5.1 Differentiation of Polynomials: Test Pack

AQA 8365 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Differentiate $y = x^6$ with respect to x.

(Total for Question 1 is 1 mark)

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2 For each curve, expand the brackets first and then find dy/dx.

(a) $y = (x + 4)(x - 2)$ **(3)**

(b) $y = (2x - 1)(3x + 5)$ **(3)**

(Total for Question 2 is 6 marks)

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3 Which of the following is the correct derivative of $y = 5x^4 - 3x^2 + 8$?

(a) Select the correct option. **(1)**

- ☐ A) $20x^3 - 6x$
- ☐ B) $20x^3 - 3x$
- ☐ C) $5x^3 - 6x$
- ☐ D) $20x^3 - 6x + 8$

(Total for Question 3 is 1 mark)

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4 A curve has equation $y = x^3 - 5x + 2$. Calculate the gradient of the curve at the point where $x = 3$.

(Total for Question 4 is 2 marks)

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5 Expand the brackets, then find dy/dx for $y = (x + 5)(x - 3)$.

(Total for Question 5 is 2 marks)

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6 A curve has equation $y = 2x^3 + 3x^2 - 12x + 7$. Find the x-coordinates of the stationary points of the curve and determine, using the second derivative, whether each is a maximum or a minimum point.

(Total for Question 6 is 6 marks)

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7 Differentiate $y = -3x^3 - 7x^2 + 5x + 4$ with respect to x.

(Total for Question 7 is 3 marks)

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8 A curve has equation $y = kx^2 - 9x + 1$. The gradient of the curve at the point where $x = 3$ is 3. Find the value of k.

(Total for Question 8 is 4 marks)

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9 A curve has equation $y = kx^2 + 5x - 5$. The gradient of the curve at the point where $x = -1$ is 1. Find the value of k.

(Total for Question 9 is 4 marks)

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10 The curve C has equation $y = x^3 - 5x^2 + 2x - 1$. Show that the gradient of C at the point where $x = 4$ is 10.

(Total for Question 10 is 3 marks)

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11 A curve has equation $y = x^4 - 8x^2 + 3$. Find the coordinates of all the stationary points of the curve, and use the second derivative to determine the nature of each.

(Total for Question 11 is 7 marks)

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12 The cost, C pounds, of producing x items of a product is modelled by $C = x^3 - 30x^2 + 225x + 400$, for $0 < x \leq 20$.

(a) Find dC/dx. **(2)**

(b) Find the value of x that minimises the cost, and find this minimum cost. **(6)**

(Total for Question 12 is 8 marks)

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13 A framer is making a rectangular picture frame from a single length of moulding 100 cm long, bent to form all four outer edges of the frame. Let x cm be the width of the frame and let A cm^2 be the area enclosed by the frame.

(a) Show that $A = 50x - x^2$. **(2)**

(b) Find the value of x that gives the maximum area, and calculate this maximum area. **(2)**

(Total for Question 13 is 4 marks)

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Mark scheme · 5.1 Differentiation of Polynomials: Test Pack

Question 1

$dy/dx = 6x^5$ cao	**B1**

Answer: $dy/dx = 6x^5$

Question 2

(a) expands brackets to obtain a quadratic	**M1**
(a) correct expansion $y = x^2 + 2x - 8$ oe	**A1**
(a) $dy/dx = 2x + 2$ cao ft from expansion	**A1**

(a) Answer: $dy/dx = 2x + 2$

(b) expands brackets to obtain a quadratic	**M1**
(b) correct expansion $y = 6x^2 + 7x - 5$ oe	**A1**
(b) $dy/dx = 12x + 7$ cao ft from expansion	**A1**

(b) Answer: $dy/dx = 12x + 7$

Question 3

(a) A cao	**B1**

(a) Answer: A) $20x^3 - 6x$

Question 4

differentiates to obtain $dy/dx = 3x^2 - 5$ and substitutes $x = 3$	**M1**
gradient = 22 cao	**A1**

Answer: gradient = 22

Question 5

expands brackets to obtain $y = x^2 + 2x - 15$ oe	**M1**
$dy/dx = 2x + 2$ cao ft from expansion	**A1**

