5.1 Differentiation of Polynomials: 2-hour Pack

AQA 8365 · Calculator allowed · about 120 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 $y = x(4x^2 - 6x + 3)$. Find dy/dx.

(Total for Question 1 is 4 marks)

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2 A curve has equation $y = 2x^3 - 32x + 4$. Find the values of x at the points on the curve where the gradient is -8.

(Total for Question 2 is 5 marks)

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3 $y = -4x^4 - 4x^3 + 8x^2 - 6x - 4$. Find dy/dx and d^2y/dx^2.

(Total for Question 3 is 4 marks)

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4 $y = (3x - 5)^2$. Find dy/dx.

(Total for Question 4 is 4 marks)

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5 A curve has equation $y = -4x^3 + 3x^2 + 2x + 1$. Find the gradient of the curve at the point where $x = 2$.

(Total for Question 5 is 3 marks)

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6 $y = (2x - 4)^2$. Find dy/dx.

(Total for Question 6 is 4 marks)

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7 A curve has equation $y = 2x^3 + 3x^2 - 39x - 1$. Find the values of x at the points on the curve where the gradient is -3.

(Total for Question 7 is 5 marks)

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8 $y = (3x + 1)(x + 6)$. Find dy/dx.

(Total for Question 8 is 4 marks)

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9 $y = (3x + 3)^2$. Find dy/dx.

(Total for Question 9 is 4 marks)

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10 A curve has equation $y = kx^2 - 2x - 1$. The gradient of the curve at the point where $x = -2$ is -10. Find the value of k.

(Total for Question 10 is 4 marks)

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11 A curve has equation $y = x^2 + 8x - 1$. Find the gradient of the curve at the point where $x = -2$.

(Total for Question 11 is 3 marks)

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12 A curve has equation $y = x^3 - 6x^2 + 3x + 9$. Find the values of x at the points on the curve where the gradient is -6.

(Total for Question 12 is 5 marks)

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13 $f(x) = x^3 + 2x^2 - 4x + 8$. Find $f'(x)$.

(Total for Question 13 is 3 marks)

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14 $f(x) = -4x^2 + 2x$. Find $f'(x)$.

(Total for Question 14 is 3 marks)

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15 A curve has equation $y = kx^2 + x - 2$. The gradient of the curve at the point where $x = -1$ is -7. Find the value of k.

(Total for Question 15 is 4 marks)

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16 $f(x) = -4x^2 - 4x + 1$. Find $f'(x)$.

(Total for Question 16 is 3 marks)

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17 A curve has equation $y = 2x^3 - 3x^2 - 114x - 5$. Find the values of x at the points on the curve where the gradient is 6.

(Total for Question 17 is 5 marks)

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18 A curve has equation $y = 2x^3 - 6x^2 - 43x + 8$. Find the values of x at the points on the curve where the gradient is 5.

(Total for Question 18 is 5 marks)

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19 A curve has equation $y = kx^2 + 2x - 6$. The gradient of the curve at the point where $x = 1$ is 6. Find the value of k.

(Total for Question 19 is 4 marks)

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20 For each curve, find the value(s) of x for which the gradient takes the stated value.

(a) Curve: $y = x^2 - 6x + 10$. Find the value of x for which the gradient of the curve is 8. **(3)**

(b) Curve: $y = x^3 - 3x$. Find the value(s) of x for which the gradient of the curve is 9. **(3)**

(Total for Question 20 is 6 marks)

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21 A curve has equation $y = x^3 - 3x^2 - 9x + 5$. Find the coordinates of the stationary points of the curve.

(Total for Question 21 is 5 marks)

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22 A curve has equation $y = x^2 - 5x + 6$. Find the equation of the tangent to the curve at the point where $x = 4$.

(Total for Question 22 is 5 marks)

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23 A curve has equation $y = x^2 - 10x + 3$. Find the value of x for which the gradient of the curve is 4.

(Total for Question 23 is 3 marks)

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24 A curve has equation $y = x^2 - 3x + 8$. Find the equation of the tangent to the curve at the point where $x = 5$.

