



AP1D Measurements and their Errors: Depth and Exam Drill

AQA 7408 · Calculator allowed · about 145 minutes

Total Marks

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Name: _____ Date: ____ / ____ / ____

Answer ALL questions. Show all your working.

- 1** SI units, prefixes and estimation are the basis of every physical quantity used in this course.
- (a) State the SI base unit of: (i) thermodynamic temperature, (ii) amount of substance, (iii) luminous intensity. (3)
 - (b) A telecommunications signal has a frequency of 2.4 GHz. Write this frequency in hertz, using standard form. (1)
 - (c) Estimate, to one significant figure, the mass of a single mosquito in kilograms. State the order of magnitude of your estimate. (2)

(Total for Question 1 is 6 marks)

- 2** Significant figures indicate how precisely a value is known; recognising them correctly, and rounding calculated results appropriately, is an essential exam skill.
- (a) State the number of significant figures in the measurement 0.070060 m. (1)
 - (b) A calculation gives a result of 143.7568 N, using two measured quantities that were each quoted to 3 significant figures. State this result to an appropriate number of significant figures. (1)
 - (c) A student measures a potential difference $V = 6.0$ V (2 significant figures) across a resistor and a current $I = 0.24$ A (2 significant figures) through it. Calculate the resistance $R = V / I$, giving your answer to an appropriate number of significant figures. (2)
 - (d) Explain the difference between the number of decimal places and the number of significant figures in a measured value, using 0.070060 m as your example. (2)

(Total for Question 2 is 6 marks)

3 Measurements are always subject to systematic and/or random error.

- (a) A top-pan balance is not zeroed before use and reads 0.05 g with nothing on the pan. Every subsequent mass measured with this balance is too high by the same 0.05 g. Identify the type of error this illustrates, and state how the student should correct the affected readings. (2)
- (b) A student reads the position of a pointer on an analogue ammeter scale from an angle, rather than looking directly along the pointer from directly above the scale. Name the type of error this introduces, and state one way to reduce it. (2)
- (c) The student re-zeros the balance from part (a) correctly, then measures the same mass 6 times, obtaining slightly different readings each time because of air currents disturbing the balance. Explain whether this variation is a systematic or a random error, and describe how the student could reduce its effect on the reported mean value. (3)

(Total for Question 3 is 7 marks)

4 Reading an instrument scale correctly is a fundamental practical skill.

- (a) A micrometer screw gauge has a resolution of 0.01 mm. For a particular reading, the main scale shows 5.5 mm and the rotating thimble scale, on which each division represents 0.01 mm, is aligned with the line labelled 23. Calculate the diameter reading shown, and state the absolute uncertainty in this single reading. (3)
- (b) A vernier caliper has a resolution of 0.02 mm. For a particular reading, the main scale shows 12 mm and the 4th line on the vernier scale is the one that aligns exactly with a main scale line. Calculate the diameter reading shown, and state the absolute uncertainty in this single reading. (3)

(Total for Question 4 is 6 marks)

5 Uncertainties combine differently depending on whether quantities are added, subtracted, multiplied or divided.

- (a) In a titration, a burette of resolution 0.10 cm^3 is used. The initial reading is $2.40 \pm 0.05 \text{ cm}^3$ and the final reading is $27.65 \pm 0.05 \text{ cm}^3$ (the uncertainty in each reading is taken as half the resolution). Calculate the titre (volume delivered) and its absolute uncertainty. (3)
- (b) Calculate the percentage uncertainty in the titre found in part (a), giving your answer to 2 significant figures. (2)
- (c) A thermometer of resolution 1 degree C is used to record the temperature of a water bath before and after heating. The initial temperature is 19.5 ± 0.5 degrees C and the final temperature is 84.0 ± 0.5 degrees C (the uncertainty in each reading is taken as half the resolution). Calculate the temperature rise and its absolute uncertainty. (3)

(Total for Question 5 is 8 marks)

