AP4D Mechanics and Materials: Depth and Exam Drill

AQA 7408 · Calculator allowed · about 125 minutes

Total Marks

Name: ______________________________ Date: ____ / ____ / ______

Answer ALL questions. Show all your working.

1 This question tests core definitions in mechanics.

(a) State Newton's second law of motion in terms of the rate of change of momentum. **(1)**

(b) Define the moment of a force about a point. **(1)**

(c) State the principle of moments. **(1)**

(Total for Question 1 is 3 marks)

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2 A ball is kicked from level ground with an initial speed of 18 m/s at an angle of 40 degrees above the horizontal. Air resistance is negligible. Take $g = 9.81$ m/s^2.

(a) Calculate the horizontal and vertical components of the initial velocity. **(2)**

(b) Calculate the maximum height reached by the ball above the ground. **(2)**

(c) Calculate the total time of flight before the ball returns to the ground. **(1)**

(d) Calculate the horizontal range of the ball. **(2)**

(Total for Question 2 is 7 marks)

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3 A ball of mass 0.16 kg travelling at 12 m/s strikes a wall at right angles and rebounds at 9.0 m/s. The ball is in contact with the wall for 0.050 s.

(a) Calculate the change in momentum of the ball during the collision, taking the direction of the initial velocity as positive. **(2)**

(b) Calculate the magnitude of the average force exerted by the wall on the ball. **(3)**

(Total for Question 3 is 5 marks)

4 REQUIRED PRACTICAL. A student investigates the force-extension behaviour of a metal wire of length 1.80 m and diameter 0.30 mm, using masses to apply tension and a travelling microscope to measure extension. The results, all within the elastic region, are given below.
Force (N): 10, 20, 30, 40, 50
Extension (mm): 1.27, 2.55, 3.82, 5.09, 6.37

(a) State what physical quantity is represented by the gradient of a force-extension graph in the elastic region. **(1)**

(b) Use the data for the largest force to calculate the gradient of the force-extension graph. **(3)**

(c) Calculate the cross-sectional area of the wire. **(2)**

(d) Calculate the Young modulus of the wire material, using E = (gradient x L) / A. **(3)**

(e) Calculate the elastic strain energy stored in the wire when the applied force is 40 N. **(2)**

(Total for Question 4 is 11 marks)

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5 A conveyor belt in a warehouse carries crates of mass 40 kg up a ramp inclined at 15 degrees to the horizontal. The ramp is 6.0 m long and a constant friction force of 45 N opposes the motion of each crate. Crates are loaded onto the belt at a rate of 3 per minute and the belt moves at constant velocity. The motor driving the belt has an efficiency of 78%. Take g = 9.81 m/s^2.

(a) Calculate the vertical height gained by a crate travelling the full length of the ramp. **(2)**

(b) Calculate the total work done on one crate against gravity and against friction as it travels up the full length of the ramp. **(3)**

(c) Given that crates are loaded at a rate of 3 per minute, calculate the useful mechanical power output of the belt. **(3)**

(d) Given that the motor is 78% efficient, calculate the electrical power that must be supplied to the motor. **(2)**

(Total for Question 5 is 10 marks)

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6 Two ice skaters, initially at rest and in contact, push apart on frictionless ice. Skater A has mass 55 kg and skater B has mass 75 kg. After the push, skater A moves off at 3.2 m/s.

(a) Explain, with reference to Newton's third law, why the total momentum of the two-skater system remains zero after the push. **(2)**

(b) Calculate the speed at which skater B moves off. **(3)**

(c) Calculate the total kinetic energy generated during the push-off. **(3)**

(Total for Question 6 is 8 marks)

7 A student determines the density of an irregularly shaped metal alloy sample. Its mass is measured on a balance as 245.6 ± 0.1 g, and its volume is found by water displacement as 31.5 ± 0.5 cm^3.

