3.6 Further Mechanics and Thermal Physics

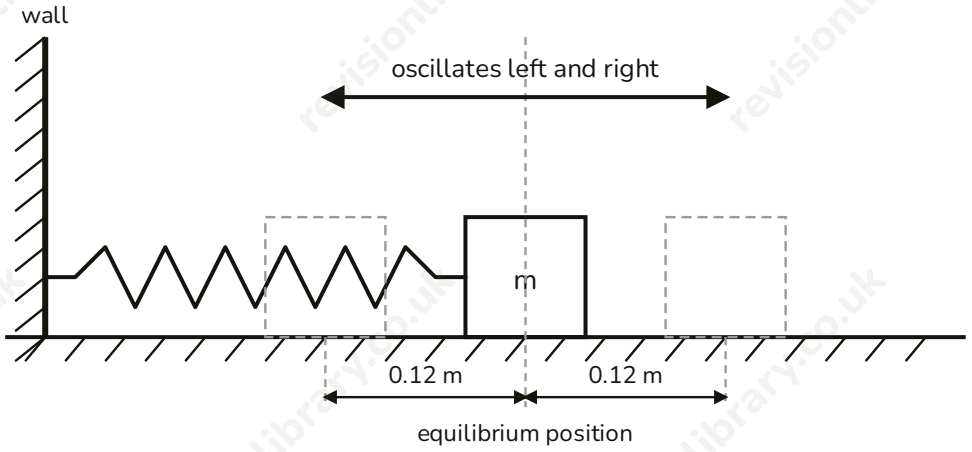
AQA 7408 · Calculator allowed · about 205 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A mass attached to a spring oscillates with simple harmonic motion (SHM) of amplitude 0.12 m and angular frequency $\omega = 8.0$ rad/s. Use the Physics Equations Sheet where needed ($v_{max} = \omega \times A$ and $a_{max} = \omega^2 \times A$).



(a) State two conditions that must both be satisfied for a system to be undergoing simple harmonic motion. **(2)**

(b) Using $v_{max} = \omega \times A$ and $a_{max} = \omega^2 \times A$, calculate the maximum speed and the maximum acceleration of the mass. **(4)**

(c) Using $T = 2\pi/\omega$ and $T = 2\pi \times \sqrt{m/k}$, calculate the period of oscillation, and hence the spring constant k of the spring if the mass is 0.25 kg. **(3)**

(Total for Question 1 is 9 marks)

2 An ice cube of mass 25 g (0.025 kg) at 0°C is dropped into a drink. The specific latent heat of fusion of ice is 3.34×10^5 J/kg.

(a) State what is meant by the specific latent heat of fusion of a substance. **(1)**

(b) Calculate the energy needed to completely melt the ice cube, and hence calculate the minimum power of a heater that would melt this mass of ice in 2.0 minutes, assuming all of the heater's output goes into melting the ice. **(4)**

(c) Explain, in terms of the behaviour of molecules, why the temperature of the ice-water mixture remains constant while the ice is melting, even though energy is continuously being supplied. **(2)**

(Total for Question 2 is 7 marks)

3 A fixed mass of an ideal gas occupies a volume of 2.40×10^{-3} m^3 at a pressure of 1.05×10^5 Pa and a temperature of 290 K. Use the Physics Equations Sheet where needed ($pV = nRT$, with $R = 8.31$ J/(mol K); $N = n \times N_A$, with $N_A = 6.02 \times 10^{23}$ /mol).

(a) State Boyle's law. **(1)**

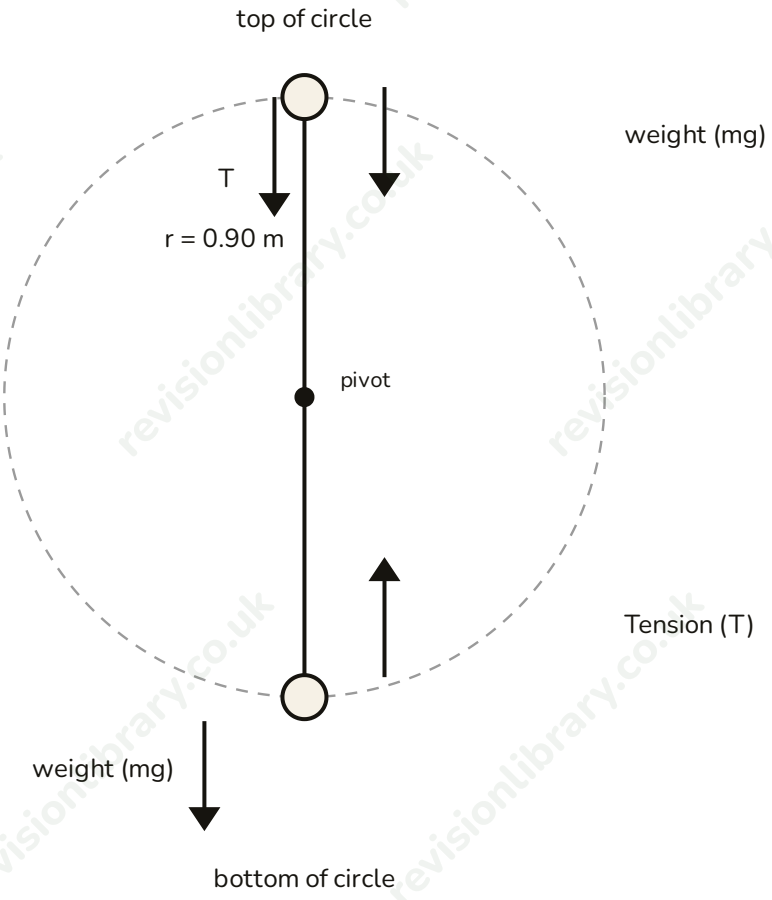
(b) The gas is compressed to a volume of 1.60×10^{-3} m^3 and heated to a temperature of 340 K. Using $pV = nRT$, calculate the new pressure of the gas. **(4)**

