5.2 Applications of Differentiation: Gradients, Tangents and Turning Points: 1-hour Pack

AQA 8365 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A curve has equation $y = -2x^3 - 6x^2 + 18x - 1$. Find the values of x for which y is decreasing.

(Total for Question 1 is 4 marks)

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2 A curve has equation $y = x^2 + 9x + 2$. At the point P on the curve, the tangent is perpendicular to the line $y = -x/7 + 5$. Find the coordinates of P.

(Total for Question 2 is 4 marks)

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3 A curve has equation $y = x^2 + 8x + 26$. Find the coordinates of the turning point of the curve and state whether it is a maximum or a minimum.

(Total for Question 3 is 4 marks)

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4 A curve has equation $y = x^2 + 3x + 2$. At the point P on the curve, the tangent is perpendicular to the line $y = x/7 + 5$. Find the coordinates of P.

(Total for Question 4 is 4 marks)

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5 A curve has equation $y = -x^2 - 4x + 6$. Find the coordinates of the turning point of the curve and state whether it is a maximum or a minimum.

(Total for Question 5 is 4 marks)

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6 A curve has equation $y = 3x^2 + 12x + 18$. Find the coordinates of the turning point of the curve and state whether it is a maximum or a minimum.

(Total for Question 6 is 4 marks)

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7 A curve has equation $y = x^3 + kx^2 + 3x - 2$. Find the range of values of k for which the curve has no stationary points.

(Total for Question 7 is 4 marks)

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8 A curve has equation $y = -3x^2 + 42x - 144$. Find the coordinates of the turning point of the curve and state whether it is a maximum or a minimum.

(Total for Question 8 is 4 marks)

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9 A curve has equation $y = x^3 - 7x$. Find dy/dx.

(Total for Question 9 is 1 mark)

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10 The profit, P thousand pounds, made by a company from selling x thousand items of a product is modelled by $P = -2x^2 + 40x - 50$, for $x > 0$.

(a) Find dP/dx. **(1)**

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(b) Find the number of items (in thousands) that should be sold to maximise profit, and find this maximum profit. **(4)**

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(c) Use the second derivative to justify that this value of x gives a maximum profit. **(2)**

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(Total for Question 10 is 7 marks)

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11 The temperature, T degrees Celsius, of a chemical reaction t minutes after it begins is modelled by $T = -2t^3 + 21t^2 - 60t + 40$, for $0 \leq t \leq 8$.

(a) Find dT/dt. **(1)**

(b) Find the values of t, where $0 \leq t \leq 8$, at which the temperature is momentarily not changing. **(3)**

(c) Determine, using the second derivative, whether the temperature is a maximum or a minimum at each of these times. **(3)**

(Total for Question 11 is 7 marks)

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12 Curve C1 has equation $y = x^2 - 2x + 5$. Curve C2 has equation $y = -x^2 + 6x - 3$.

(a) Find the gradient function of each curve. **(2)**

(b) Find the value of x for which the two curves have the same gradient. **(2)**

(c) Show that the two curves also meet at this value of x, and state what this means geometrically. **(3)**

(Total for Question 12 is 7 marks)

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Mark scheme · 5.2 Applications of Differentiation: Gradients, Tangents and Turning Points: 1-hour Pack

Question 1

differentiates	**M1**
$dy/dx = -6x^2 - 12x + 18 < 0$, with the critical values found	**M1**
critical values $x = -3$ and $x = 1$	**A1**
$x < -3$ or $x > 1$ cao	**A1**

Answer: $x < -3$ or $x > 1$

Question 2

differentiates: $dy/dx = 2x + 9$	**M1**
tangent gradient = 7 (negative reciprocal of the given gradient)	**M1**
$x = -1$	**A1**
(-1, -6) cao	**A1**

Answer: (-1, -6)

Question 3

differentiates and sets equal to 0	**M1**
$x = -4$	**A1**
$y = 10$	**A1**
minimum	**B1**

