2.6 Functions: Composite and Inverse Functions: Test Pack

Total Marks

AQA 8365 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 $f(x) = 2x + 3$ and $g(x) = x - 1$.

(a) Find $f(4)$. **(1)**

.....

(b) Find $g(7)$. **(1)**

.....

(c) Find $fg(2)$. **(2)**

.....

(Total for Question 1 is 4 marks)

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2 $f(x) = 3x + 2$ and $g(x) = x - 4$. Find $f(5)$.

(Total for Question 2 is 1 mark)

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3 $f(x) = 4x - 3$ and $g(x) = x^2$. Find $fg(2)$.

(Total for Question 3 is 1 mark)

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4 Explain why $f(x) = x^2$, for all real values of x, does not have an inverse function. State a suitable restricted domain for f(x) so that an inverse function does exist.

(Total for Question 4 is 2 marks)

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5 $f(x) = 3x - 1$ and $g(x) = x + 4$. Find fg(x), giving your answer in the form $ax + b$.

(Total for Question 5 is 2 marks)

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6 $f(x) = (2x - 1)/3$. Find $f^{-1}(x)$.

(Total for Question 6 is 2 marks)

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7 $g(x) = x^2$, for the domain $x \leq 0$. State the range of g(x) for this domain.

(Total for Question 7 is 1 mark)

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8 $f(x) = 4x - 7$ and $g(x) = x^2 + 1$.

(a) Find $fg(x)$. **(2)**

.....

(b) Find $gf(x)$. **(3)**

.....

(c) Solve $fg(x) = 33$. **(3)**

.....

(Total for Question 8 is 8 marks)

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9 $f(x) = (x + 4)/(x - 1)$, where $x \neq 1$. Show that f is a self-inverse function.

(Total for Question 9 is 4 marks)

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10 $f(x) = 2x - 3$ and $g(x) = x^2 - 2x$. Solve $gf(x) = 0$.

(Total for Question 10 is 5 marks)

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11 $f(x) = (3x - 2)/5$ and $g(x) = x + 1$. Determine whether there is a value of x for which $fg(x) = gf(x)$. Justify your answer fully.

(Total for Question 11 is 4 marks)

12 $f(x) = -x + 5$ and $g(x) = -4x - 3$. Find and simplify $gg(x)$.

(Total for Question 12 is 3 marks)

13 $f(x) = 5x + 3$ and $g(x) = x^2 + 2$. Work out the value of $fg(6)$.

(Total for Question 13 is 3 marks)

14 Find the inverse function $f^{-1}(x)$ for $f(x) = (x + 7)/4$.

(Total for Question 14 is 3 marks)

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15 $f(x) = 2x + 2$. Solve $f(x) = f^{-1}(x)$.

(Total for Question 15 is 4 marks)

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16 Find the inverse function $f^{-1}(x)$ for $f(x) = (x + 7)/(x + 5)$.

(Total for Question 16 is 4 marks)

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Mark scheme · 2.6 Functions: Composite and Inverse Functions: Test Pack

Question 1

(a) f(4) = 11 cao **B1**

(a) Answer: 11

(b) g(7) = 6 cao **B1**

(b) Answer: 6

(c) g(2) = 1 found or used **M1**

(c) fg(2) = 5 cao **A1**

(c) Answer: 5

Question 2

f(5) = 17 cao **B1**

Answer: 17

Question 3

fg(2) = 13 cao **B1**

Answer: 13

Question 4

correct reason, e.g. f(x) is many-to-one since f(2) = f(-2) = 4, so f fails the horizontal line test / two x-values give the same output **B1**

a valid restricted domain stated, oe, e.g. $x \geq 0$ (or $x \leq 0$) **B1**

Answer: f is many-to-one (e.g. f(2) = f(-2) = 4); restrict domain to $x \geq 0$ (or $x \leq 0$)

Question 5

substitute g(x) into f, e.g. 3(x+4) - 1 **M1**

fg(x) = 3x + 11 cao **A1**

Answer: fg(x) = 3x + 11

Question 6

let y = (2x-1)/3 and rearrange, e.g. 3y = 2x-1 **M1**

$f^{-1}(x)$ = (3x + 1)/2 cao **A1**

Answer: $f^{-1}(x) = (3x + 1)/2$

Question 7

$g(x) \geq 0$ oe **B1**

Answer: $g(x) \geq 0$

Question 8

(a) substitute g(x) into f, e.g. $4(x^2+1) - 7$ **M1**

(a) fg(x) = $4x^2$ - 3 cao **A1**

(a) Answer: $fg(x) = 4x^2 - 3$

(b) substitute f(x) into g, e.g. $(4x-7)^2 + 1$ **M1**

(b) expand $(4x-7)^2$ correctly to $16x^2 - 56x + 49$ **M1**

(b) gf(x) = $16x^2$ - 56x + 50 cao **A1**

(b) Answer: $gf(x) = 16x^2 - 56x + 50$

(c) set up $4x^2 - 3 = 33$, ft from (a) **M1**

(c) rearrange to $x^2 = 9$ **M1**

(c) x = 3 or x = -3 (both values required) cao **A1**

(c) Answer: x = 3 or x = -3

Question 9

let y = (x+4)/(x-1) and cross-multiply: y(x-1) = x + 4 **M1**

collect x terms, e.g. yx - x = y + 4, i.e. x(y-1) = y+4 **M1**

$f^{-1}(x)$ = (x+4)/(x-1) obtained correctly **A1**

conclusion that $f^{-1}(x) = f(x)$, so f is self-inverse cso **A1**

Answer: $f^{-1}(x) = (x+4)/(x-1) = f(x)$, so f is self-inverse

Question 10

substitute f(x) into g, e.g. $(2x-3)^2 - 2(2x-3)$ **M1**

expand and simplify to $4x^2 - 16x + 15$ (or unsimplified equivalent) **M1**

correct method to solve the quadratic, e.g. factorise (2x-3)(2x-5) = 0 or use the quadratic formula **M1**

x = 1.5 cao **A1**

x = 2.5 cao **A1**

Answer: x = 1.5 or x = 2.5

Question 11

fg(x) = (3(x+1)-2)/5 = (3x+1)/5 found **M1**

gf(x) = (3x-2)/5 + 1 = (3x+3)/5 found **M1**

attempt to equate fg(x) and gf(x) and simplify, e.g. 3x+1 = 3x+3 **M1**

correct conclusion: no such x exists, since this simplifies to 1 = 3, a contradiction cso **A1**

Answer: No value of x exists; fg(x) = gf(x) leads to the contradiction 1 = 3

Question 12

substitutes one function into the other **M1**

expands brackets **M1**

16x + 9 cao **A1**

Answer: 16x + 9

Question 13

g(6) = 38 **M1**

f(38) evaluated **M1**

193 cao **A1**

Answer: 193

Question 14

sets y = f(x) and multiplies by the denominator **M1**

isolates x **M1**

4x - 7 oe cao **A1**

Answer: $f^{-1}(x) = 4x - 7$

Question 15

$f^{-1}(x)$ = (x - 2)/2 or equivalent **M1**

sets $f(x) = f^{-1}(x)$ and clears the fraction **M1**

collects x terms **M1**

x = -2 cao **A1**

Answer: x = -2

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Question 16

$y(x - d) = x + b$	**M1**
collects x terms and factorises	**M1**
$x(y - 1) = ...$	**M1**
$(-5x + 7)/(x - 1)$ oe cao	**A1**

Answer: $f^{-1}(x) = (-5x + 7)/(x - 1)$

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