(c) Using the initial conditions ($p = 1.05 \times 10^5$ Pa, $V = 2.40 \times 10^{-3}$ m^3, $T = 290$ K), calculate the number of moles of gas present, and hence the number of gas molecules. **(3)**

(Total for Question 3 is 8 marks)

4 A bucket of water of mass 0.60 kg is swung in a vertical circle of radius 0.90 m at the end of a light rope. Take g = 9.81 m/s^2. Use the Physics Equations Sheet where needed.

Bucket swung in a vertical circle



(a) State the condition, in terms of the tension in the rope, for the water to just remain in the bucket as it passes the top of the circle. **(1)**

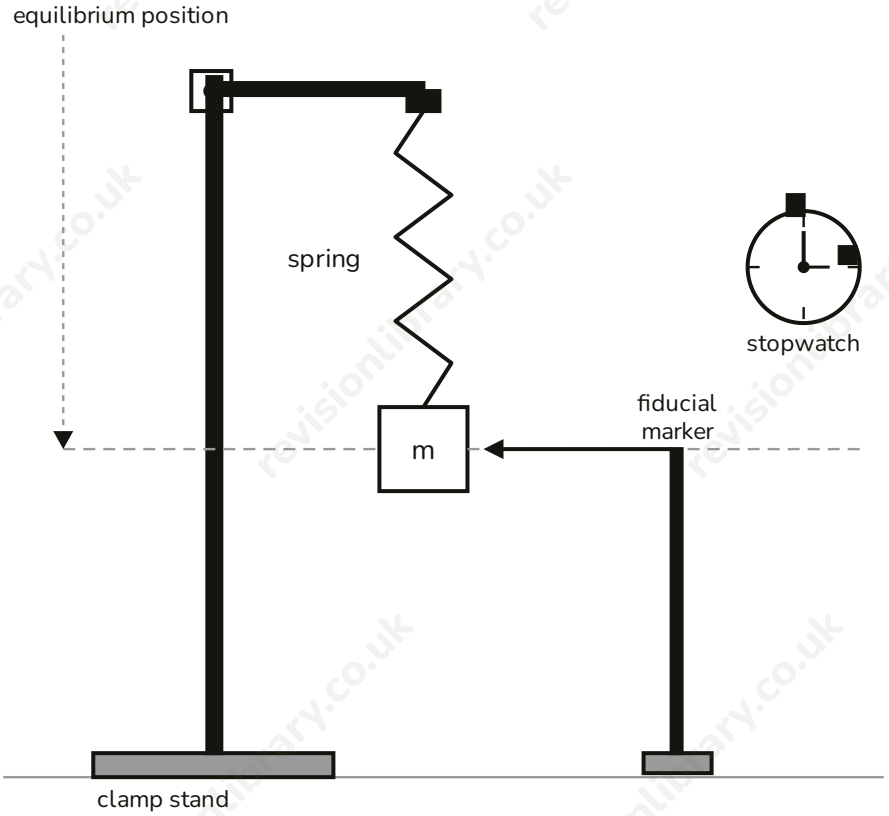
(b) Show that the minimum speed of the bucket at the top of the circle, for the water to just stay in, is about 3.0 m/s. **(3)**

(c) The bucket passes the bottom of the circle with speed 4.5 m/s. Calculate the tension in the rope at this point. **(4)**

(d) State and explain the effect on the tension at the bottom of the circle if the bucket were instead swung faster. **(2)**

(Total for Question 4 is 10 marks)

5 REQUIRED PRACTICAL. A student carries out the required practical investigation into simple harmonic motion, using a mass-spring system to determine the spring constant k from the relationship $T^2 = (4pi^2/k) \times m$, where T is the period of oscillation and m is the mass attached to the spring.



(a) Describe how the student should measure the period of oscillation of the mass-spring system accurately, including how random error in each timing is reduced. **(3)**

(b) The student obtains the following processed data for a graph of T^2 (s^2) against m (kg): at m = 0.10 kg, T^2 = 0.247 s^2; at m = 0.50 kg, T^2 = 1.234 s^2. Using these two points, calculate the gradient of the graph of T^2 against m, and hence determine the spring constant k. **(5)**

(c) Explain how the student could reduce the percentage uncertainty in the final value obtained for k. **(2)**

(Total for Question 5 is 10 marks)

6 A mass of 0.25 kg oscillates with SHM with amplitude 0.12 m and angular frequency $\omega = 8.0$ rad/s (the same system as in question 1). Use the Physics Equations Sheet where needed (total energy $E = 1/2 \times m \times \omega^2 \times A^2$, and $v = \omega \times \sqrt{A^2 - x^2}$).

