2.6 Functions: Composite and Inverse Functions: 1-hour Pack

Total Marks

AQA 8365 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 $f(x) = 5x - 1$ and $g(x) = 2x + 3$.

(a) Find $fg(x)$, giving your answer in the form $ax + b$. **(3)**

.....

(b) Find $gf(x)$, giving your answer in the form $ax + b$. **(3)**

.....

(c) State, with a reason, whether $fg(x) = gf(x)$ for all values of x. **(1)**

(Total for Question 1 is 7 marks)

---

2 $f(x) = (x + 2)/3$.

(a) Find $f^{-1}(x)$. **(3)**

.....

(b) By first finding $f(5)$, verify that $f^{-1}(f(5)) = 5$. **(2)**

(Total for Question 2 is 5 marks)

© Revision Library | revisionlibrary.co.uk

3 $f(x) = (x + 5)/2$. Find $f^{-1}(x)$.

(Total for Question 3 is 2 marks)

---

4 $f(x) = 2x - 7$ and $g(x) = x^2 + 3$. Find $fg(x)$.

(Total for Question 4 is 2 marks)

---

5 $f(x) = x^2 - 5$, for the domain $x \geq 0$. Find $f^{-1}(x)$, stating its domain.

(Total for Question 5 is 2 marks)

---

6 $f(x) = 2x + 1$ and $g(x) = x^2 - 4$.

(a) Find $fg(x)$, giving your answer in the form $ax^2 + b$. **(1)**

(b) Find $gf(x)$, giving your answer in the form $ax^2 + bx + c$. **(1)**

(c) State, with a reason, whether $fg(x) = gf(x)$ for all values of x. **(1)**

(Total for Question 6 is 3 marks)

---

7 $f(x) = (2x - 1)/(x + 3)$, where $x \neq -3$. Find $f^{-1}(x)$, stating any value which x cannot take.

(Total for Question 7 is 4 marks)

© Revision Library | revisionlibrary.co.uk

8 $f(x) = 3x + 2$ and $g(x) = x - 5$.

(a) Find $fg(x)$. **(2)**

(b) Find $(fg)^{-1}(x)$. **(3)**

(c) Find $f^{-1}(x)$ and $g^{-1}(x)$, and hence show that $g^{-1}(f^{-1}(x)) = (fg)^{-1}(x)$. **(3)**

(Total for Question 8 is 8 marks)

---

9 $f(x) = (3x + 2)/(x - 1)$, where $x \neq 1$. Find $f^{-1}(x)$, stating the value which x cannot take.

(Total for Question 9 is 3 marks)

---

10 $f(x) = x^2 - 3$, for the domain $x \leq 0$.

(a) State the range of $f(x)$ for this domain. **(1)**

(b) Find $f^{-1}(x)$, stating its domain. **(3)**

(Total for Question 10 is 4 marks)

© Revision Library | revisionlibrary.co.uk

11 $f(x) = 2x - 1$ and $g(x) = 3/(x + 2)$, where $x \neq -2$.

(a) Find $fg(x)$, giving your answer as a single fraction. **(3)**

.....

(b) State the value of x for which $fg(x)$ is undefined. **(1)**

(c) Find $g^{-1}(x)$. **(3)**

.....

(Total for Question 11 is 7 marks)

---

12 $f(x) = ax + b$, where a and b are constants. Given that $f(2) = 9$ and $f^{-1}(1) = -2$, find the values of a and b.

(Total for Question 12 is 4 marks)

---

13 $f(x) = 2x + 1$ and $g(x) = x^2 + 2$. Solve $fg(x) = 103$.

(Total for Question 13 is 4 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 2.6 Functions: Composite and Inverse Functions: 1-hour Pack

Question 1

(a) substitute g(x) into f, e.g. 5(2x + 3) - 1 **M1**

(a) expand correctly, e.g. 10x + 15 - 1 **M1**

(a) fg(x) = 10x + 14 cao **A1**

(a) Answer: fg(x) = 10x + 14

(b) substitute f(x) into g, e.g. 2(5x - 1) + 3 **M1**

(b) expand correctly, e.g. 10x - 2 + 3 **M1**

(b) gf(x) = 10x + 1 cao **A1**

(b) Answer: gf(x) = 10x + 1

(c) No, because the constant terms differ (14 ≠ 1), ft from (a) and (b) **B1**

(c) Answer: No, since 10x + 14 ≠ 10x + 1

Question 2

(a) let y = (x+2)/3 and multiply both sides by 3, i.e. 3y = x + 2 **M1**

(a) rearrange to x = 3y - 2 **M1**

(a) $f^{-1}(x) = 3x - 2$ cao **A1**

(a) Answer: $f^{-1}(x) = 3x - 2$

(b) f(5) = 7/3 (or awrt 2.33) found **M1**

(b) $f^{-1}(7/3) = 3(7/3) - 2 = 5$, shown correctly cso **A1**

(b) Answer: $f^{-1}(f(5)) = 5$, verified

Question 3

let y = (x+5)/2 and rearrange, e.g. 2y = x+5 **M1**

$f^{-1}(x) = 2x - 5$ cao **A1**

Answer: $f^{-1}(x) = 2x - 5$

Question 4

substitute g(x) into f, e.g. $2(x^2+3) - 7$ **M1**

$fg(x) = 2x^2 - 1$ cao **A1**

Answer: $fg(x) = 2x^2 - 1$

Question 5

$y = x^2-5$ rearranged to $x^2 = y+5$, taking the positive root since x ≥ 0 **M1**

