2.8 Direct and Inverse Proportion: 2-hour Pack

AQA 8365 · Calculator allowed · about 120 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 y is directly proportional to the cube of x. When x = 8, y = 6144. Find the value of y when x = 7.

(Total for Question 1 is 3 marks)

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2 y is inversely proportional to x. When x = 6, y = 100. Find the value of y when x = 10.

(Total for Question 2 is 3 marks)

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3 y is directly proportional to the square of x. x is increased by 20%. Work out the percentage change in y. State whether it is an increase or a decrease.

(Total for Question 3 is 3 marks)

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4 A table shows corresponding values of x and y. It is given that y is proportional to x^2.

x: 9, 4, 8

y: 648, 128, p

Work out the value of p.

(Total for Question 4 is 3 marks)

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5 y is directly proportional to the square of x and inversely proportional to z. When x = 2 and z = 3, y = 12. Find the value of y when x = 4 and z = 6.

(Total for Question 5 is 4 marks)

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6 y is inversely proportional to the square root of x. When x = 16, y = 15. Find the value of y when x = 25.

(Total for Question 6 is 3 marks)

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7 y is directly proportional to the square of x. When x = 2, y = 44. Find the positive value of x when y = 275.

(Total for Question 7 is 4 marks)

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8 y is directly proportional to the cube of x. When x = 8, y = 9216. Find the value of y when x = 9.

(Total for Question 8 is 3 marks)

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9 y is inversely proportional to the square of x. When x = 2, y = 90. Find the positive value of x when y = 10.

(Total for Question 9 is 4 marks)

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10 y is inversely proportional to x. When x = 4, y = 18. Find the value of y when x = 2.

(Total for Question 10 is 3 marks)

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11 y is directly proportional to the square of x. When x = 11, y = 1573. Find the positive value of x when y = 832.

(Total for Question 11 is 4 marks)

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12 A table shows corresponding values of x and y. It is given that y is proportional to x^2.

x: 12, 15, 2

y: 288, 450, p

Work out the value of p.

(Total for Question 12 is 3 marks)

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13 y is inversely proportional to x. When x = 8, y = 30. Find the value of y when x = 10.

(Total for Question 13 is 3 marks)

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14 y is directly proportional to $\sqrt{x}$. When x = 9, y = 15.

(a) Find a formula for y in terms of x. **(2)**

(b) Find the value of y when x = 25. **(2)**

(Total for Question 14 is 4 marks)

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15 m is inversely proportional to n^2. When n = 3, m = 8. Find the value of m when n = 6.

(Total for Question 15 is 3 marks)

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16 p is inversely proportional to q^2. Given that p = 5 when q = 2, state the value of the constant of proportionality k.

(Total for Question 16 is 1 mark)

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17 m is inversely proportional to n. When n = 6, m = 15. Find the value of m when n = 10.

(Total for Question 17 is 2 marks)

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18 y is directly proportional to $\sqrt{x}$. When x = 25, y = 40. Find the value of y when x = 64.

(Total for Question 18 is 2 marks)

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19 The time, T hours, taken to fill a water tank is inversely proportional to the number of pumps, n, used. Using 4 pumps takes 5 hours.

(a) Find a formula for T in terms of n. **(2)**

.....

(b) Find the time taken to fill the tank using 8 pumps. **(1)**

.....

(c) State one assumption needed for this model to be valid. **(1)**

(Total for Question 19 is 4 marks)

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20 y is directly proportional to x^3. When x = 2, y = 40.

(a) Find the value of y when x = 5. **(3)**

(b) Find the value of x when y = 1080. **(2)**

(Total for Question 20 is 5 marks)

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21 The time, h hours, taken to paint a fence is inversely proportional to the number of painters, p, working on it. Using 3 painters takes 8 hours. Find how long it would take using 4 painters, assuming all painters work at the same constant rate.

(Total for Question 21 is 2 marks)

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22 y is directly proportional to x^3. When x = 2, y = 48. Find the value of x when y = 750.

(Total for Question 22 is 2 marks)

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23 The force, F newtons, between two magnets is inversely proportional to the square of the distance, d cm, between them. When d = 2, F = 45.

(a) Find F in terms of d. **(2)**

.....

(b) Find F when d = 6. **(1)**

.....

(c) Find the distance when F = 20. **(2)**

.....

(d) State what happens to F as d increases. **(1)**

(Total for Question 23 is 6 marks)

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24 The energy, E joules, stored in a stretched spring is directly proportional to the square of its extension, x cm. When x = 4, E = 48. Find the value of E when x = 7.

(Total for Question 24 is 3 marks)

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25 The force, F newtons, needed to tow a boat at a steady speed is directly proportional to the square of the speed, v m/s. A speed of 4 m/s requires a force of 20 N. Find the force needed to tow the boat at a speed of 10 m/s.

