5.1 Differentiation of Polynomials: 30-minute Pack

AQA 8365 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Differentiate $y = 5x^3$ with respect to x.

(Total for Question 1 is 1 mark)

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2 For each curve, find the value of the gradient at the given point.

(a) Curve: $y = x^3 - 4x^2 + 5$. Calculate the gradient of the curve at the point where x = 2. **(3)**

(b) Curve: $y = 2x^2 - 7x + 3$. Calculate the gradient of the curve at the point where x = -1. **(3)**

(Total for Question 2 is 6 marks)

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3 Expand the brackets, then find dy/dx for $y = (3x - 2)(2x + 1)$.

(Total for Question 3 is 2 marks)

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4 A curve has equation $y = x^3 - 2x + 1$. The point P(2, 5) lies on the curve. Find the equation of the normal to the curve at P, giving your answer in the form $x + ky + c = 0$.

(Total for Question 4 is 5 marks)

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5 A curve has equation $y = x^3 + 6x^2 + 15x - 2$. Show that this curve is an increasing function for all values of x.

(Total for Question 5 is 4 marks)

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6 $f(x) = 5x^3 - 2x^2 + 8x + 2$. Find $f'(x)$.

(Total for Question 6 is 3 marks)

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7 A farmer has 40 metres of fencing to enclose a rectangular field. One side of the field runs along an existing straight wall, so fencing is only needed for the other three sides. Let x metres be the width of the field, measured perpendicular to the wall, and let A m^2 be the area enclosed.

(a) Show that $A = 40x - 2x^2$. **(2)**

(b) Find the value of x that gives the maximum area, and calculate this maximum area. **(5)**

(Total for Question 7 is 7 marks)

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Mark scheme · 5.1 Differentiation of Polynomials: 30-minute Pack

Question 1

$dy/dx = 15x^2$ cao	**B1**

Answer: $dy/dx = 15x^2$

Question 2

(a) differentiates to obtain $dy/dx = 3x^2 - 8x$	**M1**
(a) substitutes $x = 2$ into dy/dx	**M1**
(a) gradient = -4 cao	**A1**

(a) Answer: gradient = -4

(b) differentiates to obtain $dy/dx = 4x - 7$	**M1**
(b) substitutes $x = -1$ into dy/dx	**M1**
(b) gradient = -11 cao	**A1**

(b) Answer: gradient = -11

Question 3

expands brackets to obtain $y = 6x^2 - x - 2$ oe	**M1**
$dy/dx = 12x - 1$ cao ft from expansion	**A1**

Answer: $dy/dx = 12x - 1$

Question 4

differentiates to obtain $dy/dx = 3x^2 - 2$	**M1**
gradient of tangent at $x = 2$ is 10	**A1**
uses normal gradient = -1/(tangent gradient) = -1/10, dependent on tangent gradient found (dM1)	**M1**
forms normal equation using $y - 5 = -1/10\ (x - 2)$	**M1**
$x + 10y - 52 = 0$ oe (e.g. $y = -x/10 + 26/5$)	**A1**

Answer: $x + 10y - 52 = 0$

Question 5

differentiates to obtain $dy/dx = 3x^2 + 12x + 15$	**M1**
attempts to complete the square on $3x^2 + 12x + 15$	**M1**
$dy/dx = 3(x + 2)^2 + 3$ oe, correctly completed	**A1**
concludes that since $(x+2)^2 \geq 0$ for all x, $dy/dx \geq 3 > 0$ for all x, so the curve is increasing for all x (cso)	**A1**

Answer: $dy/dx = 3(x+2)^2 + 3$, which is always at least 3, so the curve is increasing for all values of x (shown)

Question 6

multiplies by the power and reduces the power by 1 for at least one term	**M1**
at least two terms correct	**A1**
$f'(x) = 15x^2 - 4x + 8$ cao	**A1**

Answer: $f'(x) = 15x^2 - 4x + 8$

Question 7

(a) expresses the length of the third fenced side as (40 - 2x) and multiplies by x	**M1**
(a) $A = 40x - 2x^2$ fully shown cso	**A1**

(a) Answer: $A = 40x - 2x^2$ (shown)

(b) differentiates to obtain $dA/dx = 40 - 4x$, ft from part (a)	**M1**
(b) sets $dA/dx = 0$ and solves for x, dependent on previous M1 (dM1)	**M1**
(b) x = 10	**A1**
(b) substitutes $x = 10$ back into $A = 40x - 2x^2$, dependent on $x = 10$ (dM1)	**M1**
(b) maximum area = 200 m²	**A1**

(b) Answer: x = 10 m, maximum area = 200 m^2

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