calipers of resolution 0.02 mm. The readings obtained are: 40.00 mm, 40.08 mm, 39.96 mm, 44.50 mm, 40.04 mm. The mass of the cube, measured on an electronic balance of resolution 0.1 g, is 28.3 ± 0.05 g.

Side length (mm)
40.00
40.08
39.96
44.50
40.04

(a) Identify the anomalous reading in the table of side-length measurements, and suggest one possible cause for it. **(2)**

(b) Calculate the mean side length of the cube, excluding the anomalous reading. **(2)**

(c) Estimate the absolute uncertainty in the mean side length using half the range of the four valid readings, and hence calculate the percentage uncertainty in the side length. **(2)**

(d) Calculate the volume of the cube in m^3, and the percentage uncertainty in this volume. **(3)**

(e) Calculate the density of the cube in kg/m^3, and express it with its absolute uncertainty, to an appropriate number of significant figures. **(4)**

(f) The accepted density range for this species of pine is 350-600 kg/m^3. Comment on the accuracy of the student's result, and suggest why a wooden sample might reasonably have an accepted density range rather than a single fixed value. **(2)**

(Total for Question 19 is 15 marks)

Mark scheme · 3.1 Measurements and their Errors

Question 1

(a) kilogram, kg **B1**

(a) second, s **B1**

(a) ampere, A **B1**

(a) Answer: (i) kilogram (kg); (ii) second (s); (iii) ampere (A).

(b) 2.5 x 10^{-7} m (cao) **B1**

(b) Answer: 2.5 x 10^{-7} m

(c) recognises 1 nF = 10^{3} pF (i.e. multiplies by 1000) **M1**

(c) 4700 pF, or equivalently 4.7 x 10^{3} pF (cao) **A1**

(c) Answer: 4700 pF (4.7 x 10^{3} pF)

(d) any sensible estimate in the range 5 x 10^{-5} m to 2 x 10^{-4} m **B1**

(d) correctly states the order of magnitude of their estimate (e.g. 10^{-4} m) oe **B1**

(d) Answer: Approximately 1 x 10^{-4} m; order of magnitude 10^{-4} m (any value from 5 x 10^{-5} to 2 x 10^{-4} m with a correctly stated matching order of magnitude is acceptable).

Question 2

(a) 2 (significant figures) cao **B1**

(a) Answer: 2 significant figures

(b) converts diameter to metres: d = 4.2 x 10^{-4} m **M1**

(b) correct substitution and evaluation: A = π x (4.2 x 10^{-4})^2 / 4 = 1.385 x 10^{-7} m^2 **M1**

(b) 1.4 x 10^{-7} m^2 (awrt, to 2 s.f., matching the 2 s.f. of the diameter) **A1**

(b) Answer: A = 1.4 x 10^{-7} m^2

(c) 12.3 N (cao) **B1**

(c) Answer: 12.3 N

(d) the overall result cannot be known more precisely/certainly than its least precise input measurement oe **B1**

(d) quoting extra significant figures would imply a false level of precision/accuracy that the data does not support **B1**

(d) Answer: A calculated result is only as precise as the least precise value used to find it; extra significant figures would falsely suggest greater precision than the measurements justify.

Question 3

(a) the smallest change in the quantity being measured that causes a detectable/observable change in the instrument's reading oe **B1**

(a) Answer: The smallest change in a quantity that an instrument can detect.

(b) how close a measured value is to the true (accepted) value of the quantity oe **B1**

(b) Answer: How close a measurement is to the true value of the quantity being measured.

(c) Student A: precise (small spread/good repeatability) but not accurate, because of a systematic error offsetting all readings from the true value **B1**

(c) Student B: accurate (mean value close to/matches the true value) but not precise, because of a large random error causing a wide spread of readings **B1**

(c) Answer: Student A is precise but not accurate (small spread, offset from true value by a systematic error). Student B is accurate but not precise (correct mean, but a large random spread).

