2.7 Algebraic Proof: 1-hour Pack

AQA 8365 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Let n be an integer. Prove that $n^2 + 3n$ is always even.

(Total for Question 1 is 1 mark)

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2 Let a = 2n and b = 2m, where n and m are integers, so a and b are even numbers. Prove that ab is always a multiple of 4.

(Total for Question 2 is 1 mark)

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3 Let two odd numbers be written as 2n + 1 and 2m + 1, where n and m are integers. Prove that their sum is always even.

(Total for Question 3 is 1 mark)

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4 Prove that $x^2 + 2x + 5 > 0$ for all real values of x.

(Total for Question 4 is 2 marks)

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5 Let n be an integer. Prove that three consecutive multiples of 5 always sum to a multiple of 15.

(Total for Question 5 is 1 mark)

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6 n is an integer. Prove that the sum of 13 consecutive integers n, n + 1, n + 2, ... is always a multiple of 13.

(Total for Question 6 is 4 marks)

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7 n is a positive integer. Prove that $n(n + 14) + 49$ is always a square number.

(Total for Question 7 is 4 marks)

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8 Show that $(n + 5)^2 - (n + 3)^2$ is always a multiple of 2.

(Total for Question 8 is 4 marks)

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9 n is an integer. Prove that $(n + 9)(n + 2)$ is always an even number.

(Total for Question 9 is 3 marks)

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10 n is an integer. Prove that the sum of 3 consecutive integers n, n + 1, n + 2, ... is always a multiple of 3.

(Total for Question 10 is 4 marks)

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11 5n and 5(n + 1) are two consecutive multiples of 5, where n is an integer. Prove that the sum of their squares is a multiple of 25.

(Total for Question 11 is 4 marks)

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12 n is a positive integer. Prove that n(n + 2) + 1 is always a square number.

(Total for Question 12 is 4 marks)

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13 Show that (n - 4)(n + 9) - n(n + 5) is a constant, and find its value.

(Total for Question 13 is 4 marks)

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14 Show that $(n + 8)^2 - (n + 1)^2$ is always a multiple of 7.

(Total for Question 14 is 4 marks)

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15 4n and 4(n + 1) are two consecutive multiples of 4, where n is an integer. Prove that the difference between their squares is always an odd multiple of 16.

(Total for Question 15 is 4 marks)

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16 n is an integer. Prove that the sum of 11 consecutive integers n, n + 1, n + 2, ... is always a multiple of 11.

(Total for Question 16 is 4 marks)

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17 n is a positive integer. Prove that n(n + 6) + 9 is always a square number.

(Total for Question 17 is 4 marks)

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Mark scheme · 2.7 Algebraic Proof: 1-hour Pack

Question 1

cso: factorises $n^2+3n = n(n+3)$ and correctly reasons that if n is even the product is even, and if n is odd then n+3 is even so the product is even, so n(n+3) is always even	**B1**

Answer: $n^2 + 3n = n(n+3)$, always even

Question 2

cso: ab = (2n)(2m) = 4nm, a multiple of 4 since n, m are integers	**B1**

Answer: ab = 4nm, a multiple of 4

Question 3

cso: (2n+1)+(2m+1) = 2n+2m+2 = 2(n+m+1), always even	**B1**

Answer: (2n+1) + (2m+1) = 2(n+m+1), always even

Question 4

completes the square correctly, $x^2 + 2x + 5 = (x+1)^2 - 1 + 5$	**M1**
cso: simplifies to $(x+1)^2 + 4$, and since $(x+1)^2 \geq 0$ for all real x, $(x+1)^2+4 \geq 4 > 0$, so $x^2+2x+5 > 0$ always	**A1**

Answer: $x^2 + 2x + 5 = (x+1)^2 + 4 > 0$ for all real x

Question 5

cso: 5n + (5n+5) + (5n+10) = 15n + 15 = 15(n+1), a multiple of 15	**B1**

Answer: 5n + (5n+5) + (5n+10) = 15(n+1), a multiple of 15

Question 6

sum = n + (n + 1) + ... + (n + 12) written down	**M1**
collects to 13n + 78	**M1**
13(n + 6)	**A1**
conclusion: 13 is a factor, so the sum is a multiple of 13	**C1**

Answer: 13(n + 6)

Question 7

expands: $n^2 + 14n + 49$	**M1**
recognises $7^2 = 49$ and 2 x 7 = 14	**M1**
$(n + 7)^2$	**A1**
$(n + 7)^2$ is the square of the integer n + 7	**C1**

Answer: $(n + 7)^2$

Question 8

expands both brackets: $n^2 + 10n + 25$ and $n^2 + 6n + 9$	**M1**
n^2 terms cancel: 4n + 16	**M1**
2(2n + 8) or equivalent factorised form	**A1**
conclusion linking the factorised form to the claim	**C1**

Answer: 2(2n + 8)

Question 9

n even: (n + 2) is even, so the product is even	**M1**
n odd: (n + 9) is even, so the product is even	**M1**
both cases covered, so the product is always even	**A1**

Answer: always even

Question 10

sum = n + (n + 1) + ... + (n + 2) written down	**M1**
collects to 3n + 3	**M1**
3(n + 1)	**A1**
conclusion: 3 is a factor, so the sum is a multiple of 3	**C1**

Answer: 3(n + 1)

Question 11

$(5n)^2 + (5(n + 1))^2$	**M1**
$25n^2 + 25(n^2 + 2n + 1)$ or equivalent	**M1**
$5^2(2n^2 + 2n + 1)$	**A1**
25 is a factor of every term, so the sum is a multiple of 25	**C1**

Answer: $5^2(2n^2 + 2n + 1)$

Question 12

expands: $n^2 + 2n + 1$	**M1**
recognises $1^2 = 1$ and 2 x 1 = 2	**M1**
$(n + 1)^2$	**A1**
$(n + 1)^2$ is the square of the integer n + 1	**C1**

Answer: $(n + 1)^2$

Question 13

expands the first product (3 terms correct)	**M1**
expands n(...) and subtracts	**M1**
n^2 and n terms cancel; result -36	**A1**
states the value is independent of n	**C1**

Answer: -36

Question 14

expands both brackets: $n^2 + 16n + 64$ and $n^2 + 2n + 1$	**M1**
n^2 terms cancel: 14n + 63	**M1**
7(2n + 9) or equivalent factorised form	**A1**
conclusion linking the factorised form to the claim	**C1**

Answer: 7(2n + 9)

Question 15

$(4(n + 1))^2 - (4n)^2$	**M1**
$16(n^2 + 2n + 1) - 16n^2$ or equivalent	**M1**
$4^2(2n + 1)$	**A1**
2n + 1 is odd, so the result is an odd multiple of 16	**C1**

Answer: $4^2(2n + 1)$

Question 16

sum = n + (n + 1) + ... + (n + 10) written down	**M1**
collects to 11n + 55	**M1**
11(n + 5)	**A1**
conclusion: 11 is a factor, so the sum is a multiple of 11	**C1**

Answer: 11(n + 5)

Question 17

expands: $n^2 + 6n + 9$	**M1**
recognises $3^2 = 9$ and 2 x 3 = 6	**M1**
$(n + 3)^2$	**A1**
$(n + 3)^2$ is the square of the integer n + 3	**C1**

Answer: $(n + 3)^2$

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