Binomial Expansion: 30-minute Pack

AQA 8365 · Calculators not allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 For $F(x)=(1-x)(2+x)^4$: (a) expand $(2+x)^4$ in ascending powers of x; (b) show the contributing products and find the coefficients of x^2 and x^3 in F(x); (c) state the degree and leading coefficient of F(x), explaining how the highest powers combine.

(Total for Question 1 is 6 marks)

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2 Let $G(x)=(1+bx)^3(1-2x)$. Its coefficient of x is 4. (a) Form an equation and find b. (b) Using that value, expand $(1+bx)^3$, then expand G(x) fully, showing why its x^2 term vanishes. (c) State the degree and leading coefficient of G(x).

(Total for Question 2 is 6 marks)

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3 Find the coefficient of x^2 in $(2+x)^5$. Show the number of ways to choose the two x factors and the contribution from each choice.

(Total for Question 3 is 3 marks)

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4 Find the third term of $(3-2x)^4$ when written in ascending powers of x. Show which power of x is needed and the coefficient calculation.

(Total for Question 4 is 3 marks)

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5 Expand $(1-2x)^4$ fully in ascending powers of x. Explain why the coefficient of x^3 is negative even though the exponent 4 is even.

(Total for Question 5 is 4 marks)

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6 A solid cube has side length $(5+t)$ cm, where $t \geq 0$. Write its volume as an expanded polynomial in t. Hence write an expression for the increase in volume compared with a cube of side 5 cm, and find that increase when $t=1$.

(Total for Question 6 is 4 marks)

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Mark scheme · Binomial Expansion: 30-minute Pack

Question 1

Expansion of $(2+x)^4$ has $16 + 32x + 24x^2$.	**A1**
Expansion has final terms $8x^3 + x^4$.	**A1**
Identifies x^2 contributions 24 and -32, and x^3 contributions 8 and -24.	**M1**
Obtains both x^2 coefficient -8 and x^3 coefficient -16.	**A1**
Highest-power product is $(-x)*x^4=-x^5$.	**B1**
States degree 5 and leading coefficient -1. Accept equivalent forms and valid alternative methods for the corresponding marks.	**B1**

Answer: $(2+x)^4 = 16 + 32x + 24x^2 + 8x^3 + x^4$; coefficient of x^2: -8; coefficient of x^3: -16; degree 5; leading coefficient -1.

Question 2

Forms 3b-2=4 from the two linear contributions.	**M1**
b=2.	**A1**
Expansion $(1+2x)^3 = 1 + 6x + 12x^2 + 8x^3$.	**A1**
Shows quadratic cancellation $12x^2 - 2x*6x = 0$.	**M1**
Remaining product terms give $G(x)=1 + 4x - 16x^3 - 16x^4$.	**A1**
Degree 4 and leading coefficient -16 stated. Accept equivalent forms and valid alternative methods for the corresponding marks.	**B1**

Answer: b = 2; $(1+2x)^3 = 1 + 6x + 12x^2 + 8x^3$; $G(x) = 1 + 4x - 16x^3 - 16x^4$. The x^2 coefficient is 12 - 12 = 0; degree 4; leading coefficient -16.

Question 3

10 ways to choose the two x factors.	**B1**
Each choice contributes $2^3=8$ to the coefficient.	**M1**
Combines the contributions to obtain 80. Accept equivalent forms and valid alternative methods for the corresponding marks.	**A1**

Answer: 80

Question 4

Identifies x^2 as the power in the third term, counting the constant term first.	**B1**
Uses $6*3^2*(-2)^2$ for its coefficient.	**M1**
$216x^2$ stated as the term, including x^2. Accept equivalent forms and valid alternative methods for the corresponding marks.	**A1**

Answer: $216x^2$

Question 5

Constant and linear terms 1 - 8x.	**A1**
Quadratic and quartic terms $24x^2 + 16x^4$.	**A1**
Cubic term $-32x^3$.	**A1**
Explains that each x^3 contribution selects three negative -2x factors and one positive 1, so its sign is negative. Accept equivalent forms and valid alternative methods for the corresponding marks.	**B1**

Answer: $1 - 8x + 24x^2 - 32x^3 + 16x^4$. The x^3 term uses three negative factors, so its coefficient is negative.

Question 6

Volume constant and linear terms 125 + 75t.	**A1**
Volume quadratic and cubic terms $15t^2 + t^3$.	**A1**
Subtracts the original volume 125 to give increase $75t + 15t^2 + t^3$.	**M1**
Increase at t=1 is 91 cm^3. Accept equivalent forms and valid alternative methods for the corresponding marks.	**A1**

Answer: Volume = $125 + 75t + 15t^2 + t^3$ cm^3; increase = $75t + 15t^2 + t^3$ cm^3; at t=1, increase = 91 cm^3.

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