(Total for Question 25 is 3 marks)

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26 y is directly proportional to x^2. When x = 2, y = 20.

(a) Find a formula for y in terms of x. **(2)**

(b) Separately, y is inversely proportional to w. When x = 2 (so y = 20), it is also known that w = 25 at this point. Find a formula for y in terms of w. **(2)**

(c) Hence find the value of w when x = 5. **(3)**

(Total for Question 26 is 7 marks)

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27 The volume, V cm^3, of a certain type of crystal is directly proportional to the square of its width, w cm, and directly proportional to its height, h cm. A crystal with width 4 cm and height 10 cm has volume 320 cm^3.

(a) Find a formula for V in terms of w and h. **(2)**

(b) Find the volume of a crystal with width 5 cm and height 6 cm. **(2)**

(c) A crystal has height 8 cm and volume 144 cm^3. Find its width, w cm. **(2)**

(Total for Question 27 is 6 marks)

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28 y is directly proportional to x. When $x = 4$, $y = 20$. Find, in exact surd form, the positive value of x for which $y = x^2 - 9$.

(Total for Question 28 is 4 marks)

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29 y is directly proportional to x^3 for $x \geq 0$. The graph of y against x^3 is a straight line through the origin. When $x = 2$, $y = 56$.

(a) State the gradient of the graph of y against x^3. **(1)**

(b) Find the value of y when $x = 1/2$. **(2)**

(c) A second variable, z, is such that z is inversely proportional to x, and $z = 4$ when $x = 7$. Find the value of x (other than $x = 0$) for which $y = z$, giving your answer correct to 3 significant figures. **(4)**

(Total for Question 29 is 7 marks)

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Mark scheme · 2.8 Direct and Inverse Proportion: 2-hour Pack

Question 1

writes $y = kx^3$ or equivalent	**M1**
finds $k = 12$	**M1**
$y = 4116$ cao	**A1**

Answer: $y = 4116$

Question 2

writes $y = k/x$ or equivalent	**M1**
finds $k = 600$	**M1**
$y = 60$ cao	**A1**

Answer: $y = 60$

Question 3

new $x = 6/5x$, so y is multiplied by 6/5 to the power 2	**M1**
multiplier = 36/25	**M1**
y increases by 44% cao	**A1**

Answer: y increases by 44%

Question 4

uses the given relationship with a constant k	**M1**
finds $k = 8$	**M1**
$p = 512$ cao	**A1**

Answer: $p = 512$

Question 5

$y = kx^2/z$ or equivalent	**M1**
substitutes to find $k = 9$	**M1**
substitutes the new values	**M1**
$y = 24$ cao	**A1**

Answer: $y = 24$

Question 6

writes $y = k/\sqrt{x}$ or equivalent	**M1**
finds $k = 60$	**M1**
$y = 12$ cao	**A1**

Answer: $y = 12$

Question 7

writes $y = kx^2$ or equivalent	**M1**
finds $k = 11$	**M1**
substitutes y and solves for x	**M1**
$x = 5$ cao	**A1**

Answer: $x = 5$

Question 8

writes $y = kx^3$ or equivalent	**M1**
finds $k = 18$	**M1**
$y = 13122$ cao	**A1**

Answer: $y = 13122$

Question 9

writes $y = k/x^2$ or equivalent	**M1**
finds $k = 360$	**M1**
substitutes y and solves for x	**M1**
$x = 6$ cao	**A1**

Answer: $x = 6$

Question 10

writes $y = k/x$ or equivalent	**M1**
finds $k = 72$	**M1**
$y = 36$ cao	**A1**

Answer: $y = 36$

Question 11

writes $y = kx^2$ or equivalent	**M1**
finds $k = 13$	**M1**
substitutes y and solves for x	**M1**
$x = 8$ cao	**A1**

Answer: $x = 8$

Question 12

uses the given relationship with a constant k	**M1**
finds $k = 2$	**M1**
$p = 8$ cao	**A1**

Answer: $p = 8$

Question 13

writes $y = k/x$ or equivalent	**M1**
finds $k = 240$	**M1**
$y = 24$ cao	**A1**

Answer: $y = 24$

Question 14

(a) finds $k = 5$ (from $15 = k\sqrt{9}$, i.e. $15 = 3k$)	**M1**
(a) $y = 5\sqrt{x}$ cao	**A1**

(a) Answer: $y = 5\sqrt{x}$

(b) substitutes $x = 25$ into their formula from part (a)	**M1**
(b) $y = 25$ cao	**A1**

(b) Answer: $y = 25$

Question 15

finds $k = 72$ (from $k = 8 \times 3^2$)	**M1**
substitutes $n = 6$ into $m = 72/n^2$	**M1**
$m = 2$ cao	**A1**

Answer: $m = 2$

Question 16

20 cao	**B1**

Answer: $k = 20$

Question 17

finds the constant of proportionality, $k = 90$ (from $k = 6 \times 15$)	**M1**
$m = 9$ cao	**A1**