(a) Calculate the total energy of the oscillation. **(3)**

(b) Using $v = \omega \times \sqrt{A^2 - x^2}$, calculate the speed of the mass, and hence its kinetic energy, when its displacement from equilibrium is $x = 0.06$ m. **(4)**

(c) Without further calculation of v, state the potential energy of the system at this displacement, and comment on how the total energy is shared between kinetic and potential energy as the mass moves from the equilibrium position to maximum displacement. **(3)**

(Total for Question 6 is 10 marks)

7 A Ferris wheel cabin of mass 380 kg moves at constant speed in a vertical circle of radius 22 m, completing one revolution every 40 s. Take $g = 9.81\ m/s^2$.

(a) Calculate the angular velocity of the cabin. **(2)**

(b) Calculate the linear speed of the cabin. **(2)**

(c) Calculate the centripetal acceleration of the cabin. **(2)**

(d) Calculate the force exerted by the wheel structure on the cabin at the bottom and at the top of the circle. **(4)**

(Total for Question 7 is 10 marks)

8 A 0.150 kg block of ice, initially at -8.0°C, is heated steadily until it becomes water at 20°C. Specific heat capacity of ice = 2100 J/(kg °C); specific latent heat of fusion of ice = 334,000 J/kg; specific heat capacity of water = 4200 J/(kg °C).

(a) Calculate the energy required to warm the ice from -8.0°C to 0°C. **(2)**

(b) Calculate the energy required to melt the ice at 0°C. **(2)**

(c) Calculate the energy required to warm the resulting water from 0°C to 20°C. **(2)**

(d) Calculate the total energy required for the whole process, and state one assumption made in this calculation. **(3)**

(Total for Question 8 is 9 marks)

9 A container of volume 0.020 m^3 holds 5.4 x 10^{24} molecules of nitrogen gas at a pressure of 2.0 x 10^5 Pa. Each nitrogen molecule has a mass of 4.65 x 10^{-26} kg. Use the Physics Equations Sheet where needed ($pV = (1/3) \times N \times m \times \langle c^2 \rangle$; average molecular kinetic energy $E_k = (3/2) \times k \times T$, with the Boltzmann constant $k = 1.38 \times 10^{-23}$ J/K).

(a) State two assumptions of the kinetic theory model of an ideal gas. **(2)**

(b) Using $pV = (1/3) \times N \times m \times \langle c^2 \rangle$, calculate the root-mean-square speed of the nitrogen molecules. **(4)**

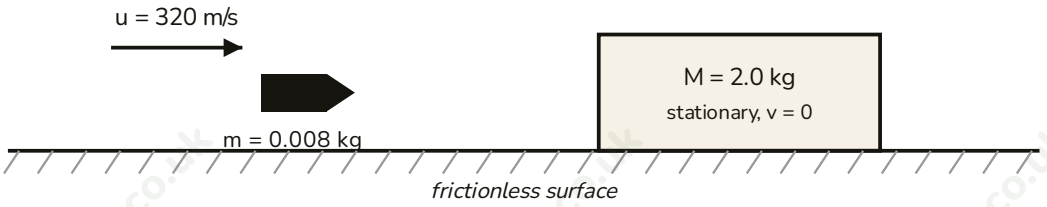
(c) Using $E_k = (3/2) \times k \times T$, calculate the average translational kinetic energy of a gas molecule at a temperature of 300 K. **(3)**

(d) A second container at the same temperature holds hydrogen gas instead of nitrogen. Without further calculation, explain which gas has the higher root-mean-square speed, and why. **(2)**

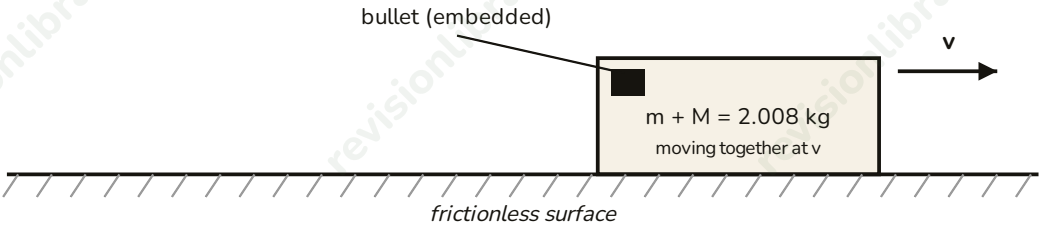
(Total for Question 9 is 11 marks)

10 A bullet of mass 8.0 g (0.008 kg) travelling horizontally at 320 m/s embeds itself in a stationary wooden block of mass 2.0 kg that is free to move. Use the Physics Equations Sheet where needed.

Before impact



After impact (bullet embedded in block)



(a) Calculate the common velocity of the bullet and block immediately after the bullet embeds itself in the block. **(3)**

(b) Calculate the kinetic energy of the bullet immediately before the impact, and the kinetic energy of the bullet-block system immediately after the impact. Hence calculate the loss of kinetic energy during the collision. **(3)**

(c) Assume the lost kinetic energy is converted into internal (thermal) energy shared between the bullet and the region of wood it embeds in, with half of this energy absorbed by the bullet. The specific heat capacity of the bullet material is 130 J/(kg K). Estimate the rise in temperature of the bullet. **(3)**

(Total for Question 10 is 9 marks)

11 A footbridge over a river in a town centre is found to sway noticeably when large groups of people cross it walking in step. Engineers are considering two possible modifications: (i) fitting mechanical dampers to the structure, or (ii) redesigning the bridge to change its natural frequency of vibration. Discuss, using your knowledge of forced oscillations, resonance and damping, how each approach would help to reduce unwanted large-amplitude oscillations of the bridge, and evaluate which approach you consider to be more effective.