(Total for Question 24 is 4 marks)

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25 A curve has equation $y = x^3 - 3x^2 - 24x + 5$. Find the coordinates of the stationary points of the curve.

(Total for Question 25 is 4 marks)

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26 A curve has equation $y = x^3 + 9x^2 + 28x - 3$. Show that this curve is an increasing function for all values of x.

(Total for Question 26 is 4 marks)

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Mark scheme · 5.1 Differentiation of Polynomials: 2-hour Pack

Question 1

expands the brackets	**M1**
$y = 4x^3 - 6x^2 + 3x$	**A1**
differentiates	**M1**
$dy/dx = 12x^2 - 12x + 3$ cao	**A1**

Answer: $dy/dx = 12x^2 - 12x + 3$

Question 2

differentiates	**M1**
$dy/dx = 6x^2 - 32$	**A1**
sets $dy/dx = -8$ and rearranges to a quadratic	**M1**
solves the quadratic	**M1**
$x = -2$ or $x = 2$ cao	**A1**

Answer: $x = -2$ or $x = 2$

Question 3

differentiates once	**M1**
$dy/dx = -16x^3 - 12x^2 + 16x - 6$	**A1**
differentiates again	**M1**
$d^2y/dx^2 = -48x^2 - 24x + 16$	**A1**

Answer: $dy/dx = -16x^3 - 12x^2 + 16x - 6$, $d^2y/dx^2 = -48x^2 - 24x + 16$

Question 4

expands the brackets	**M1**
$y = 9x^2 - 30x + 25$	**A1**
differentiates	**M1**
$dy/dx = 18x - 30$ cao	**A1**

Answer: $dy/dx = 18x - 30$

Question 5

differentiates	**M1**
$dy/dx = -12x^2 + 6x + 2$	**A1**
-34 cao	**A1**

Answer: -34

Question 6

expands the brackets	**M1**
$y = 4x^2 - 16x + 16$	**A1**
differentiates	**M1**
$dy/dx = 8x - 16$ cao	**A1**

Answer: $dy/dx = 8x - 16$

Question 7

differentiates	**M1**
$dy/dx = 6x^2 + 6x - 39$	**A1**
sets $dy/dx = -3$ and rearranges to a quadratic	**M1**
solves the quadratic	**M1**
$x = -3$ or $x = 2$ cao	**A1**

Answer: $x = -3$ or $x = 2$

Question 8

expands the brackets	**M1**
$y = 3x^2 + 19x + 6$	**A1**
differentiates	**M1**
$dy/dx = 6x + 19$ cao	**A1**

Answer: $dy/dx = 6x + 19$

Question 9

expands the brackets	**M1**
$y = 9x^2 + 18x + 9$	**A1**
differentiates	**M1**
$dy/dx = 18x + 18$ cao	**A1**

Answer: $dy/dx = 18x + 18$

Question 10

differentiates	**M1**
$dy/dx = 2kx - 2$	**A1**
substitutes $x = -2$ and sets equal to -10	**M1**
$k = 2$ cao	**A1**

Answer: $k = 2$

Question 11

differentiates	**M1**
$dy/dx = 2x + 8$	**A1**
4 cao	**A1**

Answer: 4

Question 12

differentiates	**M1**
$dy/dx = 3x^2 - 12x + 3$	**A1**
sets $dy/dx = -6$ and rearranges to a quadratic	**M1**
solves the quadratic	**M1**
$x = 1$ or $x = 3$ cao	**A1**

Answer: $x = 1$ or $x = 3$

Question 13

multiplies by the power and reduces the power by 1 for at least one term	**M1**
at least two terms correct	**A1**
$f'(x) = 3x^2 + 4x - 4$ cao	**A1**

Answer: $f'(x) = 3x^2 + 4x - 4$

Question 14

multiplies by the power and reduces the power by 1 for at least one term	**M1**
at least two terms correct	**A1**
$f'(x) = -8x + 2$ cao	**A1**

Answer: $f'(x) = -8x + 2$

Question 15

differentiates	**M1**
$dy/dx = 2kx + 1$	**A1**
substitutes $x = -1$ and sets equal to -7	**M1**
$k = 4$ cao	**A1**