- 6** The percentage uncertainty in a calculated quantity depends on how that quantity is combined mathematically from its measured inputs: percentage uncertainties are added for a product or quotient, and multiplied by the power for a term raised to a power (including a square root, which multiplies the percentage uncertainty by 1/2).
- (a) A heating element has a potential difference $V = 6.0 \pm 0.1$ V across it and a resistance $R = 15.0 \pm 0.3$ ohm. Calculate the percentage uncertainty in V and the percentage uncertainty in R . (2)
 - (b) The power dissipated is given by $P = V^2 / R$. Calculate the percentage uncertainty in P , and hence state the power P with its absolute uncertainty, to an appropriate number of significant figures. (3)
 - (c) A mass $m = 0.250 \pm 0.005$ kg is attached to a spring of spring constant $k = 18.0 \pm 0.5$ N/m. Calculate the percentage uncertainty in m and the percentage uncertainty in k . (2)
 - (d) The period of oscillation is given by $T = 2 \times \pi \times \sqrt{m/k}$. Calculate the percentage uncertainty in T , and hence state the period T with its absolute uncertainty, to an appropriate number of significant figures. (3)

(Total for Question 6 is 10 marks)

- 7** Required practical: A student determines the Young modulus of a metal wire. A mass of 1.200 ± 0.001 kg is hung from a wire of original length $L = 2.000 \pm 0.002$ m and diameter $d = 0.32 \pm 0.005$ mm (measured with a micrometer). The resulting extension of the wire is $e = 1.40 \pm 0.05$ mm. Take $g = 9.81$ m/s² (treated as exact). The Young modulus is given by $E = F \times L / (A \times e)$, where F is the force applied and A is the cross-sectional area of the wire.
- (a) Show that the force applied to the wire is 11.8 N. (2)
 - (b) Calculate the cross-sectional area, A , of the wire, using $A = \pi \times d^2 / 4$ and $d = 0.32$ mm. Give your answer in m². (3)
 - (c) Calculate the percentage uncertainty in F , in L , in A (using your value from part b), and in e , and hence find the percentage uncertainty in the Young modulus E . (4)
 - (d) Calculate the Young modulus, E , and express it with its absolute uncertainty, to an appropriate number of significant figures. (3)

(Total for Question 7 is 12 marks)

- 8** Required practical: A student investigates how the pressure, P , of a fixed mass of gas at constant volume varies with temperature. The gas is held in a sealed rigid flask connected to a pressure sensor, and is heated in a water bath. The student records the following pressure at five temperatures: $\theta = 20.0$ degrees C, $P = 107$ kPa; $\theta = 40.0$ degrees C, $P = 114$ kPa; $\theta = 60.0$ degrees C, $P = 121$ kPa; $\theta = 80.0$ degrees C, $P = 128$ kPa; $\theta = 100.0$ degrees C, $P = 135$ kPa. A best-fit straight line through these points has a gradient of 0.350 kPa per degree C and a pressure-axis intercept of 100 kPa. The steepest and shallowest straight lines that can still be drawn through all the error bars on the data points have gradients of 0.380 kPa per degree C and 0.320 kPa per degree C, and pressure-axis intercepts of 92 kPa and 108 kPa, respectively.
- (a) Calculate the percentage uncertainty in the gradient of the best-fit line. (3)
 - (b) Using the same method, calculate the percentage uncertainty in the pressure-axis intercept of the best-fit line. (3)
 - (c) Using the gradient and intercept of the best-fit line, calculate the temperature, in degrees C, at which the extrapolated line predicts the pressure would fall to zero. This value is an experimental estimate of absolute zero. (3)
 - (d) The extrapolated temperature in part (c) is found from a quotient of the intercept and the gradient. Using your percentage uncertainties from parts (a) and (b), calculate the absolute uncertainty in this extrapolated temperature, and comment on whether the accepted value of absolute zero (-273 degrees C) is consistent with the student's result. (3)

(Total for Question 8 is 12 marks)

