2.7 Algebraic Proof: 30-minute Pack

AQA 8365 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Let n be an integer. Prove that the sum of five consecutive integers is always a multiple of 5.

(Total for Question 1 is 1 mark)

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2 Let n be an integer. Prove that $n^2 - n$ is always even.

(Total for Question 2 is 3 marks)

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3 Let two odd numbers be written as $2n + 1$ and $2m + 1$, where n and m are integers. Prove that the product of two odd numbers is always odd.

(Total for Question 3 is 3 marks)

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4 Let n be an integer. Prove that the sum of three consecutive multiples of 4 is always a multiple of 12.

(Total for Question 4 is 3 marks)

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5 n is an integer. Prove that $(n + 7)(n + 4)$ is always an even number.

(Total for Question 5 is 3 marks)

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6 Show that $(n + 8)(n - 8) - n(n)$ is a constant, and find its value.

(Total for Question 6 is 4 marks)

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7 Show that $(x^2 - 16)/(x^2 - x - 12)$ simplifies to $(x + 4)/(x + 3)$, stating any values of x that must be excluded.

(Total for Question 7 is 2 marks)

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8 Ravi attempts to prove that $n^2 - n + 1$ is always odd for any integer n. His working is shown below:

"$n^2 - n = n(n-1)$. Since n-1 is even, n(n-1) must be even, so n(n-1)+1 is odd."

(Total for Question 8 is 2 marks)

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9 A student claims: "$n^2 + n + 41$ is a prime number for every positive integer n." Prove or disprove the student's claim.

(Total for Question 9 is 3 marks)

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10 Let m and n be positive integers with $m > n > 0$. Show that $(m^2-n^2)^2 + (2mn)^2 = (m^2+n^2)^2$.

(Total for Question 10 is 4 marks)

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Mark scheme · 2.7 Algebraic Proof: 30-minute Pack

Question 1

cso: writes the sum as n+(n+1)+(n+2)+(n+3)+(n+4) = 5n+10 = 5(n+2), a multiple of 5	**B1**

Answer: n+(n+1)+(n+2)+(n+3)+(n+4) = 5(n+2), a multiple of 5

Question 2

factorises n^2 - n as n(n-1)	**M1**
identifies n and n-1 as two consecutive integers, one of which must be even	**M1**
correct conclusion: since one of n, n-1 is even, their product n(n-1) is always even	**A1**

Answer: n^2 - n = n(n-1), always even

Question 3

writes the product (2n+1)(2m+1) and expands correctly to 4nm + 2n + 2m + 1	**M1**
factorises 4nm + 2n + 2m as 2(2nm + n + m)	**M1**
correct conclusion that 2(2nm+n+m) + 1 is of the form 2k + 1, so it is always odd	**A1**

Answer: (2n+1)(2m+1) = 2(2nm+n+m) + 1, always odd

Question 4

writes three consecutive multiples of 4 as 4n, 4n+4, 4n+8 and sums to 12n + 12	**M1**
factorises correctly to 12(n+1)	**A1**
correct conclusion that 12(n+1) is always a multiple of 12, since n+1 is an integer	**A1**

Answer: 4n + (4n+4) + (4n+8) = 12(n+1), a multiple of 12

Question 5

n even: (n + 4) is even, so the product is even	**M1**
n odd: (n + 7) is even, so the product is even	**M1**
both cases covered, so the product is always even	**A1**

Answer: always even

Question 6

expands the first product (3 terms correct)	**M1**
expands n(...) and subtracts	**M1**
n^2 and n terms cancel; result -64	**A1**
states the value is independent of n	**C1**

Answer: -64

Question 7

factorises the numerator, x^2 - 16 = (x-4)(x+4), and the denominator, x^2 - x - 12 = (x-4)(x+3)	**M1**
cso: cancels the common factor (x-4) correctly to give (x+4)/(x+3), stating x is not equal to 4 and x is not equal to -3	**A1**

Answer: (x+4)/(x+3), for x not equal to 4, -3

Question 8

identifies the error: n-1 is not always even (e.g. if n is even, such as n=4, then n-1=3 is odd), so the claim 'n-1 is even' is a false general statement	**B1**
gives a correct justification: n and n-1 are consecutive integers, so exactly one of them must always be even, meaning their product n(n-1) is always even regardless of which one is even	**B1**

Answer: The claim that n-1 is always even is false; the correct reasoning is that one of n, n-1 (consecutive integers) is always even

Question 9

selects a suitable value of n to test, e.g. n = 41	**M1**
evaluates n^2 + n + 41 correctly for the chosen n, writing it in a factorisable form, e.g. n^2+n+41 = n(n+1)+41, so at n=41: 41(42)+41 = 41(43)	**M1**
correct conclusion that 41 x 43 = 1763 has factors other than 1 and itself, so it is not prime, disproving the claim by counter-example	**A1**

Answer: False: at n=41, n^2+n+41 = 41 x 43 = 1763, which is not prime

Question 10

expands $(m^2-n^2)^2$ correctly to m^4 - $2m^2n^2$ + n^4	**M1**
expands $(2mn)^2$ correctly to $4m^2n^2$ and adds to the previous expansion	**M1**
simplifies the sum correctly to m^4 + $2m^2n^2$ + n^4	**A1**
recognises this factorises as $(m^2+n^2)^2$, matching the right-hand side, so the identity holds for all m > n > 0	**A1**

Answer: $(m^2-n^2)^2$ + $(2mn)^2$ = $(m^2+n^2)^2$ for all m > n > 0

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