AP6D Further Mechanics and Thermal Physics: Depth and Exam Drill

AQA 7408 · Calculator allowed · about 135 minutes

Total Marks

Name: ________________________________ Date: ____ / ____ / ______

Answer ALL questions. Show all your working.

1 This question tests core definitions in further mechanics and thermal physics.

(a) Define angular velocity for an object moving in a circle. **(1)**

(b) State the defining condition for an object to be moving with simple harmonic motion. **(1)**

(c) State what is meant by the internal energy of a system. **(1)**

(Total for Question 1 is 3 marks)

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2 A Ferris wheel cabin of mass 380 kg moves at constant speed in a vertical circle of radius 22 m, completing one revolution every 40 s. Take $g = 9.81$ m/s^2.

(a) Calculate the angular velocity of the cabin. **(2)**

(b) Calculate the linear speed of the cabin. **(2)**

(c) Calculate the centripetal acceleration of the cabin. **(2)**

(d) Calculate the force exerted by the wheel structure on the cabin at the bottom and at the top of the circle. **(4)**

(Total for Question 2 is 10 marks)

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3 A conical pendulum consists of a mass of 0.25 kg on the end of a string of length 0.80 m. The mass moves in a horizontal circle with the string making a constant angle of 25 degrees with the vertical. Take $g = 9.81$ m/s^2.

(a) Calculate the radius of the circular path. **(2)**

(b) By resolving forces on the mass vertically and horizontally, calculate the tension in the string and the speed of the mass. **(5)**

(c) Calculate the period of the motion. **(2)**

(Total for Question 3 is 9 marks)

4 REQUIRED PRACTICAL. A student determines g using a simple pendulum. For each length, the time for 20 complete oscillations is measured and divided by 20 to give the period T. The results are given below.

L (m): 0.40, 0.60, 0.80, 1.00

T (s): 1.27, 1.55, 1.79, 2.01

(a) Explain why the student times 20 oscillations and divides by 20, rather than timing a single oscillation. **(2)**

(b) Calculate T^2 for the length L = 1.00 m. **(2)**

(c) Using the data for L = 0.40 m and L = 1.00 m, calculate the gradient of a graph of T^2 (y-axis) against L (x-axis). **(3)**

(d) Given that $T^2 = (4\pi^2 / g)\,L$, use your gradient to calculate the experimental value of g. **(2)**

(e) Suggest one reason why this experimental value of g differs from the accepted value of 9.81 m/s^2. **(2)**

(Total for Question 4 is 11 marks)

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5 A mass of 0.40 kg is attached to a horizontal spring of spring constant 25 N/m and oscillates with simple harmonic motion of amplitude 0.15 m.

(a) Calculate the angular frequency of the oscillation. **(2)**

(b) Calculate the maximum speed of the mass during the oscillation. **(2)**

(c) Calculate the maximum acceleration of the mass. **(2)**

(d) Calculate the total energy of the oscillation. **(2)**

(e) Calculate the speed of the mass when its displacement from equilibrium is 0.080 m. **(3)**

(Total for Question 5 is 11 marks)

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6 A vehicle suspension system and a footbridge both involve oscillating systems that may be damped to differing degrees.

(a) Describe, using the terms light damping, critical damping and heavy (over) damping, how the displacement of a displaced oscillating system varies with time in each case. **(3)**

(b) State one practical application where critical damping is desirable, and explain why. **(2)**

(c) Explain, in terms of energy transfer, why the amplitude of oscillation of a lightly damped system becomes very large when the frequency of an external driving force is equal to the system's natural frequency. **(3)**

(Total for Question 6 is 8 marks)

7 A 0.150 kg block of ice, initially at -8.0 degrees C, is heated steadily until it becomes water at 20 degrees C. Specific heat capacity of ice = 2100 J/(kg degreesC); specific latent heat of fusion of ice = 334,000 J/kg; specific heat capacity of water = 4200 J/(kg degreesC).

(a) Calculate the energy required to warm the ice from -8.0 degrees C to 0 degrees C. **(2)**

(b) Calculate the energy required to melt the ice at 0 degrees C. **(2)**

(c) Calculate the energy required to warm the resulting water from 0 degrees C to 20 degrees C. **(2)**

(d) Calculate the total energy required for the whole process, and state one assumption made in this calculation. **(3)**

(Total for Question 7 is 9 marks)

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8 A fixed mass of an ideal gas has an initial pressure of 1.0 x 10^5 Pa, volume 2.4 x 10^{-3} m^3 and temperature 290 K. The gas is compressed to a volume of 1.6 x 10^{-3} m^3 and heated to a temperature of 350 K. The molar gas constant R = 8.31 J/(mol K).