- 9** Required practical: A student determines the density of a small cube cut from a sample of pine wood. The side length of the cube is measured five times, at different points and orientations, using vernier calipers of resolution 0.02 mm. The readings obtained are: 40.00 mm, 40.08 mm, 39.96 mm, 44.50 mm, 40.04 mm. The mass of the cube, measured on an electronic balance of resolution 0.1 g, is 28.3 ± 0.05 g.
- (a) Identify the anomalous reading in the table of side-length measurements, and suggest one possible cause for it. (2)
 - (b) Calculate the mean side length of the cube, excluding the anomalous reading. (2)
 - (c) Estimate the absolute uncertainty in the mean side length using half the range of the four valid readings, and hence calculate the percentage uncertainty in the side length. (2)
 - (d) Calculate the volume of the cube in m^3 , and the percentage uncertainty in this volume. (3)
 - (e) Calculate the density of the cube in kg/m^3 , and express it with its absolute uncertainty, to an appropriate number of significant figures. (4)
 - (f) The accepted density range for this species of pine is 350 - 600 kg/m^3 . Comment on the accuracy of the student's result, and suggest why a wooden sample might reasonably have an accepted density range rather than a single fixed value. (2)

(Total for Question 9 is 15 marks)

- 10** A student determines the terminal velocity of steel ball bearings falling through a viscous oil held in a tall, wide measuring cylinder. Two rubber bands are placed around the outside of the cylinder to mark two points a measured 400 mm apart, positioned well below the point at which each ball reaches terminal velocity. For each of five different ball bearing diameters, the student drops one ball bearing into the oil near the centre of the cylinder, starts a hand-held stopwatch by eye as the ball passes the upper band, and stops it as the ball passes the lower band. Each ball is timed once only. The terminal velocity for each ball is then calculated as 400 mm divided by the measured time. The student's five calculated terminal velocities are all slightly, but consistently, higher than a set of accepted reference values for oil of this viscosity. Evaluate the procedure described for a systematic weakness that could explain why the calculated terminal velocities are consistently too high, propose an improved procedure, and justify how your improvement would reduce this systematic uncertainty.

(Total for Question 10 is 6 marks)

- 11** Synoptic question. A trolley of mass $m = 0.842 \pm 0.002$ kg (found using a top-pan balance) is released from rest on a horizontal, effectively frictionless air track and passes through a single light gate fitted with an opaque interrupt card of length $d = 50.0 \pm 0.5$ mm. The light gate's electronic timer records a time of $t = 24.4 \pm 0.3$ ms for the card to pass through the light beam. Assume the trolley's speed does not change significantly over this short interval, so its instantaneous speed at the light gate is $v = d / t$.

- (a) Calculate the speed, v , of the trolley at the light gate. (2)
- (b) Calculate the percentage uncertainty in v . (3)
- (c) The kinetic energy of the trolley is given by $E_k = \frac{1}{2} m v^2$. Calculate the percentage uncertainty in E_k , explaining how the power in the formula affects your calculation. (2)
- (d) Calculate the kinetic energy, E_k , of the trolley and express it with its absolute uncertainty, to an appropriate number of significant figures. (3)

(Total for Question 11 is 10 marks)

Mark scheme · AP1D Measurements and their Errors: Depth and Exam Drill

Question 1

- (a) **B1** kelvin, K
- (a) **B1** mole, mol
- (a) **B1** candela, cd
- (a) **Answer: (i) kelvin (K); (ii) mole (mol); (iii) candela (cd).**
- (b) **B1** 2.4×10^9 Hz (cao)
- (b) **Answer: 2.4×10^9 Hz**
- (c) **B1** any sensible estimate in the range 1×10^{-6} kg to 5×10^{-6} kg
- (c) **B1** correctly states the order of magnitude of their estimate (e.g. 10^{-6} kg) oe
- (c) **Answer: Approximately 2×10^{-6} kg; order of magnitude 10^{-6} kg (any value from 1×10^{-6} kg to 5×10^{-6} kg with a correctly stated matching order of magnitude is acceptable).**

