2.6 Functions: Composite and Inverse Functions: 30-minute Pack

AQA 8365 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 $f(x) = x/2 + 4$.

(a) Find $f^{-1}(x)$. **(3)**

(b) Find $f^{-1}(10)$. **(1)**

(Total for Question 1 is 4 marks)

2 $f(x) = 3x - 1$ and $g(x) = x + 4$. Find $gf(x)$, giving your answer in the form $ax + b$.

(Total for Question 2 is 2 marks)

3 $g(x) = x^2 - 4$, for the domain $x \geq 0$.

(a) State the range of $g(x)$ for this domain. **(1)**

(b) Find $g^{-1}(x)$, stating its domain. **(3)**

(Total for Question 3 is 4 marks)

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4 $f(x) = ax + b$, where a and b are constants. Given that $f(2) = 7$ and $f^{-1}(3) = 1$, find the values of a and b.

(Total for Question 4 is 5 marks)

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5 $f(x) = (x + 6)/(x - 1)$, where $x \neq 1$. Show that f is a self-inverse function.

(Total for Question 5 is 3 marks)

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6 $f(x) = 3x - 3$. Solve $f(x) = f^{-1}(x)$.

(Total for Question 6 is 4 marks)

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7 $f(x) = 3x - 2$ and $g(x) = x^2 + 5x$. Solve $gf(x) = 0$.

(Total for Question 7 is 5 marks)

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Mark scheme · 2.6 Functions: Composite and Inverse Functions: 30-minute Pack

Question 1

(a) let $y = x/2 + 4$ and rearrange to isolate x, e.g. $x/2 = y - 4$	**M1**
(a) $x = 2(y - 4)$ or equivalent correct rearrangement	**M1**
(a) $f^{-1}(x) = 2x - 8$ oe (e.g. $2(x - 4)$)	**A1**

(a) Answer: $f^{-1}(x) = 2x - 8$

(b) $f^{-1}(10) = 12$ ft from part (a)	**B1**

(b) Answer: 12

Question 2

substitute f(x) into g, e.g. $(3x-1) + 4$	**M1**
$gf(x) = 3x + 3$ cao	**A1**

Answer: $gf(x) = 3x + 3$

Question 3

(a) $g(x) \geq -4$ oe	**B1**

(a) Answer: $g(x) \geq -4$

(b) $y = x^2 - 4$ rearranged to $x^2 = y + 4$	**M1**
(b) $g^{-1}(x) = \sqrt{x + 4}$ (positive root taken since $x \geq 0$)	**A1**
(b) domain stated as $x \geq -4$ oe, ft from part (a)	**B1**

(b) Answer: $g^{-1}(x) = \sqrt{x + 4}$, domain $x \geq -4$

Question 4

interpret $f^{-1}(3) = 1$ as $f(1) = 3$	**M1**
form two equations: $2a + b = 7$ and $a + b = 3$	**M1**
solve simultaneously, e.g. subtract to find a	**M1**
$a = 4$ cao	**A1**
$b = -1$ cao	**A1**

Answer: $a = 4$, $b = -1$

Question 5

let $y = (x+6)/(x-1)$ and cross-multiply: $y(x-1) = x + 6$	**M1**
collect x terms, e.g. $yx - x = y + 6$, i.e. $x(y-1) = y+6$	**M1**
$f^{-1}(x) = (x+6)/(x-1) = f(x)$, so f is self-inverse cso	**A1**

Answer: $f^{-1}(x) = (x+6)/(x-1) = f(x)$, so f is self-inverse

Question 6

$f^{-1}(x) = (x + 3)/3$ or equivalent	**M1**
sets $f(x) = f^{-1}(x)$ and clears the fraction	**M1**
collects x terms	**M1**
$x = 3/2$ cao	**A1**

Answer: $x = 3/2$

Question 7

substitute f(x) into g, e.g. $(3x-2)^2 + 5(3x-2)$	**M1**
expand and simplify to $9x^2 + 3x - 6$ (or unsimplified equivalent)	**M1**
divide by the common factor 3 and factorise: $3x^2 + x - 2 = 0$, i.e. $(3x - 2)(x + 1) = 0$	**M1**
$x = 2/3$ cao	**A1**
$x = -1$ cao	**A1**

Answer: $x = 2/3$ or $x = -1$

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