Answer: $m = 9$

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Question 18

finds k = 8 (from 40 = k $\sqrt{25}$, i.e. 40 = 5k) **M1**

y = 64 cao **A1**

Answer: y = 64

Question 19

(a) finds k = 20 (from k = 5 x 4) **M1**

(a) T = 20/n cao **A1**

(a) Answer: T = 20/n

(b) T = 2.5 hours cao **B1**

(b) Answer: T = 2.5 hours

(c) valid assumption, e.g. all pumps work at the same constant rate and do not interfere with each other oe **B1**

(c) Answer: All pumps work at the same constant rate

Question 20

(a) finds k = 5 (from 40 = k$(2)^3$, i.e. 40 = 8k) **M1**

(a) substitutes x = 5 into y = $5x^3$ **M1**

(a) y = 625 cao **A1**

(a) Answer: y = 625

(b) sets up 1080 = $5x^3$ and rearranges to x^3 = 216 **M1**

(b) x = 6 cao **A1**

(b) Answer: x = 6

Question 21

finds k = 24 (from k = 3 x 8) **M1**

h = 6 hours cao **A1**

Answer: h = 6 hours

Question 22

finds k = 6 (from 48 = k$(2)^3$, i.e. 48 = 8k) and sets up 750 = $6x^3$ **M1**

x = 5 cao **A1**

Answer: x = 5

Question 23

(a) finds k = 180 (from k = 45 x 2^2) **M1**

(a) F = 180/d^2 cao **A1**

(a) Answer: F = 180/d^2

(b) F = 5 cao **B1**

(b) Answer: F = 5

(c) sets up 20 = 180/d^2 and solves d^2 = 9 **M1**

(c) d = 3 cm cao **A1**

(c) Answer: d = 3 cm

(d) F decreases (towards zero) oe, since F is inversely proportional to d^2 **B1**

(d) Answer: F decreases towards zero as d increases

Question 24

finds k = 3 (from 48 = k$(4)^2$, i.e. 48 = 16k) **M1**

substitutes x = 7 into E = $3x^2$ **M1**

E = 147 cao **A1**

Answer: E = 147

Question 25

finds k = 1.25 (from 20 = k$(4)^2$, i.e. 20 = 16k) **M1**

substitutes v = 10 into F = $1.25v^2$ **M1**

F = 125 cao **A1**

Answer: F = 125 N

Question 26

(a) finds k = 5 (from 20 = k$(2)^2$, i.e. 20 = 4k) **M1**

(a) y = $5x^2$ cao **A1**

(a) Answer: y = $5x^2$

(b) finds the second constant of proportionality, k2 = 500 (from k2 = 20 x 25) **M1**

(b) y = 500/w cao **A1**

(b) Answer: y = 500/w

(c) finds y = 125 using their formula from part (a) (y = 5$(5)^2$) **M1**

(c) substitutes their y = 125 into their formula from part (b) to form 125 = 500/w **M1**

(c) w = 4 cao **A1**

(c) Answer: w = 4

Question 27

(a) finds k = 2 (from 320 = k$(4)^2$(10), i.e. 320 = 160k) **M1**

(a) V = $2w^2$ h cao **A1**

(a) Answer: V = $2w^2$ h

(b) substitutes w = 5, h = 6 into their formula **M1**

(b) V = 300 cm^3 cao **A1**

(b) Answer: V = 300 cm^3

(c) sets up 144 = $2w^2$(8) and solves w^2 = 9 **M1**

(c) w = 3 cm cao **A1**

(c) Answer: w = 3 cm

Question 28

finds k = 5 (from 20 = 4k), so y = 5x **M1**

sets up 5x = x^2 - 9 and rearranges to x^2 - 5x - 9 = 0 **M1**

applies the quadratic formula with a = 1, b = -5, c = -9, giving x = $(5 \pm \sqrt{25 + 36})/2$ **M1**

x = $(5 + \sqrt{61})/2$ cao (exact surd form, rejecting the negative root) **A1**

Answer: x = $(5 + \sqrt{61})/2$

Question 29

(a) gradient = 7 cao **B1**

(a) Answer: Gradient = 7

(b) substitutes x = 1/2 into y = $7x^3$ **M1**

(b) y = 7/8 (or 0.875) cao **A1**

(b) Answer: y = 7/8

(c) finds the constant for z, k2 = 28 (from k2 = 4 x 7), so z = 28/x **M1**

(c) sets up $7x^3$ = 28/x **M1**

(c) multiplies both sides by x and divides by 7 to obtain x^4 = 4 **M1**

(c) x = 1.41 (3sf) cao (exact value $\sqrt{2}$) **A1**

(c) Answer: x = $\sqrt{2}$ = 1.41 (3sf)

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