Answer: (-4, 10) minimum

Question 4

differentiates: $dy/dx = 2x + 3$	**M1**
tangent gradient = -7 (negative reciprocal of the given gradient)	**M1**
$x = -5$	**A1**
(-5, 12) cao	**A1**

Answer: (-5, 12)

Question 5

differentiates and sets equal to 0	**M1**
$x = -2$	**A1**
$y = 10$	**A1**
maximum	**B1**

Answer: (-2, 10) maximum

Question 6

differentiates and sets equal to 0	**M1**
$x = -2$	**A1**
$y = 6$	**A1**
minimum	**B1**

Answer: (-2, 6) minimum

Question 7

$dy/dx = 3x^2 + 2kx + 3$	**M1**
discriminant $(2k)^2 - 4(3)(3) < 0$	**M1**
$k^2 < 9$	**A1**
$-3 < k < 3$ cao	**A1**

Answer: $-3 < k < 3$

Question 8

differentiates and sets equal to 0	**M1**
$x = 7$	**A1**
$y = 3$	**A1**
maximum	**B1**

Answer: (7, 3) maximum

Question 9

$dy/dx = 3x^2 - 7$ cao	**B1**

Answer: $dy/dx = 3x^2 - 7$

Question 10

(a) $dP/dx = -4x + 40$ cao	**B1**

(a) Answer: $dP/dx = -4x + 40$

(b) sets their $dP/dx = 0$ and solves for x, ft from part (a)	**M1**
(b) $x = 10$	**A1**
(b) substitutes $x = 10$ back into $P = -2x^2 + 40x - 50$, dependent on previous M1 (dM1)	**M1**
(b) maximum profit = 150 (i.e. £150,000)	**A1**

(b) Answer: x = 10 (10,000 items), maximum profit = £150,000

(c) $d2P/dx2 = -4$	**B1**
(c) since $d2P/dx2 < 0$, this confirms a maximum	**B1**

(c) Answer: $d2P/dx2 = -4 < 0$, so $x = 10$ gives a maximum

Question 11

(a) $dT/dt = -6t^2 + 42t - 60$ cao	**B1**

(a) Answer: $dT/dt = -6t^2 + 42t - 60$

(b) sets their $dT/dt = 0$ and simplifies, e.g. dividing by -6 to give $t^2 - 7t + 10 = 0$, ft from part (a)	**M1**
(b) factorises or solves the quadratic, e.g. $(t-2)(t-5) = 0$	**M1**
(b) $t = 2$ and $t = 5$ (both values) cao	**A1**

(b) Answer: t = 2 and t = 5

(c) finds the second derivative $d2T/dt2 = -12t + 42$	**M1**
(c) evaluates at both $t = 2$ (= 18) and $t = 5$ (= -18), ft from part (b)	**A1**
(c) $t = 2$ gives a minimum temperature ($d2T/dt2 = 18 > 0$); $t = 5$ gives a maximum temperature ($d2T/dt2 = -18 < 0$)	**A1**

(c) Answer: Minimum at t = 2 (T = -12 C); maximum at t = 5 (T = 15 C)

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Question 12

(a) dy1/dx = 2x - 2 **B1**

(a) dy2/dx = -2x + 6 **B1**

(a) Answer: dy1/dx = 2x - 2; dy2/dx = -2x + 6

(b) sets 2x - 2 = -2x + 6, ft from part (a) **M1**

(b) x = 2 cao **A1**

(b) Answer: x = 2

(c) substitutes x = 2 into both y1 and y2, ft from part (b) **M1**

(c) y1(2) = 5 and y2(2) = 5, so both curves pass through (2, 5), fully shown cso **A1**

(c) correct geometric statement: since the curves meet at (2, 5) and have the same gradient there, they touch (are tangent to each other) at this point oe **A1**

(c) Answer: Both curves pass through (2, 5) with the same gradient, so the curves are tangent to each other at (2, 5)

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