# Binomial Expansion

AQA 8365 · Calculators not allowed · about 45 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

**Answer ALL questions. Show all your working.**

**1** State the constant term in the expansion of $(3-x)^4$.

**(Total for Question 1 is 1 mark)**

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**2** The coefficient of $x^2$ in $(1+ax)^4$ is 54, where a is positive. Form an equation for a and solve it. Then show the calculation for the coefficient of $x^3$.

**(Total for Question 2 is 4 marks)**

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**3** For $F(x)=(1-x)(2+x)^4$: (a) expand $(2+x)^4$ in ascending powers of x; (b) show the contributing products and find the coefficients of $x^2$ and $x^3$ in F(x); (c) state the degree and leading coefficient of F(x), explaining how the highest powers combine.

**(Total for Question 3 is 6 marks)**


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**4** Let $G(x)=(1+bx)^3(1-2x)$. Its coefficient of x is 4. (a) Form an equation and find b. (b) Using that value, expand $(1+bx)^3$, then expand G(x) fully, showing why its $x^2$ term vanishes. (c) State the degree and leading coefficient of G(x).

**(Total for Question 4 is 6 marks)**

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**5** State whether the coefficient of $x^3$ in $(2-x)^5$ is positive or negative.

**(Total for Question 5 is 1 mark)**

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**6** Expand $(x+4)^2$ and collect like terms.

**(Total for Question 6 is 2 marks)**

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**7** Expand $(1-3x)^3$ in ascending powers of x. Show how you obtain the coefficient of $x^2$.

**(Total for Question 7 is 2 marks)**

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**8** Find the coefficient of $x^2$ in $(2+x)^5$. Show the number of ways to choose the two x factors and the contribution from each choice.

**(Total for Question 8 is 3 marks)**


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**9** Find the third term of $(3-2x)^4$ when written in ascending powers of x. Show which power of x is needed and the coefficient calculation.

**(Total for Question 9 is 3 marks)**

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**10** Expand $(1-2x)^4$ fully in ascending powers of x. Explain why the coefficient of $x^3$ is negative even though the exponent 4 is even.

**(Total for Question 10 is 4 marks)**

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**11** A solid cube has side length (5+t) cm, where $t \geq 0$. Write its volume as an expanded polynomial in t. Hence write an expression for the increase in volume compared with a cube of side 5 cm, and find that increase when t=1.

**(Total for Question 11 is 4 marks)**

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**12** Expand both $(2+x)^4$ and $(2-x)^4$. Hence simplify $(2+x)^4-(2-x)^4$. Explain why every even power, including the constant term, cancels.

**(Total for Question 12 is 4 marks)**


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# Mark scheme · Binomial Expansion

## Question 1

81 stated as the constant term. Accept equivalent forms and valid alternative methods for the corresponding marks. **B1**

**Answer: 81**

## Question 2

Forms $6a^2=54$. **M1**

$a=3$, using the condition a is positive to reject -3. **A1**

Uses $4*3^3$ for the $x^3$ coefficient. **M1**

Coefficient of $x^3$ is 108. Accept equivalent forms and valid alternative methods for the corresponding marks. **A1**

**Answer: a = 3; coefficient of $x^3$ is 108.**

## Question 3

Expansion of $(2+x)^4$ has $16 + 32x + 24x^2$. **A1**

Expansion has final terms $8x^3 + x^4$. **A1**

Identifies $x^2$ contributions 24 and -32, and $x^3$ contributions 8 and -24. **M1**

Obtains both $x^2$ coefficient -8 and $x^3$ coefficient -16. **A1**

Highest-power product is $(-x)*x^4=-x^5$. **B1**

States degree 5 and leading coefficient -1. Accept equivalent forms and valid alternative methods for the corresponding marks. **B1**

**Answer: $(2+x)^4 = 16 + 32x + 24x^2 + 8x^3 + x^4$; coefficient of $x^2$: -8; coefficient of $x^3$: -16; degree 5; leading coefficient -1.**