(a) Use the combined gas law to calculate the new pressure of the gas. **(4)**

(b) Calculate the number of moles of gas present. **(3)**

(c) Given that the molar mass of the gas is 0.032 kg/mol, calculate the root-mean-square speed of the gas molecules at the initial temperature of 290 K, using $c(rms) = \sqrt{3RT/M}$. **(4)**

(Total for Question 8 is 11 marks)

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9 The kinetic theory of gases models gas molecules as identical hard, elastic spheres in continuous random motion. It assumes the molecules have negligible volume compared with the volume of their container, that intermolecular forces are negligible except during collisions, that collisions between molecules (and with the container walls) are perfectly elastic, and that the pressure exerted on the container walls arises from the change in momentum of molecules colliding with the walls (linking directly to Newton's second and third laws). Evaluate the assumptions of this kinetic theory model, explaining how each assumption relates to the macroscopic behaviour described by the ideal gas equation, and discuss the physical conditions (of pressure and temperature) under which a real gas is most likely to deviate significantly from ideal gas behaviour.

(Total for Question 9 is 6 marks)

Mark scheme · AP6D Further Mechanics and Thermal Physics: Depth and Exam Drill

Question 1

- (a) **B1** the rate of change of angle (angular displacement) with respect to time oe, units rad/s
- **(a) Answer: The rate of change of angular displacement with respect to time.**
- (b) **B1** acceleration is directly proportional to displacement from a fixed point and is always directed towards that point (opposite in direction to the displacement) oe
- **(b) Answer: Acceleration is proportional to displacement from a fixed equilibrium point, and always directed towards that point.**
- (c) **B1** the sum of the randomly distributed kinetic and potential energies of the particles (molecules/atoms) that make up the system oe
- **(c) Answer: The sum of the randomly distributed kinetic and potential energies of all the particles in a system.**

Question 2

- (a) **M1** $\omega = 2\pi / T = 2\pi / 40$
- (a) **A1** $\omega = 0.157$ rad/s cao
- **(a) Answer: 0.157 rad/s**
- (b) **M1** $v = \omega r = 0.157 \times 22$
- (b) **A1** $v = 3.46$ m/s cao
- **(b) Answer: 3.46 m/s**
- (c) **M1** use $a = \omega^2 r = 0.157^2 \times 22$ (or v^2/r)
- (c) **A1** $a = 0.543$ m/s^2 cao
- **(c) Answer: 0.543 m/s^2**
- (d) **M1** at the bottom: N - mg = m a(centripetal), so N = mg + m a
- (d) **A1** N(bottom) = 380 x 9.81 + 380 x 0.543 = 3934 N cao
- (d) **M1** at the top: mg - N = m a(centripetal), so N = mg - m a
- (d) **A1** N(top) = 3728 - 206 = 3522 N cao
- **(d) Answer: N(bottom) = 3934 N, N(top) = 3522 N**

Question 3

- (a) **M1** r = L sin(25) = 0.80 x sin(25)
- (a) **A1** r = 0.338 m cao
- **(a) Answer: 0.338 m**
- (b) **M1** resolve vertically: T cos(25) = mg, so T = mg/cos(25)
- (b) **A1** T = 2.71 N cao
- (b) **M1** resolve horizontally: T sin(25) = m v^2 / r
- (b) **M1** substitute and rearrange for v: $v = \sqrt{T \sin(25) \times r / m}$
- (b) **A1** v = 1.24 m/s cao
- **(b) Answer: T = 2.71 N, v = 1.24 m/s**
- (c) **M1** use T(period) = $2\pi r / v$
- (c) **A1** T(period) = 1.71 s cao
- **(c) Answer: 1.71 s**

