EP.M109 Square, Cube and Triangular Numbers in Reasoning

Total Marks

GL ASSESSMENT GL-11PLUS-MATHS · Calculators not allowed · about 50 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Which of these is a square number?

- ☐ A) 18
- ☐ B) 20
- ☐ C) 23
- ☐ D) 25
- ☐ E) 27

(Total for Question 1 is 1 mark)

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2 Which of these numbers is a cube?

- ☐ A) 18
- ☐ B) 24
- ☐ C) 27
- ☐ D) 30
- ☐ E) 32

(Total for Question 2 is 1 mark)

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3 Which number in this list is a triangular number?

- ☐ A) 14
- ☐ B) 15
- ☐ C) 16
- ☐ D) 17
- ☐ E) 18

(Total for Question 3 is 1 mark)

4 Complete the missing terms.

(a) Continue the sequence of square numbers: 1, 4, 9, 16, 25, __, __ **(1)**

(b) Give the next cube after 27. **(1)**

(Total for Question 4 is 2 marks)

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5 Is 78 a triangular number? If it is, state the position n so that the nth triangular number is 78.

(Total for Question 5 is 2 marks)

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6 Find the sum of the first four triangular numbers.

(Total for Question 6 is 3 marks)

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7 A garden uses 49 square paving stones placed in a 7 by 7 square. The gardener enlarges the paved area so it becomes a square with one more stone on each side. How many extra stones are added?

(Total for Question 7 is 2 marks)

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8 Find the smallest whole number greater than 1 which is both a square and a triangular number.

(Total for Question 8 is 2 marks)

9 Which statement is correct?

- ☐ A) The sum of two consecutive triangular numbers is always a square.
- ☐ B) The sum of two consecutive triangular numbers is sometimes a square.
- ☐ C) The sum of two consecutive triangular numbers is never a square.
- ☐ D) Two consecutive triangular numbers add to a square only when n is odd.
- ☐ E) Two consecutive triangular numbers add to a square only when n is even.

(Total for Question 9 is 1 mark)

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10 How many square numbers are there between 1 and 300 inclusive?

(Total for Question 10 is 1 mark)

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11 A model is built by stacking cubes so that the bottom layer has 1 cube, the next has 2, then 3, and so on until the top layer. If the total number of cubes is 120, how many cubes are in the top layer?

(Total for Question 11 is 2 marks)

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12 Find the smallest whole number greater than 1 that is both a square and a cube. Explain your reasoning briefly.

(Total for Question 12 is 2 marks)

Mark scheme · EP.M109 Square, Cube and Triangular Numbers in Reasoning

Question 1

- **B1** correct option D (25) cao
- **Answer: D) 25 ($5^2 = 25$). The other options are not perfect squares (18, 20, 23, 27). The common pupil slip would be confusing near-square numbers such as 27 being close to 25 or 36, but 25 is the exact square.**

Question 2

- **B1** correct option C (27) cao
- **Answer: C) 27 ($3^3 = 27$). Distractors: 32 is 2^5 or 4^? common slip; 27 is the familiar small cube.**

Question 3

- **B1** correct option B (15) cao
- **Answer: B) 15 (the 5th triangular number: 1+2+3+4+5 = 15). Other options are common near-values or result from adding a wrong term.**

Question 4

- (a) **B1** 36 and 49 seen as the next two squares (6^2 and 7^2) cao
- **(a) Answer: 36, 49**
- (b) **B1** 64 seen (4^3) cao
- **(b) Answer: 64**

Question 5

- **M1** Method to test n(n+1)/2 = 78 seen, e.g. form $n^2 + n - 156 = 0$ and check discriminant
- **A1** n = 12 cao
- **Answer: Yes. n = 12 so the 12th triangular number is 78 (12 x 13 / 2 = 78). Working check: n(n+1)/2 = 78 -> $n^2 + n - 156 = 0$. Discriminant = 1 + 624 = 625 = 25^2, n = (-1 + 25)/2 = 12.**

Question 6

- **M1** First four triangular numbers identified: 1, 3, 6, 10
- **M1** Attempt to add them, or show partial sums (e.g. 1+3 = 4, 4+6 = 10, 10+10 = 20)
- **A1** Total 20 cao
- **Answer: 20 (1 + 3 + 6 + 10 = 20). Working check: T1 = 1, T2 = 3, T3 = 6, T4 = 10; sum = 20.**

Question 7

- **M1** Method to find new square size (8 x 8 = 64) or difference between consecutive squares seen
- **A1** 15 cao
- **Answer: 15 extra stones. Working check: original 7 x 7 = 49, new 8 x 8 = 64, extra = 64 - 49 = 15.**

Question 8

- **M1** Method showing search or recognition that triangular numbers: 1, 3, 6, 10, 15, 21, 28, 36... and checking for square
- **A1** 36 cao
- **Answer: 36. Working check: triangular sequence reaches 36 at T8 (1+2+...+8 = 36) and 36 = 6^2 so it is both triangular and a square. This is the smallest >1 with that property.**

Question 9

- **B1** correct option A (always true) cao
- **Answer: A) Always true. Working check: T(n) + T(n+1) = n(n+1)/2 + (n+1)(n+2)/2 = (n+1)(2n+2)/2 = $(n+1)^2$, which is a perfect square for every n. Examples: T1+T2 = 1+3 = 4 = 2^2; T2+T3 = 3+6 = 9 = 3^2; T3+T4 = 6+10 = 16 = 4^2. Distractors represent under-generalisation or parity confusion.**

Question 10

- **B1** 17 cao
- **Answer: 17. Working check: largest square ≤ 300 is $17^2 = 289$. Squares from 1^2 to 17^2 inclusive give 17 squares.**

Question 11

- **M1** Method using n(n+1)/2 = 120 or identifying triangular position seen
- **A1** 15 cao
- **Answer: 15. Working check: $n(n+1)/2 = 120 \rightarrow n^2 + n - 240 = 0$. Discriminant = $1 + 960 = 961 = 31^2$, $n = (-1 + 31)/2 = 15$. So the top layer has 15 cubes.**

Question 12

- **M1** Method that recognises a number that is both a square and a cube must be a 6th power, or shows search 1, 64,... with justification
- **A1** 64 cao
- **Answer: 64. Working check: If a number is both a square and a cube then it is t^6 for some integer t. The smallest $t > 1$ is 2, giving $2^6 = 64$. Also $64 = 8^2$ and 4^3, so it is both a square and a cube.**