(a) Calculate the density of the alloy in kg/m^3. **(3)**

(b) Calculate the percentage uncertainty in the calculated density. **(3)**

(c) State which measurement, the mass or the volume, limits the overall precision of the density value, and explain why. **(2)**

(Total for Question 7 is 8 marks)

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8 A block is placed on a ramp whose angle of incline can be increased. For the first surface, the block just begins to slide when the ramp angle reaches 22 degrees. A second block, on a different, rougher surface, just begins to slide at an angle of 31 degrees.

(a) By resolving the weight of the block into components parallel and perpendicular to the ramp surface, and considering the forces at the point of slipping, show that the coefficient of static friction is given by $\mu = \tan(\theta_c)$, where θ_c is the angle at which sliding just begins. **(3)**

(b) Calculate the coefficient of static friction for the first surface. **(2)**

(c) Calculate the coefficient of static friction for the second surface, and state which of the two surfaces provides more grip. **(2)**

(Total for Question 8 is 7 marks)

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9 A uniform horizontal beam AB of length 4.0 m rests on supports at each end, A and B. The beam itself has a weight of 60 N acting at its midpoint, and an additional load of 90 N is placed on the beam 1.0 m from end A. The beam remains in equilibrium.

(a) State the condition, in terms of moments, that must be satisfied for the beam to be in equilibrium. **(1)**

(b) By taking moments about end A, calculate the reaction force at support B. **(3)**

(c) Using vertical equilibrium, calculate the reaction force at support A. **(2)**

(Total for Question 9 is 6 marks)

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10 Car manufacturers design the front of a car to include a crumple zone: a section of the body structure that deforms plastically during a collision, absorbing energy, rather than remaining rigid. Evaluate how principles of work and energy, and the mechanical properties of the materials used, both contribute to reducing the risk of injury to occupants in a frontal collision. In your answer, refer to the work-energy relationship for bringing the car to rest, and to how the area under a force-extension (or stress-strain) graph for the crumple zone material represents energy absorbed through plastic deformation.

(Total for Question 10 is 6 marks)

Mark scheme · AP4D Mechanics and Materials: Depth and Exam Drill

Question 1

- (a) **B1** the resultant force acting on an object is equal to (proportional to) the rate of change of its momentum oe
- **(a) Answer: The resultant force on an object equals the rate of change of its momentum.**
- (b) **B1** the product of the force and the perpendicular distance from the point (pivot/line of action) to the point oe
- **(b) Answer: The moment of a force about a point is the product of the force and the perpendicular distance from the point to the line of action of the force.**
- (c) **B1** for a system in equilibrium, the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about that same point oe
- **(c) Answer: For a body in equilibrium, the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about that point.**

Question 2

- (a) **M1** vx = 18 cos(40) = 13.8 m/s
- (a) **A1** vy = 18 sin(40) = 11.6 m/s cao
- **(a) Answer: vx = 13.8 m/s, vy = 11.6 m/s**
- (b) **M1** use $v^2 = u^2 - 2gh$ with u = vy, v = 0
- (b) **A1** h = 6.82 m cao
- **(b) Answer: 6.82 m**
- (c) **B1** t = 2 vy / g = 2 x 11.57 / 9.81 = 2.36 s cao
- **(c) Answer: 2.36 s**
- (d) **M1** use range = vx x t
- (d) **A1** range = 32.5 m cao
- **(d) Answer: 32.5 m**

Question 3

- (a) **M1** change in momentum = m(v - u) = 0.16 x (-9.0 - 12)
- (a) **A1** = -3.36 kg m/s (magnitude 3.36 kg m/s) cao
- **(a) Answer: -3.36 kg m/s (magnitude 3.4 kg m/s)**
- (b) **M1** quote impulse = change in momentum = F x t
- (b) **M1** substitute F = 3.36 / 0.050
- (b) **A1** F = 67.2 N cao
- **(b) Answer: 67.2 N**