Question 2

- (a) **B1** 5 (significant figures) cao
- (a) **Answer: 5 significant figures**
- (b) **B1** 144 N (cao)
- (b) **Answer: 144 N**
- (c) **M1** $R = 6.0 / 0.24 = 25.0$ ohm
- (c) **A1** 25 ohm, quoted to 2 s.f. to match the precision of the input data (cao)
- (c) **Answer: 25 ohm (2 s.f.)**
- (d) **B1** 0.070060 m has 6 decimal places (digits after the decimal point) but only 5 significant figures, because the leading zeros are not significant
- (d) **B1** decimal places depend on how the number is written/the unit used, whereas significant figures reflect the actual precision of the measurement regardless of the unit oe
- (d) **Answer: 0.070060 m has 6 decimal places but only 5 significant figures; decimal places depend on how the value is written (and its unit), while significant figures reflect the true precision of the measurement.**

Question 3

- (a) **B1** systematic (zero) error
- (a) **B1** subtract 0.05 g from every reading taken with the uncorrected balance (or re-zero the balance before use) oe
- (a) **Answer: This is a systematic (zero) error; the student should subtract 0.05 g from every reading taken (or zero the balance before use).**
- (b) **B1** parallax error
- (b) **B1** view the scale directly from above/in line with the pointer (e.g. using a mirror behind the scale to check the pointer and its reflection align) oe
- (b) **Answer: This is a parallax error; it can be reduced by viewing the scale directly from above the pointer, for example using a mirror scale to check the pointer and its reflection coincide.**
- (c) **B1** random error, because the readings vary unpredictably above and below the true value rather than being consistently offset in one direction
- (c) **B1** the air currents cause small, unpredictable fluctuations in each individual reading
- (c) **B1** repeating the measurement many times and calculating the mean reduces the effect of this random variation on the reported value oe
- (c) **Answer: This is a random error, since the air currents cause the readings to scatter unpredictably above and below the true value rather than shifting them consistently in one direction; taking many repeat readings and calculating the mean reduces its effect on the reported mass.**

Question 4

- (a) **M1** thimble contribution = $23 \times 0.01 \text{ mm} = 0.23 \text{ mm}$
- (a) **A1** diameter = $5.5 + 0.23 = 5.73 \text{ mm}$ (cao)
- (a) **B1** uncertainty = $\pm 0.005 \text{ mm}$ (half the resolution)
- **(a) Answer: $5.73 \pm 0.005 \text{ mm}$**
- (b) **M1** vernier contribution = $4 \times 0.02 \text{ mm} = 0.08 \text{ mm}$
- (b) **A1** diameter = $12 + 0.08 = 12.08 \text{ mm}$ (cao)
- (b) **B1** uncertainty = $\pm 0.01 \text{ mm}$ (half the resolution)
- **(b) Answer: $12.08 \pm 0.01 \text{ mm}$**

Question 5

- (a) **M1** titre = $27.65 - 2.40 = 25.25 \text{ cm}^3$
- (a) **M1** for a difference of two readings, absolute uncertainties are added: uncertainty = $0.05 + 0.05 = 0.10 \text{ cm}^3$
- (a) **A1** titre = $25.25 \pm 0.10 \text{ cm}^3$ (cao)
- **(a) Answer: $25.25 \pm 0.10 \text{ cm}^3$**
- (b) **M1** percentage uncertainty = $(0.10 / 25.25) \times 100$
- (b) **A1** 0.40% (awrt, ft from a)
- **(b) Answer: 0.40%**
- (c) **M1** temperature rise = $84.0 - 19.5 = 64.5 \text{ degrees C}$
- (c) **M1** for a difference of two readings, absolute uncertainties are added: uncertainty = $0.5 + 0.5 = 1.0 \text{ degrees C}$
- (c) **A1** temperature rise = $64.5 \pm 1.0 \text{ degrees C}$ (cao)
- **(c) Answer: $64.5 \pm 1.0 \text{ degrees C}$**