(d) ± 0.5 mm (half the smallest scale division) oe **B1**

(d) Answer: ± 0.5 mm

Question 4

(a) 5 (significant figures) cao **B1**

(a) Answer: 5 significant figures

(b) 144 N (cao) **B1**

(b) Answer: 144 N

(c) R = 6.0 / 0.24 = 25.0 ohm **M1**

(c) 25 ohm, quoted to 2 s.f. to match the precision of the input data (cao) **A1**

(c) Answer: 25 ohm (2 s.f.)

(d) 0.070060 m has 6 decimal places (digits after the decimal point) but only 5 significant figures, because the leading zeros are not significant **B1**

(d) decimal places depend on how the number is written/the unit used, whereas significant figures reflect the actual precision of the measurement regardless of the unit oe **B1**

(d) Answer: 0.070060 m has 6 decimal places but only 5 significant figures; decimal places depend on how the value is written (and its unit), while significant figures reflect the true precision of the measurement.

Question 5

(a) systematic (zero) error **B1**

(a) subtract 0.05 g from every reading taken with the uncorrected balance (or re-zero the balance before use) oe **B1**

(a) Answer: This is a systematic (zero) error; the student should subtract 0.05 g from every reading taken (or zero the balance before use).

(b) parallax error **B1**

(b) view the scale directly from above/in line with the pointer (e.g. using a mirror behind the scale to check the pointer and its reflection align) oe **B1**

(b) Answer: This is a parallax error; it can be reduced by viewing the scale directly from above the pointer, for example using a mirror scale to check the pointer and its reflection coincide.

(c) random error, because the readings vary unpredictably above and below the true value rather than being consistently offset in one direction **B1**

(c) the air currents cause small, unpredictable fluctuations in each individual reading **B1**

(c) repeating the measurement many times and calculating the mean reduces the effect of this random variation on the reported value oe **B1**

(c) Answer: This is a random error, since the air currents cause the readings to scatter unpredictably above and below the true value rather than shifting them consistently in one direction; taking many repeat readings and calculating the mean reduces its effect on the reported mass.

Question 6

(a) thimble contribution = 23 x 0.01 mm = 0.23 mm **M1**

(a) diameter = 5.5 + 0.23 = 5.73 mm (cao) **A1**

(a) uncertainty = ± 0.005 mm (half the resolution) **B1**

(a) Answer: 5.73 ± 0.005 mm

(b) vernier contribution = 4 x 0.02 mm = 0.08 mm **M1**

(b) diameter = 12 + 0.08 = 12.08 mm (cao) **A1**

(b) uncertainty = ± 0.01 mm (half the resolution) **B1**

(b) Answer: 12.08 ± 0.01 mm

Question 7

(a) identifies absolute uncertainty as half the resolution: ± 0.05 mm **M1**

(a) 0.11% (awrt) **A1**

(a) Answer: 0.11%

(b) mean = (45.0 + 45.2 + 44.8 + 45.1 + 44.9) / 5 = 45.0 mm **M1**

(b) uncertainty = half the range = (45.2 - 44.8) / 2 = 0.2 mm **M1**

(b) 45.0 ± 0.2 mm (cao) **A1**

(b) Answer: 45.0 ± 0.2 mm

(c) the repeated readings show random variation (e.g. from positioning the calipers slightly differently each time) that a single reading cannot reveal **M1**

(c) this random variation between repeats is larger than the instrument's resolution alone, so the range-based uncertainty is greater than the resolution-based uncertainty **A1**

(c) Answer: Repeating the measurement reveals random variation between readings that is bigger than the instrument's resolution, so the spread-based uncertainty in (b) exceeds the resolution-based uncertainty in (a).