$f^{-1}(x) = \sqrt{x+5}$, domain x ≥ -5 **A1**

Answer: $f^{-1}(x) = \sqrt{x + 5}$, domain x ≥ -5

Question 6

(a) $fg(x) = 2x^2 - 7$ cao **B1**

(a) Answer: $fg(x) = 2x^2 - 7$

(b) $gf(x) = 4x^2 + 4x - 3$ cao **B1**

(b) Answer: $gf(x) = 4x^2 + 4x - 3$

(c) No, because the coefficients of x^2 differ (2 vs 4), ft from (a) and (b) **B1**

(c) Answer: No, since $2x^2 - 7 \neq 4x^2 + 4x - 3$

Question 7

let y = (2x-1)/(x+3) and cross-multiply: y(x+3) = 2x - 1 **M1**

collect x terms on one side, e.g. yx - 2x = -1 - 3y, i.e. x(y-2) = -1-3y **M1**

$f^{-1}(x) = (3x+1)/(2-x)$ oe **A1**

x ≠ 2 stated **B1**

Answer: $f^{-1}(x) = (3x+1)/(2-x)$, x ≠ 2

Question 8

(a) substitute g(x) into f, e.g. 3(x-5) + 2 **M1**

(a) fg(x) = 3x - 13 cao **A1**

(a) Answer: fg(x) = 3x - 13

(b) let y = 3x - 13, ft from (a) **M1**

(b) rearrange to x = (y+13)/3 **M1**

(b) $(fg)^{-1}(x) = (x+13)/3$ cao **A1**

(b) Answer: $(fg)^{-1}(x) = (x+13)/3$

(c) $f^{-1}(x) = (x-2)/3$ and $g^{-1}(x) = x+5$ both found **M1**

(c) substitute $f^{-1}(x)$ into g^{-1}, e.g. (x-2)/3 + 5 **M1**

(c) simplifies to (x+13)/3, matching $(fg)^{-1}(x)$ from (b) cso **A1**

(c) Answer: $g^{-1}(f^{-1}(x)) = (x+13)/3 = (fg)^{-1}(x)$, as required

Question 9

let y = (3x+2)/(x-1) and cross-multiply: y(x-1) = 3x+2 **M1**

collect x terms on one side, e.g. yx - 3x = y + 2, i.e. x(y-3) = y+2 **M1**

$f^{-1}(x) = (x+2)/(x-3)$, x ≠ 3 **A1**

Answer: $f^{-1}(x) = (x+2)/(x-3)$, x ≠ 3

Question 10

(a) f(x) ≥ -3 oe **B1**

(a) Answer: f(x) ≥ -3

(b) $y = x^2 - 3$ rearranged to $x^2 = y + 3$ **M1**

(b) $f^{-1}(x) = -\sqrt{x+3}$ (negative root taken since x ≤ 0) **A1**

(b) domain x ≥ -3 stated, ft from part (a) **B1**

(b) Answer: $f^{-1}(x) = -\sqrt{x+3}$, domain x ≥ -3

Question 11

(a) substitute g(x) into f, e.g. 2(3/(x+2)) - 1 **M1**

(a) write over a common denominator, e.g. (6 - (x+2))/(x+2) **M1**

(a) fg(x) = (4-x)/(x+2) oe **A1**

(a) Answer: fg(x) = (4-x)/(x+2)

(b) x = -2 **B1**

(b) Answer: x = -2

(c) let y = 3/(x+2) and cross-multiply: y(x+2) = 3 **M1**

(c) rearrange to x = (3-2y)/y **M1**

(c) $g^{-1}(x) = (3-2x)/x$, x ≠ 0 cao **A1**

(c) Answer: $g^{-1}(x) = (3-2x)/x$

© Revision Library | revisionlibrary.co.uk

Question 12

interpret $f^{-1}(1) = -2$ as $f(-2) = 1$	**M1**
form two equations: $2a + b = 9$ and $-2a + b = 1$	**M1**
solve simultaneously, e.g. subtract to find a	**M1**
$a = 2$ and $b = 5$ (both values) cao	**A1**

Answer: a = 2, b = 5

Question 13

$fg(x) = 2(x^2 + 2) + 1$	**M1**
$2x^2 - 98 = 0$ or $x^2 = 49$	**M1**
square roots both sides	**M1**
$x = 7$ or $x = -7$ cao	**A1**

Answer: x = 7 or x = -7

© Revision Library | revisionlibrary.co.uk