Question 6

- (a) **B1** percentage uncertainty in $V = (0.1/6.0) \times 100 = 1.7\%$ (awrt)
- (a) **B1** percentage uncertainty in $R = (0.3/15.0) \times 100 = 2.0\%$
- **(a) Answer: V: 1.7%; R: 2.0%**
- (b) **M1** since V is squared, its percentage uncertainty doubles: $\%unc(P) = (2 \times 1.67\%) + 2.0\% = 5.3\%$ (ft from a)
- (b) **M1** $P = V^2/R = 6.0^2/15.0 = 2.4 \text{ W}$
- (b) **A1** absolute uncertainty = $2.4 \times 0.053 = 0.13$, so $P = (2.4 \pm 0.1) \text{ W}$ (ft, awrt)
- **(b) Answer: $P = (2.4 \pm 0.1) \text{ W}$**
- (c) **B1** percentage uncertainty in $m = (0.005/0.250) \times 100 = 2.0\%$
- (c) **B1** percentage uncertainty in $k = (0.5/18.0) \times 100 = 2.8\%$ (awrt)
- **(c) Answer: m: 2.0%; k: 2.8%**
- (d) **M1** for a square root, the percentage uncertainty is multiplied by 1/2: $\%unc(T) = 0.5 \times (2.0\% + 2.8\%) = 2.4\%$ (ft from c)
- (d) **M1** $T = 2 \times \pi \times \sqrt{0.250/18.0} = 0.740 \text{ s}$
- (d) **A1** absolute uncertainty = $0.740 \times 0.024 = 0.018$, so $T = (0.740 \pm 0.018) \text{ s}$ (ft, awrt)
- **(d) Answer: $T = (0.740 \pm 0.018) \text{ s}$**

Question 7

- (a) **M1** $F = mg = 1.200 \times 9.81$
- (a) **A1** $F = 11.772 \text{ N} = 11.8 \text{ N}$ (3 s.f.) (cso, answer given)
- (a) **Answer: $F = 11.8 \text{ N}$**
- (b) **M1** converts diameter to metres: $d = 3.2 \times 10^{-4} \text{ m}$
- (b) **M1** correct substitution and evaluation: $A = \pi \times (3.2 \times 10^{-4})^2 / 4 = 8.04 \times 10^{-8} \text{ m}^2$
- (b) **A1** $8.0 \times 10^{-8} \text{ m}^2$ (awrt, to 2 s.f., matching the precision of d)
- (b) **Answer: $A = 8.0 \times 10^{-8} \text{ m}^2$**
- (c) **M1** percentage uncertainty in $F = (0.001/1.200) \times 100 = 0.083\%$, and in $L = (0.002/2.000) \times 100 = 0.10\%$
- (c) **M1** percentage uncertainty in $d = (0.005/0.32) \times 100 = 1.56\%$, so (since A depends on d^2) percentage uncertainty in $A = 2 \times 1.56\% = 3.1\%$
- (c) **M1** percentage uncertainty in $e = (0.05/1.40) \times 100 = 3.6\%$
- (c) **A1** for a product/quotient of several quantities percentage uncertainties add: $\%unc(E) = 0.083 + 0.10 + 3.1 + 3.6 = 6.9\%$ (ft)
- (c) **Answer: 6.9%**
- (d) **M1** correct substitution: $E = (11.772 \times 2.000) / (8.04 \times 10^{-8} \times 1.40 \times 10^{-3})$
- (d) **M1** $E = 2.09 \times 10^{11} \text{ Pa}$ (209 GPa) (awrt, ft from b)
- (d) **A1** absolute uncertainty $= 2.09 \times 10^{11} \times 0.069 = 1.4 \times 10^{10} \text{ Pa}$, so $E = (2.09 \pm 0.14) \times 10^{11} \text{ Pa}$ (ft, awrt)
- (d) **Answer: $E = (2.09 \pm 0.14) \times 10^{11} \text{ Pa}$**