Question 8

(a) 2.0% (cao) **B1**

(a) Answer: 2.0%

(b) for a cubed quantity, the percentage uncertainty is multiplied by 3: %unc(V) = 3 x %unc(r) **M1**

(b) 6.0% (ft from a) **A1**

(b) Answer: 6.0%

(c) $V = (4/3) \times \pi \times (2.50)^3 = 65.4\ cm^3$ **M1**

(c) absolute uncertainty = $65.4 \times 0.060 = 3.9\ cm^3$ (ft from b) **M1**

(c) $V = (65 \pm 4)\ cm^3$ (awrt, appropriate s.f.) **A1**

(c) Answer: $V = (65 \pm 4)\ cm^3$

Question 9

(a) a systematic error affects all readings by the same amount and in the same direction (consistently too high or too low), and is not reduced by repeating/averaging **B1**

(a) a random error causes readings to scatter unpredictably above and below the true value, and its effect can be reduced by repeating readings and averaging **B1**

(a) Answer: Systematic errors shift every reading consistently in the same direction and cannot be reduced by averaging; random errors scatter readings unpredictably and can be reduced by taking repeats and averaging.

(b) this is a systematic (zero) error **B1**

(b) the student should subtract 0.20 N from every subsequent reading (or re-zero/calibrate the instrument before use) **B1**

(b) Answer: It is a systematic (zero) error; the student should subtract 0.20 N from every reading taken (or zero the instrument before use).

(c) the absolute uncertainty from human reaction time (e.g. approximately ± 0.2 s) when starting/stopping the stopwatch is essentially the same whichever quantity is timed **M1**

(c) timing 20 oscillations gives a total time about 20 times larger than timing 1 oscillation, while the absolute uncertainty stays roughly fixed **M1**

(c) so the percentage uncertainty (absolute uncertainty / measured value) is reduced by a factor of about 20 compared with timing a single oscillation **A1**

(c) Answer: The fixed reaction-time uncertainty becomes a much smaller fraction of the (about 20 times larger) total time, so the percentage uncertainty in the period is roughly 20 times smaller than if a single oscillation were timed.

Question 10

(a) titre = $27.65 - 2.40 = 25.25\ cm^3$ **M1**

(a) for a difference of two readings, absolute uncertainties are added: uncertainty = $0.05 + 0.05 = 0.10\ cm^3$ **M1**

(a) titre = $25.25 \pm 0.10\ cm^3$ (cao) **A1**

(a) Answer: $25.25 \pm 0.10\ cm^3$

(b) percentage uncertainty = (0.10 / 25.25) x 100 **M1**

(b) 0.40% (awrt, ft from a) **A1**

(b) Answer: 0.40%

(c) temperature rise = 84.0 - 19.5 = 64.5°C **M1**

(c) for a difference of two readings, absolute uncertainties are added: uncertainty = 0.5 + 0.5 = 1.0°C **M1**

(c) temperature rise = 64.5 ± 1.0°C (cao) **A1**

(c) Answer: 64.5 ± 1.0°C

Question 11

(a) percentage uncertainty in V = (0.1/6.0) x 100 = 1.7% (awrt) **B1**

(a) percentage uncertainty in R = (0.3/15.0) x 100 = 2.0% **B1**

(a) Answer: V: 1.7%; R: 2.0%

(b) since V is squared, its percentage uncertainty doubles: %unc(P) = (2 x 1.67%) + 2.0% = 5.3% (ft from a) **M1**

(b) $P = V^2/R = 6.0^2/15.0 = 2.4$ W **M1**

(b) absolute uncertainty = 2.4 x 0.053 = 0.13, so P = (2.4 ± 0.1) W (ft, awrt) **A1**

(b) Answer: P = (2.4 ± 0.1) W

(c) percentage uncertainty in m = (0.005/0.250) x 100 = 2.0% **B1**

(c) percentage uncertainty in k = (0.5/18.0) x 100 = 2.8% (awrt) **B1**

(c) Answer: m: 2.0%; k: 2.8%

(d) for a square root, the percentage uncertainty is multiplied by 1/2: %unc(T) = 0.5 x (2.0% + 2.8%) = 2.4% (ft from c) **M1**