(Total for Question 11 is 6 marks)

12 A conical pendulum consists of a mass of 0.25 kg on the end of a string of length 0.80 m. The mass moves in a horizontal circle with the string making a constant angle of 25 degrees with the vertical. Take $g = 9.81\ m/s^2$.

(a) Calculate the radius of the circular path. **(2)**

(b) By resolving forces on the mass vertically and horizontally, calculate the tension in the string and the speed of the mass. **(5)**

(c) Calculate the period of the motion. **(2)**

(Total for Question 12 is 9 marks)

13 REQUIRED PRACTICAL. A student determines g using a simple pendulum. For each length, the time for 20 complete oscillations is measured and divided by 20 to give the period T. The results are given below.

L (m): 0.40, 0.60, 0.80, 1.00

T (s): 1.27, 1.55, 1.79, 2.01

(a) Explain why the student times 20 oscillations and divides by 20, rather than timing a single oscillation. **(2)**

(b) Calculate T^2 for the length L = 1.00 m. **(2)**

(c) Using the data for L = 0.40 m and L = 1.00 m, calculate the gradient of a graph of T^2 (y-axis) against L (x-axis). **(3)**

(d) Given that $T^2 = (4 \pi^2 / g) L$, use your gradient to calculate the experimental value of g. **(2)**

(e) Suggest one reason why this experimental value of g differs from the accepted value of 9.81 m/s^2. **(2)**

(Total for Question 13 is 11 marks)

14 A mass of 0.40 kg is attached to a horizontal spring of spring constant 25 N/m and oscillates with simple harmonic motion of amplitude 0.15 m.

(a) Calculate the angular frequency of the oscillation. **(2)**

(b) Calculate the maximum speed of the mass during the oscillation. **(2)**

(c) Calculate the maximum acceleration of the mass. **(2)**

(d) Calculate the total energy of the oscillation. **(2)**

(e) Calculate the speed of the mass when its displacement from equilibrium is 0.080 m. **(3)**

(Total for Question 14 is 11 marks)

15 A vehicle suspension system and a footbridge both involve oscillating systems that may be damped to differing degrees.

(a) Describe, using the terms light damping, critical damping and heavy (over) damping, how the displacement of a displaced oscillating system varies with time in each case. **(3)**

(b) State one practical application where critical damping is desirable, and explain why. **(2)**

(c) Explain, in terms of energy transfer, why the amplitude of oscillation of a lightly damped system becomes very large when the frequency of an external driving force is equal to the system's natural frequency. **(3)**

(Total for Question 15 is 8 marks)

16 The kinetic theory of gases models gas molecules as identical hard, elastic spheres in continuous random motion. It assumes the molecules have negligible volume compared with the volume of their container, that intermolecular forces are negligible except during collisions, that collisions between molecules (and with the container walls) are perfectly elastic, and that the pressure exerted on the container walls arises from the change in momentum of molecules colliding with the walls (linking directly to Newton's second and third laws). Evaluate the assumptions of this kinetic theory model, explaining how each assumption relates to the macroscopic behaviour described by the ideal gas equation, and discuss the physical conditions (of pressure and temperature) under which a real gas is most likely to deviate significantly from ideal gas behaviour.

(Total for Question 16 is 6 marks)

Mark scheme · 3.6 Further Mechanics and Thermal Physics

Question 1

(a) the acceleration (or restoring force) is proportional to the displacement from equilibrium, oe **B1**

(a) the acceleration (or restoring force) is always directed towards the equilibrium position (opposite in direction to the displacement), oe **B1**

(a) Answer: Acceleration is proportional to displacement from equilibrium, and is always directed towards the equilibrium position ($a = -\omega^2 x$).

(b) correct substitution into $v_{max} = \omega \times A$ **M1**

(b) $v_{max} = 0.96$ m/s, cao **A1**

(b) correct substitution into $a_{max} = \omega^2 \times A$ **M1**

(b) $a_{max} = 7.68$ m/s^2, cao **A1**

(b) Answer: $v_{max} = 0.96$ m/s; $a_{max} = 7.68$ m/s^2

(c) $T = 2\pi/\omega = 2\pi/8.0 = 0.785$ s (awrt) **M1**

(c) rearrangement of $T = 2\pi \times \sqrt{m/k}$ to make k the subject (e.g. $k = 4\pi^2 \times m/T^2$, or equivalently $k = m \times \omega^2$), with correct substitution **M1**

(c) k = 16.0 N/m, cao **A1**

(c) Answer: T = 0.785 s; k = 16.0 N/m

Question 2

(a) the energy required to change the state of 1 kg of a substance from solid to liquid, without any change in temperature, oe **B1**

(a) Answer: The energy needed to melt 1 kg of a substance (solid to liquid) at constant temperature.