Question 4

- (a) **B1** the fixed reaction-time uncertainty in starting/stopping the stopwatch is spread over a much longer total time
- (a) **B1** this reduces the percentage uncertainty in the value obtained for a single period, compared with timing one oscillation directly oe
- **(a) Answer: Timing 20 oscillations and dividing by 20 spreads the fixed reaction-time uncertainty of the stopwatch over a much longer total time, reducing the percentage uncertainty in the period of a single oscillation.**
- (b) **M1** $T^2 = 2.01^2$
- (b) **A1** $T^2 = 4.04\ s^2$ cao
- **(b) Answer: 4.04 s^2**
- (c) **M1** T^2 at L=0.40 is $1.27^2 = 1.61\ s^2$; T^2 at L=1.00 is $2.01^2 = 4.04\ s^2$
- (c) **M1** gradient = (4.04 - 1.61) / (1.00 - 0.40)
- (c) **A1** gradient = 4.05 s^2/m cao
- **(c) Answer: 4.05 s^2/m**
- (d) **M1** $g = 4\pi^2$ / gradient
- (d) **A1** g = 9.76 m/s^2 cao
- **(d) Answer: 9.76 m/s^2**
- (e) **B2** a valid systematic error, e.g. the amplitude of swing was too large so the small-angle approximation (needed for $T = 2\pi\sqrt{L/g}$) no longer holds accurately; or the length was measured to the centre of the bob incorrectly (e.g. omitting half the bob diameter), giving a consistent error in L; or air resistance/timing reaction delay introduced a small systematic effect oe (1 mark for a plausible source, 1 mark for linking it to the direction/nature of the discrepancy)
- **(e) Answer: A plausible cause is a systematic error, such as the pendulum being swung with too large an amplitude (breaking the small-angle approximation used in the formula) or the length being measured incorrectly (e.g. not including the radius of the bob to its centre of mass).**

Question 5

- (a) **M1** $\omega = \sqrt{k/m} = \sqrt{25/0.40}$
- (a) **A1** ω = 7.91 rad/s cao
- **(a) Answer: 7.91 rad/s**
- (b) **M1** v(max) = ω A = 7.91 x 0.15
- (b) **A1** v(max) = 1.19 m/s cao
- **(b) Answer: 1.19 m/s**
- (c) **M1** a(max) = $\omega^2 A = 7.91^2 \times 0.15$
- (c) **A1** a(max) = 9.38 m/s^2 cao
- **(c) Answer: 9.38 m/s^2**
- (d) **M1** E(total) = $1/2\ k A^2 = 0.5 \times 25 \times 0.15^2$
- (d) **A1** E(total) = 0.281 J cao
- **(d) Answer: 0.281 J**
- (e) **M1** quote $v = \omega\sqrt{A^2 - x^2}$
- (e) **M1** substitute $v = 7.91 \times \sqrt{0.15^2 - 0.080^2}$
- (e) **A1** v = 1.00 m/s cao
- **(e) Answer: 1.00 m/s**

Question 6

- (a) **B1** light damping: the amplitude of oscillation decreases (decays) gradually over many cycles, while the system continues to oscillate
- (a) **B1** critical damping: the system returns to equilibrium in the shortest possible time without oscillating (overshooting)
- (a) **B1** heavy (over) damping: the system returns to equilibrium without oscillating, but takes longer to do so than in the critically damped case oe
- **(a) Answer: Light damping: amplitude gradually decays while the system keeps oscillating. Critical damping: the system returns to equilibrium in the shortest possible time without oscillating. Heavy damping: the system returns to equilibrium without oscillating, but more slowly than critical damping.**
- (b) **B1** car suspension systems (shock absorbers) oe
- (b) **B1** critical damping allows the suspension to absorb a bump and return the car body to its normal position as quickly as possible without oscillating (bouncing), maintaining control and comfort oe
- **(b) Answer: Car suspension systems are critically damped so that after hitting a bump the car body returns to its normal position as quickly as possible without continuing to bounce, maintaining ride comfort and control.**
- (c) **B1** when the driving frequency equals the natural frequency, the system is in resonance
- (c) **B1** energy is transferred from the driver to the oscillating system most efficiently (the driving force is always in the right direction to do positive work on the system) at this condition
- (c) **B1** with only light damping, very little of this input energy is dissipated each cycle, so energy (and hence amplitude) builds up to a large value over successive cycles oe
- **(c) Answer: At resonance (driving frequency equal to natural frequency), energy is transferred from the driver to the system most efficiently each cycle; since light damping removes very little energy per cycle, energy accumulates over many cycles, producing a large amplitude.**