(d) $T = 2 \times \pi \times \sqrt{0.250/18.0} = 0.740$ s **M1**

(d) absolute uncertainty = 0.740 x 0.024 = 0.018, so T = (0.740 ± 0.018) s (ft, awrt) **A1**

(d) Answer: T = (0.740 ± 0.018) s

Question 12

(a) uncertainty = half the difference between the steepest and least steep gradients = (26.0 - 23.0)/2 = 1.5 N/m **M1**

(a) percentage uncertainty = (1.5/24.5) x 100 **M1**

(a) 6.1% (awrt) **A1**

(a) Answer: 6.1%

(b) central value stated as 24.5 N/m **B1**

(b) uncertainty of 1.5 N/m quoted to the same precision (1 d.p.) as the central value: 24.5 ± 1.5 N/m **B1**

(b) Answer: 24.5 ± 1.5 N/m

(c) each error bar shows the range of uncertainty in the measured value of that data point **B1**

(c) the steepest and least steep straight lines that still pass through every error bar are drawn on the graph, and the gradient of each is calculated **B1**

(c) the uncertainty in the gradient is half the difference between these two gradients **B1**

(c) Answer: Error bars show the uncertainty range on each point; drawing the steepest and shallowest lines that still pass through all the error bars, and taking half the difference between their gradients, gives the uncertainty in the gradient.

(d) using many data points averages out the effect of random error/uncertainty in any one reading oe **B1**

(d) any anomalous point is more easily identified/its effect is reduced, giving a more representative/reliable value of k than relying on a single pair of readings **B1**

(d) Answer: The gradient uses many data points, so random errors and any single anomalous reading have much less effect, giving a more reliable value of k than a single F-x pair.

Question 13

(a) T = (time for 20 oscillations) / 20 = 37.0/20 **M1**

(a) 1.85 s (cso, answer given) **A1**

(a) Answer: T = 1.85 s

(b) percentage uncertainty in total time = (0.4/37.0) x 100 = 1.1% **M1**

(b) dividing by the constant 20 to find T does not change the percentage uncertainty **M1**

(b) percentage uncertainty in T = 1.1% (ft) **A1**

(b) Answer: 1.1%

(c) 2.2% (ft from b, doubled for the squared term) **B1**

(c) Answer: 2.2%

(d) for a product/quotient, percentage uncertainties add: $\%unc(g) = \%unc(l) + \%unc(T^2)$ **M1**

(d) 2.4% (awrt, ft from c) **A1**

(d) Answer: 2.4%

(e) correct substitution: $g = 4 \times \pi^2 \times 0.850 / (1.85)^2$ **M1**

(e) g = 9.80 m/s^2 (awrt) **M1**

(e) absolute uncertainty = 9.80 x 0.024 = 0.24 m/s^2 (ft from d) **M1**

(e) g = (9.80 ± 0.24) m/s^2 (awrt, appropriate s.f.; accept (9.8 ± 0.2) m/s^2) **A1**

(e) Answer: g = (9.80 ± 0.24) m/s^2

(f) the range given by the result is 9.80 - 0.24 = 9.56 m/s^2 to 9.80 + 0.24 = 10.04 m/s^2 (ft from e) **B1**

(f) since 9.81 m/s^2 lies within this range, the result is consistent with the accepted value **B1**

(f) Answer: Yes; 9.81 m/s^2 lies within the range 9.56 m/s^2 to 10.04 m/s^2, so the result is consistent with the accepted value.