(b) correct substitution into Q = mL, using mass in kg **M1**

(b) Q = 8350 J, cao **A1**

(b) P = Q/t, using t = 2.0 x 60 = 120 s, ft **M1**

(b) P = 69.6 W (awrt), cao **A1**

(b) Answer: Q = 8350 J; P = 69.6 W (3 s.f.)

(c) the energy supplied does work against the intermolecular forces holding molecules in the fixed lattice structure, increasing their potential energy, rather than increasing their kinetic energy, oe **B1**

(c) since temperature is a measure of the average kinetic energy of the molecules, and this does not change during melting, the temperature remains constant until all the ice has melted, oe **B1**

(c) Answer: Energy supplied breaks intermolecular bonds (increasing potential energy) rather than increasing molecular kinetic energy, so since temperature depends on average kinetic energy, it stays constant until melting is complete.

Question 3

(a) for a fixed mass of gas at constant temperature, the pressure is inversely proportional to the volume (pV = constant), oe **B1**

(a) Answer: For a fixed mass of gas at constant temperature, pressure is inversely proportional to volume (pV = constant).

(b) recognises that n and R are constant, so $p1V1/T1 = p2V2/T2$, oe **M1**

(b) correct substitution of all given values **M1**

(b) correctly rearranged to make p2 the subject **M1**

(b) $p2 = 1.85 \times 10^5$ Pa (awrt), cao **A1**

(b) Answer: 1.85×10^5 Pa (3 s.f.)

(c) correct substitution into n = pV/RT **M1**

(c) n = 0.105 mol (awrt) **A1**

(c) $N = n \times N_A = 6.30 \times 10^{22}$ molecules (awrt), ft **A1**

(c) Answer: n = 0.105 mol; $N = 6.30 \times 10^{22}$ molecules

Question 4

(a) at the limiting condition, the contact force of the bucket on the water is zero, so weight alone provides the water's centripetal force; in the ideal common-radius model the rope tension is also zero, but it acts on the bucket and is distinct from the contact force on the water, oe **B1**

(a) Answer: Tension = 0 N (weight alone provides the centripetal force)

(b) at minimum speed, $mg = mv^2/r$ (tension = 0), oe **M1**

(b) rearranged to $v = \sqrt{g \times r}$ **M1**

(b) cso: $v = \sqrt{9.81 \times 0.90} = 2.97$ m/s, rounding to about 3.0 m/s as given **A1**

(b) Answer: v = 2.97 m/s (approx 3.0 m/s, as given)

(c) at the bottom, $T - mg = mv^2/r$, oe **M1**

(c) centripetal term calculated: $mv^2/r = 0.60 \times 4.5^2/0.90 = 13.5$ N **M1**

(c) weight term calculated: $mg = 0.60 \times 9.81 = 5.89$ N (awrt), and added to centripetal term **M1**

(c) T = 19.4 N (awrt), cao **A1**

(c) Answer: 19.4 N (3 s.f.)

(d) the tension would be greater (increase), oe **B1**

(d) a higher speed requires a greater centripetal force ($F = mv^2/r$ increases with v^2), and this extra force must be provided by additional tension (beyond that supporting the weight), oe **B1**

(d) Answer: Tension increases, because a greater speed requires a greater centripetal force, which the rope must supply in addition to supporting the weight of the bucket.

Question 5

(a) use a fiducial marker (fixed reference point) at the equilibrium position, viewed at eye level, to judge when the mass passes through the same point each time and avoid parallax error, oe **B1**

(a) time a large number of oscillations (e.g. 20) rather than one, then divide the total time by the number of oscillations, to reduce the percentage/random error in the period, oe **B1**

(a) repeat each timing (e.g. three times) and calculate a mean value of T, oe **B1**

(a) Answer: Use a fiducial marker to define one full oscillation, time 20 oscillations rather than one and divide by 20, and repeat and average the timings.

(b) gradient = (change in T^2)/(change in m), using the two given data points **M1**

(b) correct substitution: gradient = (1.234 - 0.247)/(0.50 - 0.10) **M1**

(b) gradient = 2.47 s^2/kg (awrt) **A1**

(b) gradient = $4pi^2$/k rearranged to k = $4pi^2$/gradient, with correct substitution, ft **M1**

(b) k = 16.0 N/m (awrt), cao **A1**

(b) Answer: gradient = 2.47 s^2/kg (approx); k = 16.0 N/m

(c) use a wider range of masses (larger spread of m values), which increases the range spanned by the data and reduces the percentage uncertainty in the gradient, oe **B1**

(c) take measurements at more values of m across the range and draw a line of best fit, rather than using only two points, to average out random error, oe **B1**

(c) Answer: Use a larger range of masses and take more data points, drawing a line of best fit rather than joining just two points, to reduce the percentage uncertainty in the gradient.

Question 6

(a) correct substitution into E = 1/2 x m x ω^2 x A^2 **M1**

(a) correct arithmetic combination, e.g. 0.5 x 0.25 x 64 = 8.0 **M1**

(a) E = 0.115 J (awrt), cao **A1**

(a) Answer: 0.115 J (3 s.f.)

