2.6 Functions: Composite and Inverse Functions: 2-hour Pack

Total Marks

AQA 8365 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 $f(x) = x^3 - 2$.

(a) Find $f^{-1}(x)$. **(3)**

(b) Evaluate $f^{-1}(25)$. **(2)**

(Total for Question 1 is 5 marks)

2 $f(x) = (2x + 1)/(x - 3)$, where $x \neq 3$. Find the value(s) of x for which $f(x) = x$, giving your answers to 2 decimal places.

(Total for Question 2 is 5 marks)

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3 $f(x) = 2x - 5$ and $g(x) = x^2 + k$, where k is a constant.

(a) Find $fg(x)$ in terms of x and k. **(2)**

.....

(b) Given that $fg(3) = 21$, find the value of k. **(3)**

.....

(c) Hence find $gf(x)$, giving your answer in the form $ax^2 + bx + c$. **(3)**

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(Total for Question 3 is 8 marks)

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4 $f(x) = x - 6$ and $g(x) = -x^2 + 3$. Find and simplify $fg(x)$.

(Total for Question 4 is 3 marks)

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5 $f(x) = ax + b$, where a and b are constants. $f(-4) = 19$ and $f(-2) = 11$. Find $f(x)$.

(Total for Question 5 is 4 marks)

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6 $g(x) = 2x^2 - 8$, $x \geq 0$. Find $g^{-1}(x)$ and state its domain.

(Total for Question 6 is 4 marks)

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7 $f(x) = -4x - 3$ and $g(x) = -2x^2 + 1$. Find and simplify $gf(x)$.

(Total for Question 7 is 4 marks)

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8 $f(x) = -5x - 1$ and $g(x) = 6x + 4$. Find and simplify $fg(x)$.

(Total for Question 8 is 3 marks)

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9 $f(x) = ax + b$, where a and b are constants. $f(-3) = 3$ and $f(3) = -9$. Find $f(x)$.

(Total for Question 9 is 4 marks)

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10 $f(x) = 5x - 7$ and $g(x) = 6x - 2$. Find and simplify $fg(x)$.

(Total for Question 10 is 3 marks)

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11 Find the inverse function $f^{-1}(x)$ for $f(x) = (x + 5)/(x + 1)$.

(Total for Question 11 is 4 marks)

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12 $f(x) = ax + b$, where a and b are constants. $f(-4) = 11$ and $f(4) = -5$. Find $f(x)$.

(Total for Question 12 is 4 marks)

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13 $f(x) = -x + 4$ and $g(x) = x - 3$. Find and simplify $gg(x)$.

(Total for Question 13 is 3 marks)

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14 Find the inverse function $f^{-1}(x)$ for $f(x) = (x + 5)/(x + 4)$.

(Total for Question 14 is 4 marks)

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15 $g(x) = x^2 - 9$, $x \geq 0$. Find $g^{-1}(x)$ and state its domain.

(Total for Question 15 is 4 marks)

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16 $f(x) = -2x + 7$ and $g(x) = -3x - 1$. Find and simplify $gg(x)$.

(Total for Question 16 is 3 marks)

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17 $f(x) = ax + b$, where a and b are constants. $f(3) = 3$ and $f(5) = 9$. Find $f(x)$.

(Total for Question 17 is 4 marks)

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18 $f(x) = -4x - 5$ and $g(x) = 4x - 1$. Find and simplify $gg(x)$.

(Total for Question 18 is 3 marks)

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19 $f(x) = 2x - 5$ and $g(x) = 2x^2 - 1$. Solve $fg(x) = -3$.

(Total for Question 19 is 4 marks)

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20 $f(x) = ax + b$, where a and b are constants. $f(-4) = -9$ and $f(6) = 31$. Find $f(x)$.

(Total for Question 20 is 4 marks)

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21 $f(x) = 3x + 4$ and $g(x) = -3x + 6$. Find and simplify $fg(x)$.

(Total for Question 21 is 3 marks)

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22 $f(x) = 2x - 1$. Solve $f(x) = f^{-1}(x)$.

(Total for Question 22 is 4 marks)

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23 $g(x) = 2x^2 + 6$, $x \geq 0$. Find $g^{-1}(x)$ and state its domain.

(Total for Question 23 is 4 marks)

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24 $f(x) = 5x + 2$. Solve $f(x) = f^{-1}(x)$.

(Total for Question 24 is 4 marks)

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Mark scheme · 2.6 Functions: Composite and Inverse Functions: 2-hour Pack

Question 1

(a) $y = x^3 - 2$ rearranged to $x^3 = y + 2$	**M1**
(a) take cube roots, $x = \sqrt[3]{y+2}$	**M1**
(a) $f^{-1}(x) = \sqrt[3]{x+2}$ cao	**A1**
(a) Answer: $f^{-1}(x) = \sqrt[3]{x+2}$	
(b) 25 + 2 = 27 found, ft from (a)	**M1**
(b) $f^{-1}(25) = 3$ cao	**A1**
(b) Answer: 3	

Question 2

set (2x+1)/(x-3) = x and cross-multiply: 2x+1 = x(x-3)	**M1**
rearrange to $x^2 - 5x - 1 = 0$	**M1**
correct use of the quadratic formula with a=1, b=-5, c=-1	**M1**
x = awrt 5.19	**A1**
x = awrt -0.19	**A1**
Answer: x = 5.19 or x = -0.19 (2 dp)	