Question 14

Level 3 A full, coherent evaluation that calculates or clearly compares the percentage uncertainty in the extension for both methods, correctly identifies extension as the measurement that dominates the overall percentage uncertainty in the Young modulus for both students, and reaches a justified conclusion (with a valid supporting reason such as the use of a reference wire) that Student B's method is more precise. **5-6**

Level 2 The answer compares the resolutions or uncertainties of the two measuring instruments and reaches a conclusion about which method is more precise, but the reasoning is incomplete, lacks a numerical comparison of percentage uncertainty, or omits reference to the small size of the extension being the limiting factor. **3-4**

Level 1 A basic statement identifying one method as better (e.g. Searle's apparatus is more precise) with little or no supporting justification. **1-2**

Level 0 No relevant content, or the answer does not relate to the question. **0**

Indicative content:

- The extension itself is small (about 2 mm) in both experiments, so the absolute uncertainty in measuring the extension is the critical factor in the overall percentage uncertainty of the Young modulus.
- Student A's metre rule has a resolution of 1 mm, giving an absolute uncertainty of about 0.5 mm on an extension of about 2 mm, a percentage uncertainty of roughly 25%.
- Student B's vernier/micrometer scale has a resolution of 0.01 mm, giving an absolute uncertainty of about 0.005-0.01 mm on a similar extension, a percentage uncertainty of only around 0.5%.
- Because the extension has by far the largest percentage uncertainty of any quantity in the Young modulus calculation, it dominates the overall percentage uncertainty in E for both students, but far more severely for Student A.
- Searle's apparatus also uses a reference wire under the same conditions (temperature, sag) as the test wire, reducing systematic errors that would otherwise arise from thermal expansion or the wire settling under load.
- Conclusion: Student B's method (Searle's apparatus) gives a more precise value of the Young modulus, because its much finer resolution greatly reduces the percentage uncertainty in the extension, the limiting measurement in this experiment.

Question 15

(a) fill the displacement can with water until it overflows from the side spout, and allow the dripping to stop before starting **B1**

(a) place an empty measuring cylinder under the spout, then lower the stone gently into the can on a thread until fully submerged, without touching the sides or base **B1**

(a) the volume of water collected in the measuring cylinder equals the volume of the stone **B1**

(a) Answer: Fill the eureka can to the spout and let it stop dripping; place a measuring cylinder under the spout; lower the stone in on a thread until fully submerged; the volume of water collected equals the stone's volume.

(b) radius = diameter/2 = 0.75 cm **M1**

(b) correct substitution and evaluation: $V = \pi \times (0.75)^2 \times 4.00$ **M1**

(b) 7.07 cm^3 (awrt, cao) **A1**

(b) Answer: V = 7.07 cm^3

(c) density = mass/volume = 62.3/7.07 = 8.81 g/cm^3 (ft from b) **M1**

(c) converts g/cm^3 to kg/m^3 by multiplying by 1000 **M1**

(c) 8810 kg/m^3 (awrt, accept 8800-8820 kg/m^3) **A1**

(c) Answer: 8810 kg/m^3

(d) percentage uncertainty in diameter = (0.02/1.50) x 100 = 1.33% **M1**

(d) percentage uncertainty in height = (0.05/4.00) x 100 = 1.25% **M1**

(d) since V depends on d^2 x h, percentage uncertainty is doubled for the diameter term: %unc(V) = (2 x 1.33%) + 1.25% **M1**

(d) 3.9% (awrt) **A1**

(d) Answer: 3.9%

(e) percentage uncertainty in mass = (0.1/40.2) x 100 = 0.25% **M1**

(e) percentage uncertainty in volume = (0.5/15.0) x 100 = 3.3% **M1**

(e) percentage uncertainty in density = 0.25% + 3.3% = 3.6% (ft) **A1**

(e) the volume measurement contributes the greater uncertainty, since 3.3% is much larger than 0.25% **B1**

(e) Answer: Percentage uncertainty in density = 3.6%; the volume (displacement) measurement contributes the greater uncertainty.