(b) correct substitution into $v = \omega \times \sqrt{A^2 - x^2}$ **M1**

(b) v = 0.831 m/s (awrt), cao **A1**

(b) KE = 1/2 x m x v^2 substitution, ft from v **M1**

(b) KE = 0.0864 J (awrt), cao **A1**

(b) Answer: v = 0.831 m/s; KE = 0.0864 J

(c) PE = E_total - KE = 0.1152 - 0.0864 = 0.0288 J (awrt), ft from (a) and (b) **B1**

(c) KE is maximum at the equilibrium position (x = 0) and decreases to zero at maximum displacement (x = A); PE does the opposite, oe **B1**

(c) the total energy (KE + PE) remains constant throughout the oscillation, oe **B1**

(c) Answer: PE = 0.0288 J; KE and PE are continuously exchanged as the mass oscillates, with KE maximum (and PE zero) at equilibrium, and PE maximum (and KE zero) at maximum displacement, while the total energy stays constant.

Question 7

(a) $\omega = 2\pi / T = 2\pi / 40$ **M1**

(a) ω = 0.157 rad/s cao **A1**

(a) Answer: 0.157 rad/s

(b) v = ω r = 0.157 x 22 **M1**

(b) v = 3.46 m/s cao **A1**

(b) Answer: 3.46 m/s

(c) use a = ω^2 r = 0.157^2 x 22 (or v^2/r) **M1**

(c) a = 0.543 m/s^2 cao **A1**

(c) Answer: 0.543 m/s^2

(d) at the bottom: N - mg = m a(centripetal), so N = mg + m a **M1**

(d) N(bottom) = 380 x 9.81 + 380 x 0.543 = 3934 N cao **A1**

(d) at the top: mg - N = m a(centripetal), so N = mg - m a **M1**

(d) N(top) = 3728 - 206 = 3522 N cao **A1**

(d) Answer: N(bottom) = 3934 N, N(top) = 3522 N

Question 8

(a) Q1 = mcT = 0.150 x 2100 x 8.0 **M1**

(a) Q1 = 2520 J cao **A1**

(a) Answer: 2520 J

(b) Q2 = mL = 0.150 x 334,000 **M1**

(b) Q2 = 50,100 J cao **A1**

(b) Answer: 50,100 J

(c) Q3 = mcT = 0.150 x 4200 x 20 **M1**

(c) Q3 = 12,600 J cao **A1**

(c) Answer: 12,600 J

(d) total = Q1 + Q2 + Q3 = 2520 + 50,100 + 12,600 **M1**

(d) total = 65,220 J (65.2 kJ) cao **A1**

(d) a valid assumption, e.g. no energy is lost to the surroundings during heating, or the specific heat capacities are constant over the temperature ranges used oe **B1**

(d) Answer: 65,220 J (65.2 kJ); assumption: no energy is lost to the surroundings.

Question 9

(a) any one correct assumption, e.g. molecules undergo perfectly elastic collisions with each other and the container walls, oe **B1**

(a) any other correct assumption, e.g. the volume of the molecules is negligible compared with the volume of the container; the duration of a collision is negligible compared with the time between collisions; there are negligible intermolecular forces except during collisions; molecules move in continuous random motion, oe (any two distinct assumptions for 2 marks) **B1**

(a) Answer: Any two of: collisions are perfectly elastic; molecular volume is negligible compared to the container volume; collision time is negligible compared to time between collisions; negligible forces between molecules except during collisions; molecules move in continuous random motion.

(b) rearranged correctly to $\langle c^2 \rangle = 3pV/(Nm)$ **M1**

(b) correct substitution of all given values **M1**

(b) $\langle c^2 \rangle = 4.78 \times 10^4$ m^2/s^2 (awrt), ecf allowed to next mark **A1**

(b) $c_{rms} = \sqrt{\langle c^2 \rangle} = 219$ m/s (awrt), cao, unit given **A1**

(b) Answer: $c_{rms} = 219$ m/s (3 s.f.)

(c) correct substitution into $E_k = (3/2) \times k \times T$ **M1**

(c) correct arithmetic combination, e.g. $1.5 \times 1.38 \times 10^{-23} = 2.07 \times 10^{-23}$ **M1**

(c) $E_k = 6.21 \times 10^{-21}$ J (awrt), cao, unit given **A1**

(c) Answer: 6.21×10^{-21} J (3 s.f.)

(d) hydrogen has the higher rms speed, oe **B1**

(d) at the same temperature both gases have the same average kinetic energy per molecule ($E_k = (3/2)kT$ depends only on T); since $E_k = 1/2 \times m \times \langle c^2 \rangle$, the much smaller mass of a hydrogen molecule (compared to nitrogen) means $\langle c^2 \rangle$ (and hence rms speed) must be greater to give the same E_k, oe **B1**

(d) Answer: Hydrogen has the higher rms speed, because at the same temperature both gases have equal average molecular kinetic energy, and the lower mass of hydrogen molecules means they must move faster to have the same kinetic energy.

Question 10

(a) conservation of momentum applied: m_bullet x u = (m_bullet + M_block) x v, oe **M1**

(a) correct substitution: 0.008 x 320 = (0.008 + 2.0) x v **M1**

(a) v = 1.27 m/s (awrt), cao **A1**

(a) Answer: 1.27 m/s (3 s.f.)

(b) KE before = $1/2 \times 0.008 \times 320^2 = 409.6$ J (awrt), cao **M1**

(b) KE after = $1/2 \times 2.008 \times v^2 = 1.63$ J (awrt), ft from (a) **M1**

(b) KE loss = 408 J (awrt), ft **A1**

(b) Answer: KE before = 409.6 J; KE after = 1.63 J; KE lost = 408 J (3 s.f.)