## Question 4

Forms $3b-2=4$ from the two linear contributions. **M1**

$b=2$. **A1**

Expansion $(1+2x)^3 = 1 + 6x + 12x^2 + 8x^3$. **A1**

Shows quadratic cancellation $12x^2 - 2x*6x = 0$. **M1**

Remaining product terms give $G(x)=1 + 4x - 16x^3 - 16x^4$. **A1**

Degree 4 and leading coefficient -16 stated. Accept equivalent forms and valid alternative methods for the corresponding marks. **B1**

**Answer: b = 2; $(1+2x)^3 = 1 + 6x + 12x^2 + 8x^3$; $G(x) = 1 + 4x - 16x^3 - 16x^4$. The $x^2$ coefficient is 12 - 12 = 0; degree 4; leading coefficient -16.**

## Question 5

Negative stated. Accept equivalent forms and valid alternative methods for the corresponding marks. **B1**

**Answer: Negative**

## Question 6

$x^2$ and constant term 16 correct. **A1**

The two cross products collected to give 8x. Accept equivalent forms and valid alternative methods for the corresponding marks. **A1**

**Answer: $16 + 8x + x^2$**

## Question 7

Shows the $x^2$ coefficient as $3*(-3)^2$ or the sum of three products of two -3 factors. **M1**

Complete expansion $1 - 9x + 27x^2 - 27x^3$. Accept equivalent forms and valid alternative methods for the corresponding marks. **A1**

**Answer: $1 - 9x + 27x^2 - 27x^3$**

## Question 8

10 ways to choose the two x factors. **B1**

Each choice contributes $2^3=8$ to the coefficient. **M1**

Combines the contributions to obtain 80. Accept equivalent forms and valid alternative methods for the corresponding marks. **A1**

**Answer: 80**

## Question 9

Identifies $x^2$ as the power in the third term, counting the constant term first. **B1**

Uses $6*3^2*(-2)^2$ for its coefficient. **M1**

$216x^2$ stated as the term, including $x^2$. Accept equivalent forms and valid alternative methods for the corresponding marks. **A1**

**Answer: $216x^2$**

## Question 10

Constant and linear terms $1 - 8x$. **A1**

Quadratic and quartic terms $24x^2 + 16x^4$. **A1**

Cubic term $-32x^3$. **A1**

Explains that each $x^3$ contribution selects three negative -2x factors and one positive 1, so its sign is negative. Accept equivalent forms and valid alternative methods for the corresponding marks. **B1**

**Answer: $1 - 8x + 24x^2 - 32x^3 + 16x^4$. The $x^3$ term uses three negative factors, so its coefficient is negative.**

## Question 11

Volume constant and linear terms $125 + 75t$. **A1**

Volume quadratic and cubic terms $15t^2 + t^3$. **A1**

Subtracts the original volume 125 to give increase $75t + 15t^2 + t^3$. **M1**

Increase at t=1 is 91 $cm^3$. Accept equivalent forms and valid alternative methods for the corresponding marks. **A1**

**Answer: Volume = $125 + 75t + 15t^2 + t^3$ $cm^3$; increase = $75t + 15t^2 + t^3$ $cm^3$; at t=1, increase = 91 $cm^3$.**

## Question 12

Expansion of $(2+x)^4$ correct. **A1**

Expansion of $(2-x)^4$ correct. **A1**

Difference simplifies to $64x + 16x^3$. **A1**

Replacing x by -x preserves each even-power term, so equal even-power terms subtract to zero. Accept equivalent forms and valid alternative methods for the corresponding marks. **B1**

**Answer: $(2+x)^4 = 16 + 32x + 24x^2 + 8x^3 + x^4$; $(2-x)^4 = 16 - 32x + 24x^2 - 8x^3 + x^4$; difference = $64x + 16x^3$. Even powers are unchanged by x becoming -x, so they cancel on subtraction.**


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