Question 16

Level 3 Identifies a plausible systematic weakness that would consistently bias every calculated velocity in the same direction (e.g. the 400 mm band separation being over-measured, for example through parallax when the bands were positioned or measured), links this clearly to why the effect is systematic (present in every reading) rather than random (each ball timed only once, so no repeats to reveal random scatter), proposes a specific, workable improved procedure (e.g. re-measuring the band separation carefully at eye level with a calibrated ruler/set square, and/or using light gates to time the interval), and gives a clear, correct justification of how the improvement reduces the systematic uncertainty. **5-6**

Level 2 Identifies a plausible systematic weakness in the procedure and proposes at least one sensible improvement, but the explanation of why the error is systematic (rather than random) is incomplete, or the justification of how the improvement reduces uncertainty is only partially developed. **3-4**

Level 1 Makes a basic, generally correct observation about the procedure (e.g. that a single hand-timed reading per ball is unreliable, or that the distance/timing could be measured more precisely) with little or no justification linking it to a systematic (rather than random) source of error. **1-2**

Indicative content:

- Because the same small percentage overestimate appears in every one of the five results (for different ball sizes), the source must be systematic and common to every measurement, not a random timing fluctuation.
- A likely systematic weakness: the 400 mm separation between the two rubber bands may have been measured or positioned inaccurately (e.g. through parallax when marking or measuring the bands, or using a stretched/non-taut tape measure), making the true separation smaller than 400 mm; dividing an overestimated distance by the correct time gives a velocity that is systematically too high for every ball.
- Timing the ball only once for each diameter means there is no way to identify or average out random variation in that particular reading, but this alone would not explain a consistent bias in one direction.
- Human reaction time when starting and stopping the stopwatch by eye introduces a delay at both the start and the end of the timed interval; if these delays are similar, their effects tend to cancel rather than produce a large, consistent, one-directional bias.
- Improvement: remeasure and verify the separation of the bands using a rigid ruler or vernier scale held vertically against the cylinder at eye level (avoiding parallax), removing the systematic overestimate in distance.
- Improvement: replace hand-timing with a pair of light gates positioned exactly at the bands, connected to an electronic timer; this removes human reaction-time error from the timing altogether and allows repeat drops for each ball size so a mean and its uncertainty can be found.
- Justification: correcting the measured distance removes a bias that affects every calculated velocity in the same direction (a genuine systematic correction), while light gates and repeats address the remaining random uncertainty from timing, together giving a more accurate and more precise set of results.

Question 17

(a) converts to SI units and substitutes: v = 0.0500/0.0244 **M1**

(a) v = 2.05 m/s (awrt, cao) **A1**

(a) Answer: v = 2.05 m/s

(b) percentage uncertainty in d = (0.5/50.0) x 100 = 1.0% **M1**

(b) percentage uncertainty in t = (0.3/24.4) x 100 = 1.2% (awrt) **M1**

(b) for a quotient, percentage uncertainties add: %unc(v) = 1.0 + 1.2 = 2.2% (ft) **A1**

(b) Answer: 2.2%

(c) percentage uncertainty in m = (0.002/0.842) x 100 = 0.24%; since v is squared, its percentage uncertainty doubles to 2 x 2.23% = 4.5% (ft from b) **M1**

(c) for a product, percentage uncertainties add: %unc(Ek) = 0.24 + 4.5 = 4.7% (ft) **A1**

(c) Answer: 4.7%

(d) Ek = 0.5 x 0.842 x $(2.05)^2$ **M1**

(d) Ek = 1.77 J (awrt, ft from a) **M1**

(d) absolute uncertainty = 1.77 x 0.047 = 0.083, so Ek = (1.77 ± 0.08) J (ft, awrt) **A1**

(d) Answer: Ek = (1.77 ± 0.08) J

Question 18

(a) F = mg = 1.200 x 9.81 **M1**

(a) F = 11.772 N = 11.8 N (3 s.f.) (cso, answer given) **A1**

(a) Answer: F = 11.8 N

(b) converts diameter to metres: d = 3.2 x 10^{-4} m **M1**

(b) correct substitution and evaluation: A = π x $(3.2 \times 10^{-4})^2$ / 4 = 8.04 x 10^{-8} m^2 **M1**