(c) energy absorbed by bullet = half of KE loss = 204 J (awrt), ft from (b) **M1**

(c) correct substitution into deltaT = Q/(m x c), using the bullet's mass (0.008 kg) **M1**

(c) deltaT = 196 K (or °C) (awrt), cao, ft **A1**

(c) Answer: Approximately 196 K rise in temperature (3 s.f.)

Question 11

Level 3 A detailed and coherent discussion that explains resonance occurring when the driving frequency (footsteps) matches (or is close to) the natural frequency of the bridge, causing a large amplitude of forced oscillation; explains that dampers increase the degree of damping, dissipating vibrational energy as heat and reducing (and broadening) the amplitude peak at resonance; explains that changing the bridge's natural frequency (e.g. by altering its stiffness or mass) moves the resonant frequency away from typical walking frequencies so resonance is avoided; a reasoned, well-justified evaluation is given comparing the two approaches, using correct physics terminology throughout. **5-6**

Level 2 A reasonable discussion showing understanding of resonance, with at least one of the two modifications (damping or changing natural frequency) explained correctly and linked to a reduction in amplitude; the evaluation may be brief, one-sided, or only partially justified; generally correct physics with some gaps in detail or terminology. **3-4**

Level 1 Basic, largely unlinked statements about resonance and/or damping are made, with limited explanation and little or no reference to the bridge context; no meaningful evaluation attempted. **1-2**

Level 0 No relevant content, or the answer does not relate to the question. **0**

Indicative content:

- Resonance occurs when the frequency of a periodic driving force (the footsteps of people walking in step) matches, or is very close to, the natural frequency of the bridge, producing a maximum amplitude of oscillation.
- With little damping present, the amplitude of oscillation at resonance can become very large, which explains the noticeable swaying.
- Fitting dampers increases the degree of damping in the system, dissipating vibrational energy as heat (internal energy); this reduces the amplitude of oscillation at resonance and broadens (flattens) the resonance peak, so the response is less sensitive to the exact driving frequency.
- Changing the structure (its mass and/or stiffness) alters the bridge's natural frequency, so that it no longer coincides with the typical range of frequencies produced by pedestrians walking, meaning resonance is not driven under normal use.
- Evaluation: changing the natural frequency removes the risk of resonance for expected walking frequencies but may not protect against a wider range of driving frequencies (e.g. running, wind gusts, or unusual crowd behaviour); dampers reduce the amplitude across a whole range of driving frequencies, giving more general protection.
- A well-justified conclusion might argue that using dampers (or a combination of both approaches) is the more robust practical solution, since pedestrian step frequency cannot be precisely controlled, whereas changing natural frequency alone only guards against resonance at one specific frequency.

Question 12

(a) r = L sin(25) = 0.80 x sin(25) **M1**

(a) r = 0.338 m cao **A1**

(a) Answer: 0.338 m

(b) resolve vertically: T cos(25) = mg, so T = mg/cos(25) **M1**

(b) T = 2.71 N cao **A1**

(b) resolve horizontally: T sin(25) = m v^2 / r **M1**

(b) substitute and rearrange for v: v = $\sqrt{T \sin(25) \times r / m}$ **M1**

(b) v = 1.24 m/s cao **A1**

(b) Answer: T = 2.71 N, v = 1.24 m/s

(c) use T(period) = 2 π r / v **M1**

(c) T(period) = 1.71 s cao **A1**

(c) Answer: 1.71 s

Question 13

(a) the fixed reaction-time uncertainty in starting/stopping the stopwatch is spread over a much longer total time **B1**

(a) this reduces the percentage uncertainty in the value obtained for a single period, compared with timing one oscillation directly oe **B1**

(a) Answer: Timing 20 oscillations and dividing by 20 spreads the fixed reaction-time uncertainty of the stopwatch over a much longer total time, reducing the percentage uncertainty in the period of a single oscillation.

(b) $T^2 = 2.01^2$ **M1**

(b) T^2 = 4.04 s^2 cao **A1**

(b) Answer: 4.04 s^2

(c) T^2 at L=0.40 is 1.27^2 = 1.61 s^2; T^2 at L=1.00 is 2.01^2 = 4.04 s^2 **M1**

(c) gradient = (4.04 - 1.61) / (1.00 - 0.40) **M1**

(c) gradient = 4.05 s^2/m cao **A1**

(c) Answer: 4.05 s^2/m

(d) g = 4 π^2 / gradient **M1**

(d) g = 9.76 m/s^2 cao (accept 9.75 from the rounded gradient 4.05) **A1**

(d) Answer: 9.76 m/s^2 (9.75 m/s^2 if the rounded gradient 4.05 is used)

(e) a valid systematic error, e.g. the amplitude of swing was too large so the small-angle approximation (needed for T = 2 π $\sqrt{L/g}$) no longer holds accurately; or the length was measured to the centre of the bob incorrectly (e.g. omitting half the bob diameter), giving a consistent error in L; or air resistance/timing reaction delay introduced a small systematic effect oe (1 mark for a plausible source, 1 mark for linking it to the direction/nature of the discrepancy) **B2**

(e) Answer: A plausible cause is a systematic error, such as the pendulum being swung with too large an amplitude (breaking the small-angle approximation used in the formula) or the length being measured incorrectly (e.g. not including the radius of the bob to its centre of mass).