Question 4

- (a) **B1** the stiffness (spring constant, k) of the wire oe
- **(a) Answer: The stiffness (spring constant) of the wire.**
- (b) **M1** gradient = F / x = 50 / (6.37 x 10^{-3})
- (b) **M1** convert extension to metres before dividing
- (b) **A1** gradient = 7850 N/m (accept 7840-7860) cao
- **(b) Answer: 7850 N/m**
- (c) **M1** use A = π $(d/2)^2$ with d = 0.30 x 10^{-3} m
- (c) **A1** A = 7.07 x 10^{-8} m^2 cao
- **(c) Answer: 7.07 x 10^{-8} m^2**
- (d) **M1** substitute E = (7850 x 1.80) / (7.07 x 10^{-8})
- (d) **M1** carry out the calculation
- (d) **A1** E = 2.0 x 10^{11} Pa cao
- **(d) Answer: 2.0 x 10^{11} Pa**
- (e) **M1** use elastic strain energy = 1/2 F x, with x = 5.09 x 10^{-3} m
- (e) **A1** energy = 0.102 J cao
- **(e) Answer: 0.102 J**

Question 5

- (a) **M1** h = 6.0 x sin(15)
- (a) **A1** h = 1.55 m cao
- **(a) Answer: 1.55 m**
- (b) **M1** work against gravity = mgh = 40 x 9.81 x 1.55
- (b) **M1** work against friction = F x d = 45 x 6.0 = 270 J
- (b) **A1** total work = 879 J (609 + 270) cao
- **(b) Answer: 879 J**
- (c) **M1** time per crate = 60/3 = 20 s
- (c) **M1** power = work per crate / time per crate = 879.4 / 20
- (c) **A1** power = 44.0 W cao
- **(c) Answer: 44.0 W**
- (d) **M1** input power = output power / efficiency = 44.0 / 0.78
- (d) **A1** input power = 56.4 W cao
- **(d) Answer: 56.4 W**

Question 6

- (a) **B1** by Newton's third law the force exerted by A on B is equal and opposite to the force exerted by B on A, and they act for the same time
- (a) **B1** so the impulses (changes in momentum) of A and B are equal and opposite, meaning the total momentum change of the system is zero and momentum is conserved (no external horizontal forces act) oe
- **(a) Answer: The push forces on A and B form a Newton's third law pair, equal in magnitude and opposite in direction, acting for the same time, so the impulses (and hence momentum changes) of A and B are equal and opposite; with no external horizontal force, total momentum stays at zero.**
- (b) **M1** use conservation of momentum: mA vA = mB vB (momenta equal and opposite, total zero)
- (b) **M1** substitute vB = (55 x 3.2) / 75
- (b) **A1** vB = 2.3 m/s cao
- **(b) Answer: 2.3 m/s**
- (c) **M1** KE(A) = 0.5 x 55 x 3.2^2 = 282 J
- (c) **M1** KE(B) = 0.5 x 75 x 2.347^2 = 207 J
- (c) **A1** total KE = 488 J cao
- **(c) Answer: 488 J**

Question 7

- (a) **M1** density = mass/volume = 245.6/31.5 = 7.80 g/cm^3
- (a) **M1** convert to kg/m^3 by multiplying by 1000
- (a) **A1** density = 7.80 x 10^3 kg/m^3 cao
- **(a) Answer: 7.80 x 10^3 kg/m^3**
- (b) **M1** percentage uncertainty in mass = 0.1/245.6 x 100 = 0.041%
- (b) **M1** percentage uncertainty in volume = 0.5/31.5 x 100 = 1.59%
- (b) **A1** total percentage uncertainty = 1.63% (sum of the two) cao
- **(b) Answer: 1.6%**
- (c) **B1** the volume measurement limits the precision
- (c) **B1** because its percentage uncertainty (about 1.6%) is far larger than that of the mass measurement (about 0.04%), so it dominates the combined uncertainty oe
- **(c) Answer: The volume measurement limits the precision, because its percentage uncertainty (about 1.6%) is much larger than the mass measurement's percentage uncertainty (about 0.04%), so it dominates the overall uncertainty in density.**