Question 8

- (a) **M1** uncertainty in gradient = half the difference between the steepest and shallowest gradients $= (0.380 - 0.320)/2 = 0.030 \text{ kPa per degree C}$
- (a) **M1** percentage uncertainty $= (0.030/0.350) \times 100$
- (a) **A1** 8.6% (awrt)
- (a) **Answer: 8.6%**
- (b) **M1** uncertainty in intercept = half the difference between the two extreme intercepts $= (108 - 92)/2 = 8 \text{ kPa}$
- (b) **M1** percentage uncertainty $= (8/100) \times 100$
- (b) **A1** 8.0% (cao)
- (b) **Answer: 8.0%**
- (c) **M1** rearranges $P = \text{intercept} + \text{gradient} \times \theta$ with $P = 0$ to give $\theta = -\text{intercept}/\text{gradient}$
- (c) **M1** substitutes $\theta = -100/0.350$
- (c) **A1** -286 degrees C (awrt, accept -285 to -286)
- (c) **Answer: -286 degrees C**
- (d) **M1** for a quotient, percentage uncertainties add: $\%unc(\theta) = \%unc(\text{intercept}) + \%unc(\text{gradient}) = 8.0 + 8.6 = 16.6\%$ (ft)
- (d) **M1** absolute uncertainty $= 286 \times (16.6/100) = 47 \text{ degrees C}$ (ft), giving a range of roughly -333 degrees C to -239 degrees C
- (d) **A1** the accepted value of -273 degrees C lies within this range, so the result is consistent with the accepted value, although the large uncertainty reflects how imprecise it is to extrapolate a short section of line so far beyond the measured data
- (d) **Answer: Uncertainty is approximately $\pm 47 \text{ degrees C}$, giving a range of about -333 degrees C to -239 degrees C ; since -273 degrees C lies within this range, the result is consistent with the accepted value.**

Question 9

- (a) **B1** 44.50 mm is anomalous, since it is much larger than (and inconsistent with) the other four readings, which cluster closely around 40.0 mm
- (a) **B1** a possible cause, e.g. the calipers were misread (a slipped decimal/scale misread), the jaws were not closed correctly on the cube, or the measurement was taken across a corner/diagonal rather than a face of the cube oe
- (a) **Answer: 44.50 mm is the anomalous reading; a likely cause is a misread scale or the calipers being incorrectly positioned across the cube (e.g. along a diagonal rather than a face).**
- (b) **M1** mean = $(40.00 + 40.08 + 39.96 + 40.04) / 4$
- (b) **A1** 40.02 mm (cao)
- (b) **Answer: 40.02 mm**
- (c) **M1** uncertainty = half the range = $(40.08 - 39.96) / 2 = 0.06$ mm
- (c) **A1** percentage uncertainty = $(0.06 / 40.02) \times 100 = 0.15\%$ (awrt, ft from b)
- (c) **Answer: 0.15%**
- (d) **M1** converts the mean side length to metres and cubes it: $V = (4.002 \times 10^{-2})^3$
- (d) **M1** $V = 6.41 \times 10^{-5} \text{ m}^3$ (awrt, ft from b)
- (d) **A1** since V depends on (side length)³, percentage uncertainty in $V = 3 \times 0.15\% = 0.45\%$ (ft from c)
- (d) **Answer: $V = 6.41 \times 10^{-5} \text{ m}^3$, percentage uncertainty 0.45%**
- (e) **M1** percentage uncertainty in mass = $(0.05 / 28.3) \times 100 = 0.18\%$
- (e) **M1** density = mass/volume = $0.0283 / (6.41 \times 10^{-5}) = 442 \text{ kg/m}^3$ (awrt, ft from d)
- (e) **M1** for a quotient, percentage uncertainties add: percentage uncertainty in density = $0.18 + 0.45 = 0.63\%$ (ft)
- (e) **A1** absolute uncertainty = $442 \times 0.0063 = 2.8$, so density = $(442 \pm 3) \text{ kg/m}^3$ (ft, awrt)
- (e) **Answer: density = $(442 \pm 3) \text{ kg/m}^3$**
- (f) **B1** 442 kg/m^3 lies within the stated accepted range of 350-600 kg/m^3 , so the student's result is accurate/consistent with the accepted value oe
- (f) **B1** wood is a natural material, so its density varies between (and within) samples owing to factors such as moisture content, growth rate/ring spacing and the presence of knots or voids, which is why a range rather than a single value is quoted oe
- (f) **Answer: The result (442 kg/m^3) lies within the accepted range of 350-600 kg/m^3 , so it is accurate; wood density naturally varies between samples (moisture content, growth conditions, knots), which is why a range rather than a single fixed value is quoted.**