Question 7

- (a) **M1** Q1 = mcT = 0.150 x 2100 x 8.0
- (a) **A1** Q1 = 2520 J cao
- **(a) Answer: 2520 J**
- (b) **M1** Q2 = mL = 0.150 x 334,000
- (b) **A1** Q2 = 50,100 J cao
- **(b) Answer: 50,100 J**
- (c) **M1** Q3 = mcT = 0.150 x 4200 x 20
- (c) **A1** Q3 = 12,600 J cao
- **(c) Answer: 12,600 J**
- (d) **M1** total = Q1 + Q2 + Q3 = 2520 + 50,100 + 12,600
- (d) **A1** total = 65,220 J (65.2 kJ) cao
- (d) **B1** a valid assumption, e.g. no energy is lost to the surroundings during heating, or the specific heat capacities are constant over the temperature ranges used oe
- **(d) Answer: 65,220 J (65.2 kJ); assumption: no energy is lost to the surroundings.**

Question 8

- (a) **M1** quote P1V1/T1 = P2V2/T2
- (a) **M1** rearrange to P2 = P1 V1 T2 / (T1 V2)
- (a) **M1** substitute P2 = (1.0 x 10^5 x 2.4 x 10^{-3} x 350) / (290 x 1.6 x 10^{-3})
- (a) **A1** P2 = 1.81 x 10^5 Pa cao
- **(a) Answer: 1.81 x 10^5 Pa**
- (b) **M1** use n = P1V1/(RT1)
- (b) **M1** substitute n = (1.0 x 10^5 x 2.4 x 10^{-3}) / (8.31 x 290)
- (b) **A1** n = 0.0996 mol cao
- **(b) Answer: 0.0996 mol**
- (c) **M1** quote c(rms) = $\sqrt{3RT/M}$
- (c) **M1** substitute c(rms) = $\sqrt{(3 \times 8.31 \times 290) / 0.032}$
- (c) **M1** carry out the calculation inside the square root: 7229.7/0.032 = 225,928
- (c) **A1** c(rms) = 475 m/s cao
- **(c) Answer: 475 m/s**

Question 9

- **Level 3** (5-6): Clearly explains at least three of the key kinetic theory assumptions (negligible molecular volume, negligible intermolecular forces except during collision, elastic collisions, pressure arising from momentum change on collision with walls linked to Newton's laws), correctly relates them to the ideal gas equation, and gives a well-reasoned discussion of when real gases deviate from ideal behaviour (high pressure, where molecular volume becomes significant compared with the container volume; low temperature, where molecules move more slowly and intermolecular attractive forces become significant, potentially leading to condensation), with a clear synoptic link between the molecular (mechanics-based) model and the macroscopic gas laws.
- **Level 2** (3-4): Explains at least two kinetic theory assumptions with some link to the ideal gas equation, and identifies at least one condition (high pressure or low temperature) under which real gases deviate from ideal behaviour, though the explanation of why is only partially developed.
- **Level 1** (1-2): States one or two assumptions of the kinetic theory model with little explanation of their significance, or a vague comment that real gases are not perfectly ideal without valid supporting reasoning.
- **Indicative content:**
  - Negligible molecular volume: assumes the volume actually occupied by the molecules themselves is insignificant compared with the container volume, which underpins pV = nRT holding for the full container volume.
  - Negligible intermolecular forces (except during collisions): assumes molecules do not attract or repel one another between collisions, so they travel in straight lines at constant velocity, consistent with molecules only changing momentum during brief, localised collisions.
  - Elastic collisions: kinetic energy is conserved in collisions between molecules and with the walls, so the gas does not lose energy over time purely through molecular collisions, keeping the model's internal energy consistent with temperature.
  - Pressure from momentum change: each collision with a wall changes a molecule's momentum; by Newton's second law this requires a force on the molecule from the wall, and by Newton's third law an equal and opposite force on the wall from the molecule; the sum of many such momentum changes per second, divided by wall area, gives the macroscopic gas pressure. This directly links the mechanics of individual particle collisions (momentum, Newton's laws) to the macroscopic thermal quantity of pressure.
  - Deviation at high pressure: molecules are forced closer together, so their own volume becomes a significant fraction of the container volume (violating the negligible-volume assumption), causing real gas behaviour to depart from the ideal gas law.
  - Deviation at low temperature: molecules move more slowly, so intermolecular attractive forces (previously negligible) become significant relative to the molecules' kinetic energy, pulling molecules together and reducing the pressure below the ideal prediction; at sufficiently low temperature this can lead to condensation, which the ideal gas model cannot describe at all.