Answer: $dy/dx = 2x + 2$

Question 6

differentiates to obtain $dy/dx = 6x^2 + 6x - 12$	**M1**
sets $dy/dx = 0$ and factorises or solves the quadratic	**M1**
$x = -2$ and $x = 1$ (both values) cao	**A1**
finds the second derivative $d2y/dx2 = 12x + 6$	**M1**
evaluates $d2y/dx2$ at both $x = -2$ and $x = 1$, dependent on previous M1 (dM1)	**M1**
$x = -2$ gives a maximum ($d2y/dx2 = -18 < 0$) and $x = 1$ gives a minimum ($d2y/dx2 = 18 > 0$)	**A1**

Answer: Maximum at $x = -2$; minimum at $x = 1$

Question 7

multiplies by the power and reduces the power by 1 for at least one term	**M1**
at least two terms correct	**A1**
$dy/dx = -9x^2 - 14x + 5$ cao	**A1**

Answer: $dy/dx = -9x^2 - 14x + 5$

Question 8

differentiates	**M1**
$dy/dx = 2kx - 9$	**A1**
substitutes $x = 3$ and sets equal to 3	**M1**
$k = 2$ cao	**A1**

Answer: $k = 2$

Question 9

differentiates	**M1**
$dy/dx = 2kx + 5$	**A1**
substitutes $x = -1$ and sets equal to 1	**M1**
$k = 2$ cao	**A1**

Answer: $k = 2$

Question 10

differentiates correctly to obtain $dy/dx = 3x^2 - 10x + 2$	**M1**
substitutes $x = 4$ into dy/dx	**M1**
gradient = 10 fully shown cso (answer printed, all working correct)	**A1**

Answer: 10 (shown)

Question 11

differentiates to obtain $dy/dx = 4x^3 - 16x$	**M1**
sets $dy/dx = 0$ and factorises fully, e.g. $4x(x-2)(x+2) = 0$	**M1**
$x = 0$, $x = 2$ and $x = -2$ (all three values) cao	**A1**
$y = 3$, $y = -13$ and $y = -13$ respectively (ft from the x-values)	**A1**
finds the second derivative $d2y/dx2 = 12x^2 - 16$	**M1**
evaluates $d2y/dx2$ at all three x-values, dependent on previous M1 (dM1)	**M1**
(0, 3) is a maximum ($d2y/dx2 = -16 < 0$); (2, -13) and (-2, -13) are both minima ($d2y/dx2 = 32 > 0$)	**A1**

Answer: Maximum at (0, 3); minima at (2, -13) and (-2, -13)

Question 12

(a) differentiates at least two terms correctly	**M1**
(a) $dC/dx = 3x^2 - 60x + 225$ cao	**A1**

(a) Answer: $dC/dx = 3x^2 - 60x + 225$

(b) sets $dC/dx = 0$ and simplifies, e.g. dividing by 3 to give $x^2 - 20x + 75 = 0$, ft from part (a)	**M1**
(b) factorises or solves to find both stationary values, dependent on previous M1 (dM1)	**M1**
(b) $x = 5$ and $x = 15$ (both values)	**A1**
(b) finds the second derivative $d2C/dx2 = 6x - 60$ and evaluates it at $x = 15$, dependent on $x = 15$ (dM1)	**M1**
(b) $d2C/dx2 = 30 > 0$, so $x = 15$ gives the minimum cost ($x = 5$ gives a maximum, since $d2C/dx2 = -30 < 0$)	**A1**
(b) minimum cost = £400, ft from $x = 15$	**A1**

(b) Answer: x = 15 items, minimum cost = £400

Question 13

(a) uses half the perimeter to express the other side as $(50 - x)$ and multiplies by x	**M1**
(a) $A = 50x - x^2$ fully shown cso	**A1**

(a) Answer: $A = 50x - x^2$ (shown)

(b) differentiates to obtain $dA/dx = 50 - 2x$, sets equal to 0 and solves, ft from part (a)	**M1**
(b) $x = 25$, giving a maximum area of 625 cm²	**A1**

(b) Answer: x = 25 cm, maximum area = 625 cm^2

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