Question 14

(a) ω = $\sqrt{k/m}$ = $\sqrt{25/0.40}$ **M1**

(a) ω = 7.91 rad/s cao **A1**

(a) Answer: 7.91 rad/s

(b) v(max) = ω A = 7.91 x 0.15 **M1**

(b) v(max) = 1.19 m/s cao **A1**

(b) Answer: 1.19 m/s

(c) a(max) = ω^2 A = 7.91^2 x 0.15 **M1**

(c) a(max) = 9.38 m/s^2 cao **A1**

(c) Answer: 9.38 m/s^2

(d) E(total) = 1/2 k A^2 = 0.5 x 25 x 0.15^2 **M1**

(d) E(total) = 0.281 J cao **A1**

(d) Answer: 0.281 J

(e) quote v = ω $\sqrt{A^2 - x^2}$ **M1**

(e) substitute v = 7.91 x $\sqrt{0.15^2 - 0.080^2}$ **M1**

(e) v = 1.00 m/s cao **A1**

(e) Answer: 1.00 m/s

Question 15

(a) light damping: the amplitude of oscillation decreases (decays) gradually over many cycles, while the system continues to oscillate **B1**

(a) critical damping: the system returns to equilibrium in the shortest possible time without oscillating (overshooting) **B1**

(a) heavy (over) damping: the system returns to equilibrium without oscillating, but takes longer to do so than in the critically damped case oe **B1**

(a) Answer: Light damping: amplitude gradually decays while the system keeps oscillating. Critical damping: the system returns to equilibrium in the shortest possible time without oscillating. Heavy damping: the system returns to equilibrium without oscillating, but more slowly than critical damping.

(b) car suspension systems (shock absorbers) oe **B1**

(b) critical damping allows the suspension to absorb a bump and return the car body to its normal position as quickly as possible without oscillating (bouncing), maintaining control and comfort oe **B1**

(b) Answer: Car suspension systems are critically damped so that after hitting a bump the car body returns to its normal position as quickly as possible without continuing to bounce, maintaining ride comfort and control.

(c) when the driving frequency equals the natural frequency, the system is in resonance **B1**

(c) energy is transferred from the driver to the oscillating system most efficiently (the driving force is always in the right direction to do positive work on the system) at this condition **B1**

(c) with only light damping, very little of this input energy is dissipated each cycle, so energy (and hence amplitude) builds up to a large value over successive cycles oe **B1**

(c) Answer: At resonance (driving frequency equal to natural frequency), energy is transferred from the driver to the system most efficiently each cycle; since light damping removes very little energy per cycle, energy accumulates over many cycles, producing a large amplitude.

Question 16

Level 3 Clearly explains at least three of the key kinetic theory assumptions (negligible molecular volume, negligible intermolecular forces except during collision, elastic collisions, pressure arising from momentum change on collision with walls linked to Newton's laws), correctly relates them to the ideal gas equation, and gives a well-reasoned discussion of when real gases deviate from ideal behaviour (high pressure, where molecular volume becomes significant compared with the container volume; low temperature, where molecules move more slowly and intermolecular attractive forces become significant, potentially leading to condensation), with a clear synoptic link between the molecular (mechanics-based) model and the macroscopic gas laws. **5-6**

Level 2 Explains at least two kinetic theory assumptions with some link to the ideal gas equation, and identifies at least one condition (high pressure or low temperature) under which real gases deviate from ideal behaviour, though the explanation of why is only partially developed. **3-4**

Level 1 States one or two assumptions of the kinetic theory model with little explanation of their significance, or a vague comment that real gases are not perfectly ideal without valid supporting reasoning. **1-2**

Indicative content:

- Negligible molecular volume: assumes the volume actually occupied by the molecules themselves is insignificant compared with the container volume, which underpins $pV = nRT$ holding for the full container volume.
- Negligible intermolecular forces (except during collisions): assumes molecules do not attract or repel one another between collisions, so they travel in straight lines at constant velocity, consistent with molecules only changing momentum during brief, localised collisions.
- Elastic collisions: kinetic energy is conserved in collisions between molecules and with the walls, so the gas does not lose energy over time purely through molecular collisions, keeping the model's internal energy consistent with temperature.
- Pressure from momentum change: each collision with a wall changes a molecule's momentum; by Newton's second law this requires a force on the molecule from the wall, and by Newton's third law an equal and opposite force on the wall from the molecule; the sum of many such momentum changes per second, divided by wall area, gives the macroscopic gas pressure. This directly links the mechanics of individual particle collisions (momentum, Newton's laws) to the macroscopic thermal quantity of pressure.
- Deviation at high pressure: molecules are forced closer together, so their own volume becomes a significant fraction of the container volume (violating the negligible-volume assumption), causing real gas behaviour to depart from the ideal gas law.
- Deviation at low temperature: molecules move more slowly, so intermolecular attractive forces (previously negligible) become significant relative to the molecules' kinetic energy, pulling molecules together and reducing the pressure below the ideal prediction; at sufficiently low temperature this can lead to condensation, which the ideal gas model cannot describe at all.