Question 10

- **Level 3** (5-6): Identifies a plausible systematic weakness that would consistently bias every calculated velocity in the same direction (e.g. the 400 mm band separation being over-measured, for example through parallax when the bands were positioned or measured), links this clearly to why the effect is systematic (present in every reading) rather than random (each ball timed only once, so no repeats to reveal random scatter), proposes a specific, workable improved procedure (e.g. re-measuring the band separation carefully at eye level with a calibrated ruler/set square, and/or using light gates to time the interval), and gives a clear, correct justification of how the improvement reduces the systematic uncertainty.
- **Level 2** (3-4): Identifies a plausible systematic weakness in the procedure and proposes at least one sensible improvement, but the explanation of why the error is systematic (rather than random) is incomplete, or the justification of how the improvement reduces uncertainty is only partially developed.
- **Level 1** (1-2): Makes a basic, generally correct observation about the procedure (e.g. that a single hand-timed reading per ball is unreliable, or that the distance/timing could be measured more precisely) with little or no justification linking it to a systematic (rather than random) source of error.
- **Indicative content:**
 - Because the same small percentage overestimate appears in every one of the five results (for different ball sizes), the source must be systematic and common to every measurement, not a random timing fluctuation.
 - A likely systematic weakness: the 400 mm separation between the two rubber bands may have been measured or positioned inaccurately (e.g. through parallax when marking or measuring the bands, or using a stretched/non-taut tape measure), making the true separation smaller than 400 mm; dividing an overestimated distance by the correct time gives a velocity that is systematically too high for every ball.
 - Timing the ball only once for each diameter means there is no way to identify or average out random variation in that particular reading, but this alone would not explain a consistent bias in one direction.
 - Human reaction time when starting and stopping the stopwatch by eye introduces a delay at both the start and the end of the timed interval; if these delays are similar, their effects tend to cancel rather than produce a large, consistent, one-directional bias.
 - Improvement: remeasure and verify the separation of the bands using a rigid ruler or vernier scale held vertically against the cylinder at eye level (avoiding parallax), removing the systematic overestimate in distance.
 - Improvement: replace hand-timing with a pair of light gates positioned exactly at the bands, connected to an electronic timer; this removes human reaction-time error from the timing altogether and allows repeat drops for each ball size so a mean and its uncertainty can be found.
 - Justification: correcting the measured distance removes a bias that affects every calculated velocity in the same direction (a genuine systematic correction), while light gates and repeats address the remaining random uncertainty from timing, together giving a more accurate and more precise set of results.

Question 11

- (a) **M1** converts to SI units and substitutes: $v = 0.0500/0.0244$
- (a) **A1** $v = 2.05 \text{ m/s}$ (awrt, cao)
- (a) **Answer: $v = 2.05 \text{ m/s}$**
- (b) **M1** percentage uncertainty in $d = (0.5/50.0) \times 100 = 1.0\%$
- (b) **M1** percentage uncertainty in $t = (0.3/24.4) \times 100 = 1.2\%$ (awrt)
- (b) **A1** for a quotient, percentage uncertainties add: $\%unc(v) = 1.0 + 1.2 = 2.2\%$ (ft)
- (b) **Answer: 2.2%**
- (c) **M1** percentage uncertainty in $m = (0.002/0.842) \times 100 = 0.24\%$; since v is squared, its percentage uncertainty doubles to $2 \times 2.23\% = 4.5\%$ (ft from b)
- (c) **A1** for a product, percentage uncertainties add: $\%unc(E_k) = 0.24 + 4.5 = 4.7\%$ (ft)
- (c) **Answer: 4.7%**
- (d) **M1** $E_k = 0.5 \times 0.842 \times (2.05)^2$
- (d) **M1** $E_k = 1.77 \text{ J}$ (awrt, ft from a)
- (d) **A1** absolute uncertainty $= 1.77 \times 0.047 = 0.083$, so $E_k = (1.77 \pm 0.08) \text{ J}$ (ft, awrt)
- (d) **Answer: $E_k = (1.77 \pm 0.08) \text{ J}$**