Question 3

(a) substitute g(x) into f, e.g. $2(x^2+k) - 5$	**M1**
(a) $fg(x) = 2x^2 + 2k - 5$ oe	**A1**
(a) Answer: $fg(x) = 2x^2 + 2k - 5$	
(b) substitute x=3 into fg(x), ft from (a), i.e. 2(9) + 2k - 5	**M1**
(b) simplify to 13 + 2k = 21	**M1**
(b) k = 4 cao	**A1**
(b) Answer: k = 4	
(c) substitute f(x) into g using k=4 (ft from (b)), i.e. $(2x-5)^2 + 4$	**M1**
(c) expand $(2x-5)^2$ correctly to $4x^2 - 20x + 25$	**M1**
(c) $gf(x) = 4x^2 - 20x + 29$ cao (ft)	**A1**
(c) Answer: $gf(x) = 4x^2 - 20x + 29$	

Question 4

$f(g(x)) = 1(-x^2 + 3) - 6$ or equivalent	**M1**
expands the bracket	**M1**
$-x^2 - 3$ cao	**A1**
Answer: $-x^2 - 3$	

Question 5

-4a + b = 19 and -2a + b = 11	**M1**
eliminates a or b	**M1**
a = -4 or b = 3	**A1**
f(x) = -4x + 3 cao	**A1**
Answer: f(x) = -4x + 3	

Question 6

y = g(x), makes x^2 the subject	**M1**
takes the positive square root	**M1**
$g^{-1}(x) = \sqrt{(x + 8)/2}$ cao	**A1**
domain $x \geq -8$	**B1**
Answer: $g^{-1}(x) = \sqrt{(x + 8)/2}$, $x \geq -8$	

Question 7

$g(f(x)) = -2(-4x - 3)^2 + 1$ or equivalent	**M1**
expands $(-4x - 3)^2$ correctly	**M1**
multiplies by -2	**M1**
$-32x^2 - 48x - 17$ cao	**A1**
Answer: $-32x^2 - 48x - 17$	

Question 8

substitutes one function into the other	**M1**
expands brackets	**M1**
-30x - 21 cao	**A1**
Answer: -30x - 21	

Question 9

-3a + b = 3 and 3a + b = -9	**M1**
eliminates a or b	**M1**
a = -2 or b = -3	**A1**
f(x) = -2x - 3 cao	**A1**
Answer: f(x) = -2x - 3	

Question 10

substitutes one function into the other	**M1**
expands brackets	**M1**
30x - 17 cao	**A1**
Answer: 30x - 17	

Question 11

y(x - d) = x + b	**M1**
collects x terms and factorises	**M1**
x(y - 1) = ...	**M1**
(-x + 5)/(x - 1) oe cao	**A1**
Answer: $f^{-1}(x) = (-x + 5)/(x - 1)$	

Question 12

-4a + b = 11 and 4a + b = -5	**M1**
eliminates a or b	**M1**
a = -2 or b = 3	**A1**
f(x) = -2x + 3 cao	**A1**
Answer: f(x) = -2x + 3	

Question 13

substitutes one function into the other	**M1**
expands brackets	**M1**
x - 6 cao	**A1**
Answer: x - 6	

Question 14

y(x - d) = x + b	**M1**
collects x terms and factorises	**M1**
x(y - 1) = ...	**M1**
(-4x + 5)/(x - 1) oe cao	**A1**
Answer: $f^{-1}(x) = (-4x + 5)/(x - 1)$	

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Question 15

$y = g(x)$, makes x^2 the subject	**M1**
takes the positive square root	**M1**
$g^{-1}(x) = \sqrt{(x + 9)}$ cao	**A1**
domain $x \geq -9$	**B1**

Answer: $g^{-1}(x) = \sqrt{(x + 9)}$, $x \geq -9$

Question 16

substitutes one function into the other	**M1**
expands brackets	**M1**
$9x + 2$ cao	**A1**

Answer: $9x + 2$

Question 17

$3a + b = 3$ and $5a + b = 9$	**M1**
eliminates a or b	**M1**
$a = 3$ or $b = -6$	**A1**
$f(x) = 3x - 6$ cao	**A1**

Answer: $f(x) = 3x - 6$

Question 18

substitutes one function into the other	**M1**
expands brackets	**M1**
$16x - 5$ cao	**A1**

Answer: $16x - 5$

Question 19

$fg(x) = 2(2x^2 - 1) - 5$	**M1**
$4x^2 - 4 = 0$ or $x^2 = 1$	**M1**
square roots both sides	**M1**
$x = 1$ or $x = -1$ cao	**A1**

Answer: $x = 1$ or $x = -1$

Question 20

$-4a + b = -9$ and $6a + b = 31$	**M1**
eliminates a or b	**M1**
$a = 4$ or $b = 7$	**A1**
$f(x) = 4x + 7$ cao	**A1**

Answer: $f(x) = 4x + 7$

Question 21

substitutes one function into the other	**M1**
expands brackets	**M1**
$-9x + 22$ cao	**A1**

Answer: $-9x + 22$

Question 22

$f^{-1}(x) = (x + 1)/2$ or equivalent	**M1**
sets $f(x) = f^{-1}(x)$ and clears the fraction	**M1**
collects x terms	**M1**
$x = 1$ cao	**A1**

Answer: $x = 1$

Question 23

$y = g(x)$, makes x^2 the subject	**M1**
takes the positive square root	**M1**
$g^{-1}(x) = \sqrt{(x - 6)/2}$ cao	**A1**
domain $x \geq 6$	**B1**

Answer: $g^{-1}(x) = \sqrt{(x - 6)/2}$, $x \geq 6$

Question 24

$f^{-1}(x) = (x - 2)/5$ or equivalent	**M1**
sets $f(x) = f^{-1}(x)$ and clears the fraction	**M1**
collects x terms	**M1**
$x = -1/2$ cao	**A1**

Answer: $x = -1/2$

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