5.2 Applications of Differentiation: Gradients, Tangents and Turning Points: 2-hour Pack

AQA 8365 · Calculator allowed · about 120 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A curve has equation $y = 2x^3 - 12x^2 - 30x - 9$. Find the values of x for which y is increasing.

(Total for Question 1 is 4 marks)

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2 A curve has equation $y = x^3 - 12x + 10$. Find the coordinates of the stationary points of the curve and determine whether each is a maximum or a minimum.

(Total for Question 2 is 6 marks)

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3 A curve has equation $y = x^2 + 6x + 13$. Find the coordinates of the turning point of the curve and state whether it is a maximum or a minimum.

(Total for Question 3 is 4 marks)

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4 A curve has equation $y = -3x^2 + 7x$. Find the equation of the normal to the curve at the point where $x = 1$. Give your answer in the form $ax + by = c$, where a, b and c are integers.

(Total for Question 4 is 5 marks)

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5 A curve has equation $y = x^2 - 4x + 5$. Find the equation of the normal to the curve at the point where $x = 3$. Give your answer in the form $x + 2y = c$.

(Total for Question 5 is 4 marks)

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6 Find the equation of the tangent to the curve $y = x^2 - 5x + 3$ that is parallel to the line $y = 3x - 7$.

(Total for Question 6 is 5 marks)

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7 A ball is thrown vertically upwards. Its height, h metres, above the ground after t seconds is modelled by $h = 20t - 5t^2$, for $t \geq 0$.

(a) Find dh/dt. This represents the vertical velocity of the ball. **(1)**

(b) Find the time at which the ball reaches its maximum height, and find this maximum height. **(4)**

(c) Find d2h/dt2 (the acceleration of the ball), and explain what this value represents in this context. **(2)**

(Total for Question 7 is 7 marks)

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8 A curve has equation $y = x^3 - 12x$. Find the x-coordinates of the stationary points of the curve.

(Total for Question 8 is 2 marks)

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9 A curve has equation $y = 2x^2 - 5x + 1$. Find the value of x at which the gradient of the curve equals 7.

(Total for Question 9 is 2 marks)

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10 A curve has equation $y = x^2 + 2x - 8$. Find the coordinates of the point on the curve where the gradient is zero.

(Total for Question 10 is 2 marks)

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11 A curve has equation $y = x^2 - 10x + 21$. Find the x-coordinate of the turning point of the curve.

(Total for Question 11 is 2 marks)

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12 A curve has equation $y = x^2 - 2x - 3$.

(a) Find the equation of the normal to the curve at the point where $x = 0$. **(4)**

(b) Find the area of the triangle enclosed by this normal and the two coordinate axes. **(3)**

(Total for Question 12 is 7 marks)

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13 The curve C has equation $y = x^2 - 4x + 7$. The point P(1, 4) lies on C.

(a) Confirm that P lies on C, and find the equation of the tangent to C at P. **(4)**

.....

(b) The tangent meets the y-axis at the point Q. Find the coordinates of Q. **(1)**

.....

(c) Find the area of triangle OPQ, where O is the origin. **(3)**

.....

(Total for Question 13 is 8 marks)

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14 A curve has equation $y = 3x^2 - 2$. Find the second derivative d2y/dx2.

(Total for Question 14 is 1 mark)

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15 A curve has equation $y = x^3 - 6x^2 + 2$. Find the second derivative d2y/dx2.

(Total for Question 15 is 2 marks)

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16 A curve has equation $y = x^2 - 8x + 19$. Find the coordinates of the turning point of the curve and state whether it is a minimum or a maximum.

(Total for Question 16 is 3 marks)

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17 A curve has equation $y = x^3 - 3x^2 - 9x + 10$. Show that the gradient of the curve at the point where $x = -2$ is 15.

(Total for Question 17 is 3 marks)

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18 A curve has equation $y = x^2 - 6x + 5$. Find the equation of the tangent to the curve at the point where $x = 1$.

(Total for Question 18 is 4 marks)

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19 A curve has equation $y = 2x^3 + 3x^2 - 36x + 5$. Find the range of values of x for which the curve is a decreasing function.

(Total for Question 19 is 3 marks)

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20 A curve has equation $y = (x^2 + 6)/x$, for x not equal to 0.

(a) Show that y can be written as $x + 6/x$. **(1)**

(b) Find dy/dx. **(2)**

(c) Find the coordinates of the stationary points of the curve. **(4)**

(d) Use the second derivative to determine the nature of each stationary point. **(3)**

(Total for Question 20 is 10 marks)

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21 A rectangular sheet of card measures 24 cm by 15 cm. A square of side x cm is cut from each corner, and the sides are folded up to form an open-topped box.

(a) Show that the volume, V cm^3, of the box is given by $V = 4x^3 - 78x^2 + 360x$. **(3)**

(b) Find dV/dx. **(2)**

(c) Given that $0 < x < 7.5$, find the value of x that maximises the volume of the box, and find this maximum volume. **(5)**

(Total for Question 21 is 10 marks)

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22 A curve has equation $y = 2x^3 - 15x^2 + 24x + 6$.