(b) 8.0 x 10^{-8} m^2 (awrt, to 2 s.f., matching the precision of d) **A1**

(b) Answer: A = 8.0 x 10^{-8} m^2

(c) percentage uncertainty in F = (0.001/1.200) x 100 = 0.083%, and in L = (0.002/2.000) x 100 = 0.10% **M1**

(c) percentage uncertainty in d = (0.005/0.32) x 100 = 1.56%, so (since A depends on d^2) percentage uncertainty in A = 2 x 1.56% = 3.1% **M1**

(c) percentage uncertainty in e = (0.05/1.40) x 100 = 3.6% **M1**

(c) for a product/quotient of several quantities percentage uncertainties add: %unc(E) = 0.083 + 0.10 + 3.1 + 3.6 = 6.9% (ft) **A1**

(c) Answer: 6.9%

(d) correct substitution: E = (11.772 x 2.000) / (8.04 x 10^{-8} x 1.40 x 10^{-3}) **M1**

(d) E = 2.09 x 10^{11} Pa (209 GPa) (awrt, ft from b) **M1**

(d) absolute uncertainty = 2.09 x 10^{11} x 0.069 = 1.4 x 10^{10} Pa, so E = (2.09 ± 0.14) x 10^{11} Pa (ft, awrt) **A1**

(d) Answer: E = (2.09 ± 0.14) x 10^{11} Pa

Question 19

(a) 44.50 mm is anomalous, since it is much larger than (and inconsistent with) the other four readings, which cluster closely around 40.0 mm **B1**

(a) a possible cause, e.g. the calipers were misread (a slipped decimal/scale misread), the jaws were not closed correctly on the cube, or the measurement was taken across a corner/diagonal rather than a face of the cube oe **B1**

(a) Answer: 44.50 mm is the anomalous reading; a likely cause is a misread scale or the calipers being incorrectly positioned across the cube (e.g. along a diagonal rather than a face).

(b) mean = (40.00 + 40.08 + 39.96 + 40.04) / 4 **M1**

(b) 40.02 mm (cao) **A1**

(b) Answer: 40.02 mm

(c) uncertainty = half the range = (40.08 - 39.96)/2 = 0.06 mm **M1**

(c) percentage uncertainty = (0.06/40.02) x 100 = 0.15% (awrt, ft from b) **A1**

(c) Answer: 0.15%

(d) converts the mean side length to metres and cubes it: V = $(4.002 \times 10^{-2})^3$ **M1**

(d) V = 6.41×10^{-5} m^3 (awrt, ft from b) **M1**

(d) since V depends on (side length)^3, percentage uncertainty in V = 3 x 0.15% = 0.45% (ft from c) **A1**

(d) Answer: V = 6.41×10^{-5} m^3, percentage uncertainty 0.45%

(e) percentage uncertainty in mass = (0.05/28.3) x 100 = 0.18% **M1**

(e) density = mass/volume = 0.0283/(6.41×10^{-5}) = 442 kg/m^3 (awrt, ft from d) **M1**

(e) for a quotient, percentage uncertainties add: percentage uncertainty in density = 0.18 + 0.45 = 0.63% (ft) **M1**

(e) absolute uncertainty = 442 x 0.0063 = 2.8, so density = (442 ± 3) kg/m^3 (ft, awrt) **A1**

(e) Answer: density = (442 ± 3) kg/m^3

(f) 442 kg/m^3 lies within the stated accepted range of 350-600 kg/m^3, so the student's result is accurate/consistent with the accepted value oe **B1**

(f) wood is a natural material, so its density varies between (and within) samples owing to factors such as moisture content, growth rate/ring spacing and the presence of knots or voids, which is why a range rather than a single value is quoted oe **B1**

(f) Answer: The result (442 kg/m^3) lies within the accepted range of 350-600 kg/m^3, so it is accurate; wood density naturally varies between samples (moisture content, growth conditions, knots), which is why a range rather than a single fixed value is quoted.