Question 8

- (a) **M1** at the point of slipping, the maximum friction force equals the component of weight parallel to the ramp: F(friction, max) = mg $\sin(\theta_c)$
- (a) **M1** the normal reaction force equals the component of weight perpendicular to the ramp: N = mg $\cos(\theta_c)$
- (a) **A1** since F(friction, max) = μ N, dividing gives μ = mg $\sin(\theta_c)$ / (mg $\cos(\theta_c)$) = $\tan(\theta_c)$
- **(a) Answer: μ = $\tan(\theta_c)$**
- (b) **M1** μ = tan(22 degrees)
- (b) **A1** μ = 0.40 cao
- **(b) Answer: 0.40**
- (c) **M1** μ = tan(31 degrees) = 0.60
- (c) **A1** the second surface provides more grip, since it has the larger coefficient of friction cao
- **(c) Answer: μ = 0.60; the second surface provides more grip.**

Question 9

- (a) **B1** the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about that point oe
- **(a) Answer: The sum of clockwise moments about any point equals the sum of anticlockwise moments about that point.**
- (b) **M1** set up moment equation about A: R(B) x 4.0 = 60 x 2.0 + 90 x 1.0
- (b) **M1** solve: R(B) x 4.0 = 120 + 90 = 210
- (b) **A1** R(B) = 52.5 N cao
- **(b) Answer: 52.5 N**
- (c) **M1** R(A) + R(B) = total downward force = 60 + 90 = 150 N
- (c) **A1** R(A) = 150 - 52.5 = 97.5 N cao
- **(c) Answer: 97.5 N**

Question 10

- **Level 3** (5-6): Clear, well-linked evaluation that correctly uses the work-energy relationship (work done to stop the car = initial kinetic energy = average force x stopping distance) to explain that increasing the stopping distance (via crumple zone deformation) reduces the average force and deceleration experienced by occupants, AND explains that the crumple zone material is chosen to deform plastically beyond its elastic limit, absorbing a large amount of energy (area under the force-extension/stress-strain graph, including the permanent plastic deformation region) rather than transmitting force elastically to the passenger cell. Reaches a justified conclusion connecting both mechanics (energy/force) and materials (plastic behaviour) principles.
- **Level 2** (3-4): Correctly explains one of the two principles (either the work-energy/stopping-distance argument, or the plastic deformation/energy-absorption argument) in reasonable detail, with only partial or underdeveloped treatment of the other.
- **Level 1** (1-2): Basic answer that mentions crumple zones absorb energy or reduce force, without correctly applying the work-energy relationship or explaining the role of plastic deformation in the material.
- **Indicative content:**
  - Work-energy principle: the kinetic energy of the car must be removed by the work done by the (average) resistive force over the stopping distance, W = F x d = initial KE.
  - For a fixed initial kinetic energy, increasing the stopping distance d (by allowing the front of the car to crumple over some distance) reduces the average force F needed to stop the car, and hence reduces the deceleration and force transmitted to occupants.
  - A rigid (non-deforming) car structure would stop over a much shorter distance, producing a much larger average force and deceleration, increasing injury risk.
  - Materials principle: the crumple zone is made from a material designed to deform plastically (beyond its elastic limit) during the collision, rather than behave purely elastically.
  - The energy absorbed by the material as it deforms is represented by the area under its force-extension (or stress-strain) graph; permanent (plastic) deformation absorbs much more energy than purely elastic deformation for a given force, converting kinetic energy into other forms (heat, sound, permanent structural change) rather than elastic strain energy that could otherwise be released back into the passenger cell.
  - Overall: the combination of a longer controlled stopping distance (mechanics) and a material that absorbs energy through plastic deformation (materials) both act to reduce the force and deceleration experienced by occupants, improving safety.