2.3 Quadratic Equations: Factorising, the Formula and Completing the Square: Test Pack

AQA 8365 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Factorise each expression.

(a) $x^2 + 7x + 12$ **(1)**

(b) $x^2 - 5x - 24$ **(2)**

(Total for Question 1 is 3 marks)

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2 Which of the following is the correct factorisation of $x^2 + 9x + 20$?

- ☐ A) (x+4)(x+5)
- ☐ B) (x+2)(x+10)
- ☐ C) (x-4)(x-5)
- ☐ D) (x+20)(x+1)

(Total for Question 2 is 1 mark)

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3 Solve $x^2 - 8x + 9 = 0$. Give your solutions correct to 3 significant figures.

(Total for Question 3 is 3 marks)

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4 Find the values of k for which $6x^2 + kx + 6 = 0$ has equal roots.

(Total for Question 4 is 3 marks)

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5 Write $x^2 - 4x - 4$ in the form $(x + p)^2 + q$, where p and q are integers.

(Total for Question 5 is 3 marks)

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6 A rectangular patio has width x metres and length $(x + 1)$ metres. Its area is 240 square metres. Form and solve a quadratic equation to find x, and hence find the perimeter of the rectangle.

(Total for Question 6 is 5 marks)

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7 Solve $x(x + 14) = 8x + 7$.

(Total for Question 7 is 4 marks)

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8 Show that the equation $x^2 + 4x + 9 = 0$ has no real roots.

(Total for Question 8 is 2 marks)

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9 Write down the coordinates of the minimum point of the curve $y = (x - 3)^2 + 7$.

(Total for Question 9 is 1 mark)

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10 By completing the square, express $3x^2 - 18x + 7$ in the form $a(x+p)^2 + q$, where a, p and q are constants to be found.

(Total for Question 10 is 3 marks)

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11 Solve $x^2 - 10x + 7 = 0$ by completing the square. Give your answer in simplified surd form.

(Total for Question 11 is 3 marks)

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12 Prove that $x^2 - 6x + 12 > 2$ for all real values of x.

(Total for Question 12 is 3 marks)

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13 Find the set of values of k for which the equation $x^2 + 4x + k = 0$ has two distinct real roots.

(Total for Question 13 is 3 marks)

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14 A rectangular vegetable patch has length (x+5) metres and width x metres. The area of the patch is 60 m².

(a) Show that $x^2 + 5x - 60 = 0$. **(2)**

(b) Solve the equation to find x, giving your answer correct to 2 decimal places. Reject any solution that is not valid in this context. **(4)**

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(Total for Question 14 is 6 marks)

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15 The equation $x^2 - 2kx + (k^2 + 4) = 0$, where k is a constant, is to be solved for x. Prove that this equation has no real roots for any value of k.

(Total for Question 15 is 4 marks)

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16 The equation $x^2 - 2kx + (k^2 - 9) = 0$, where k is a constant, is to be solved for x. Show that this equation always has two distinct real roots for any value of k.

(Total for Question 16 is 4 marks)

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Mark scheme · 2.3 Quadratic Equations: Factorising, the Formula and Completing the Square: Test Pack

A correct answer earns full marks unless the question asks for working, a method, proof or explanation.

Question 1

(a) (x+3)(x+4) cao **B1**

(a) Answer: (x+3)(x+4)

(b) identifies pair of numbers with product -24 and sum -5 (-8 and 3), or partial correct bracket **M1**

(b) (x-8)(x+3) cao **A1**

(b) Answer: (x-8)(x+3)

Question 2

A cao **B1**

Answer: A

Question 3

correct substitution into the quadratic formula **M1**

1.35 cao **A1**

6.65 cao **A1**

Answer: x = 1.35 or x = 6.65

Question 4

discriminant $k^2 - 4(6)(6)$ used **M1**

$k^2 = 144$ **M1**

k = 12 or k = -12 cao **A1**

Answer: k = 12 or k = -12

Question 5

$(x - 2)^2$ seen **M1**

p = -2 correct **A1**

$(x - 2)^2 - 8$ cao **A1**

Answer: $(x - 2)^2 - 8$

Question 6

x(x + 1) = 240 **M1**

$x^2 + x - 240 = 0$ **M1**

factorises or solves **M1**

x = 15 with the negative root rejected **A1**

62 m cao **A1**

Answer: 62 m

Question 7

expands the bracket **M1**

rearranges to $x^2 + 6x - 7 = 0$ **M1**

factorises or substitutes into the formula **M1**

x = -7 or x = 1 cao **A1**

Answer: x = -7 or x = 1

Question 8

discriminant = $4^2 - 4(1)(9)$ (= 16-36) **M1**

= -20, and since -20 < 0 the equation has no real roots cso **A1**

Answer: Discriminant = -20 < 0, so no real roots

Question 9

(3, 7) cao **B1**

Answer: Minimum point (3, 7)

Question 10

takes out factor 3: $3(x^2-6x)+7$ **M1**

$3[(x-3)^2-9]+7$ oe **M1**

$3(x-3)^2 - 20$ cao (a=3, p=-3, q=-20) **A1**

Answer: $3(x-3)^2 - 20$

Question 11

$(x-5)^2 - 25 + 7 (= 0)$ oe **M1**

$(x-5)^2 = 18$ **M1**

$x = 5 + 3\sqrt{2}$ and $x = 5 - 3\sqrt{2}$ fully simplified surd form cao **A1**

Answer: $x = 5 + 3\sqrt{2}$ or $x = 5 - 3\sqrt{2}$

Question 12

completes the square correctly, $x^2 - 6x + 12 = (x-3)^2 - 9 + 12$ **M1**

simplifies to $(x-3)^2 + 3$ **A1**

cso: since $(x-3)^2 \geq 0$ for all real x, $(x-3)^2 + 3 \geq 3 > 2$, so $x^2-6x+12 > 2$ for all real x **A1**

Answer: $x^2 - 6x + 12 = (x-3)^2 + 3 \geq 3 > 2$ for all real x

Question 13

discriminant = $4^2 - 4(1)(k)$ (= 16-4k) **M1**

sets discriminant > 0 and rearranges **M1**

k < 4 oe cao **A1**

Answer: k < 4

Question 14

(a) area = x(x+5) = 60 **M1**

(a) $x^2 + 5x - 60 = 0$ cso (answer given, correct expansion shown) **A1**

(a) Answer: $x^2 + 5x - 60 = 0$

(b) correct substitution into quadratic formula with a=1, b=5, c=-60 **M1**

(b) discriminant = 265 (or $\sqrt{265}$ = 16.27...) seen **A1**

(b) x = 5.64 awrt **A1**

(b) rejects x = -10.64 with reason that a length cannot be negative **B1**

(b) Answer: x = 5.64 m (2dp)

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Question 15

discriminant = $(-2k)^2 - 4(1)(k^2+4)$	**M1**
correctly expands to $4k^2 - 4k^2 - 16$	**M1**
simplifies to discriminant = -16	**A1**
since -16 < 0 for all values of k, the equation has no real roots for any k cso	**A1**

Answer: Discriminant = -16 for all k, so no real roots for any value of k

Question 16

discriminant = $(-2k)^2 - 4(1)(k^2-9)$	**M1**
correctly expands to $4k^2 - 4k^2 + 36$	**M1**
simplifies to discriminant = 36	**A1**
since 36 > 0 for all values of k, the equation always has two distinct real roots cso	**A1**

Answer: Discriminant = 36 for all k, so the equation always has two distinct real roots

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