Answer: $k = 4$

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Question 16

multiplies by the power and reduces the power by 1 for at least one term	**M1**
at least two terms correct	**A1**
f'(x) = -8x - 4 cao	**A1**

Answer: f'(x) = -8x - 4

Question 17

differentiates	**M1**
$dy/dx = 6x^2 - 6x - 114$	**A1**
sets dy/dx = 6 and rearranges to a quadratic	**M1**
solves the quadratic	**M1**
x = -4 or x = 5 cao	**A1**

Answer: x = -4 or x = 5

Question 18

differentiates	**M1**
$dy/dx = 6x^2 - 12x - 43$	**A1**
sets dy/dx = 5 and rearranges to a quadratic	**M1**
solves the quadratic	**M1**
x = -2 or x = 4 cao	**A1**

Answer: x = -2 or x = 4

Question 19

differentiates	**M1**
dy/dx = 2kx + 2	**A1**
substitutes x = 1 and sets equal to 6	**M1**
k = 2 cao	**A1**

Answer: k = 2

Question 20

(a) differentiates and sets 2x - 6 = 8	**M1**
(a) rearranges correctly to solve for x	**M1**
(a) x = 7 cao	**A1**

(a) Answer: x = 7

(b) differentiates and sets $3x^2 - 3 = 9$	**M1**
(b) rearranges to $x^2 = 4$ and takes both square roots	**M1**
(b) x = 2 or x = -2 (both values required) cao	**A1**

(b) Answer: x = 2 or x = -2

Question 21

differentiates to obtain $dy/dx = 3x^2 - 6x - 9$	**M1**
sets dy/dx = 0 and factorises or solves the quadratic	**M1**
x = -1 and x = 3 (both values) cao	**A1**
substitutes both x-values back into $y = x^3 - 3x^2 - 9x + 5$, dependent on previous M1 (dM1)	**M1**
stationary points (-1, 10) and (3, -22) (both required) cao	**A1**

Answer: Stationary points at (-1, 10) and (3, -22)

Question 22

y-coordinate at x = 4 is 2 (point (4, 2) found)	**B1**
differentiates to obtain dy/dx = 2x - 5	**M1**
gradient at x = 4 is 3	**A1**
forms tangent equation using y - 2 = 3(x - 4), dependent on gradient found (dM1)	**M1**
y = 3x - 10 oe (e.g. y - 3x + 10 = 0)	**A1**

Answer: y = 3x - 10

Question 23

differentiates and sets 2x - 10 = 4	**M1**
rearranges correctly to solve for x	**M1**
x = 7 cao	**A1**

Answer: x = 7

Question 24

finds the y-coordinate at x = 5 is 18	**M1**
differentiates to obtain dy/dx = 2x - 3 and finds the gradient at x = 5 is 7	**M1**
forms the tangent equation using y - 18 = 7(x - 5), dependent on both previous marks (dM1)	**M1**
y = 7x - 17 oe cao	**A1**

Answer: y = 7x - 17

Question 25

differentiates to obtain $dy/dx = 3x^2 - 6x - 24$	**M1**
sets dy/dx = 0 and factorises to (x - 4)(x + 2) = 0, dependent on previous M1 (dM1)	**M1**
x = 4 and x = -2 (both values) cao	**A1**
stationary points (4, -75) and (-2, 33) (both required, ft from the x-values)	**A1**

Answer: Stationary points at (4, -75) and (-2, 33)

Question 26

differentiates to obtain $dy/dx = 3x^2 + 18x + 28$	**M1**
attempts to complete the square on $3x^2 + 18x + 28$	**M1**
$dy/dx = 3(x + 3)^2 + 1$ oe, correctly completed	**A1**
concludes that since $(x+3)^2 \geq 0$ for all x, $dy/dx \geq 1 > 0$ for all x, so the curve is increasing for all x (cso)	**A1**

Answer: $dy/dx = 3(x+3)^2 + 1$, which is always at least 1, so the curve is increasing for all values of x (shown)

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