(a) Find dy/dx. **(1)**

(b) Find the x-coordinates of the stationary points of the curve. **(3)**

(c) Find the corresponding y-coordinates of each stationary point. **(2)**

(d) Use the second derivative to determine the nature of each stationary point. **(3)**

(e) Find the range of values of x for which the curve is an increasing function. **(2)**

(Total for Question 22 is 11 marks)

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Mark scheme · 5.2 Applications of Differentiation: Gradients, Tangents and Turning Points: 2-hour Pack

Question 1

differentiates **M1**

$dy/dx = 6x^2 - 24x - 30 > 0$, with the critical values found **M1**

critical values $x = -1$ and $x = 5$ **A1**

$x < -1$ or $x > 5$ cao **A1**

Answer: $x < -1$ or $x > 5$

Question 2

differentiates **M1**

$dy/dx = 3x^2 - 12$ **A1**

sets $dy/dx = 0$ and solves **M1**

$x = -2$ and $x = 2$ **A1**

$y = 26$ and $y = -6$ **A1**

natures maximum and minimum justified (second derivative or gradient test) **B1**

Answer: (-2, 26) maximum, (2, -6) minimum

Question 3

differentiates and sets equal to 0 **M1**

$x = -3$ **A1**

$y = 4$ **A1**

minimum **B1**

Answer: (-3, 4) minimum

Question 4

differentiates **M1**

gradient of tangent = 1 **A1**

gradient of normal = -1/(1) **M1**

line through (1, 4) with the normal gradient **M1**

$x + y = 5$ oe cao **A1**

Answer: $x + y = 5$

Question 5

finds the y-coordinate at $x = 3$, $y = 2$ **M1**

finds the gradient of the tangent at $x = 3$ using $dy/dx = 2x - 4$, gradient = 2 **M1**

uses normal gradient = -1/(tangent gradient) = -1/2, dependent on previous M1 (dM1) **M1**

$x + 2y = 7$ oe (e.g. $y = -0.5x + 3.5$) **A1**

Answer: $x + 2y = 7$

Question 6

identifies that the gradient of the line $y = 3x - 7$ is 3 **M1**

differentiates the curve and sets $dy/dx = 2x - 5 = 3$, dependent on previous M1 (dM1) **M1**

$x = 4$ **A1**

$y = -1$, giving the point (4, -1) **A1**

$y = 3x - 13$ oe **A1**

Answer: $y = 3x - 13$

Question 7

(a) $dh/dt = 20 - 10t$ cao **B1**

(a) Answer: $dh/dt = 20 - 10t$

(b) sets their $dh/dt = 0$, ft from part (a) **M1**

(b) t = 2 seconds **A1**

(b) substitutes $t = 2$ into $h = 20t - 5t^2$, dependent on previous M1 (dM1) **M1**

(b) maximum height = 20 metres **A1**

(b) Answer: t = 2 seconds, maximum height = 20 m

(c) $d2h/dt2 = -10$ **B1**

(c) correct interpretation: this is the (constant) acceleration due to gravity, acting downwards, causing the ball to decelerate on the way up oe **B1**

(c) Answer: -10 m/s^2, the constant deceleration due to gravity

Question 8

differentiates to obtain $dy/dx = 3x^2 - 12$ and sets equal to 0 **M1**

$x = 2$ and $x = -2$ (both values) cao **A1**

Answer: $x = 2$ or $x = -2$

Question 9

differentiates and sets $4x - 5 = 7$ **M1**

$x = 3$ cao **A1**

Answer: $x = 3$

Question 10

differentiates and sets $2x + 2 = 0$ to find $x = -1$ **M1**

point (-1, -9) cao **A1**

Answer: (-1, -9)

Question 11

differentiates to obtain $dy/dx = 2x - 10$ and sets equal to 0 **M1**

$x = 5$ cao **A1**

Answer: $x = 5$

Question 12

(a) finds the y-coordinate at $x = 0$, $y = -3$ **M1**

(a) finds the gradient of the tangent at $x = 0$ using $dy/dx = 2x - 2$, gradient = -2 **M1**

(a) uses normal gradient = -1/(tangent gradient) = 1/2, dependent on previous M1 (dM1) **M1**

(a) $y = 0.5x - 3$ oe **A1**

(a) Answer: $y = 0.5x - 3$

(b) finds the x-intercept of the normal, ft from part (a) **M1**

(b) identifies the two perpendicular distances (base and height) from the axis intercepts, dependent on previous M1 (dM1) **M1**

(b) area = 9 square units **A1**

(b) Answer: 9 square units

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Question 13

(a) confirms y(1) = 1-4+7 = 4, so P lies on C **B1**

(a) differentiates to obtain dy/dx = 2x - 4, and finds the gradient at x = 1, gradient = -2 **M1**

(a) forms the tangent equation y - 4 = -2(x - 1), dependent on previous M1 (dM1) **M1**

(a) y = -2x + 6 oe **A1**

(a) Answer: y = -2x + 6

(b) Q = (0, 6), ft from part (a) **B1**

(b) Answer: Q = (0, 6)

(c) identifies OQ = 6 as a suitable base (on the y-axis), ft from part (b) **M1**

(c) identifies the height as the horizontal distance from P to the y-axis, height = 1, dependent on previous M1 (dM1) **M1**

(c) area = 3 square units **A1**

(c) Answer: 3 square units

Question 14

d2y/dx2 = 6 cao **B1**

Answer: d2y/dx2 = 6

Question 15

differentiates once to obtain dy/dx = $3x^2$ - 12x **M1**

d2y/dx2 = 6x - 12 cao **A1**

Answer: d2y/dx2 = 6x - 12

Question 16

differentiates and sets 2x - 8 = 0 to find x = 4 **M1**

y = 3, so the point is (4, 3) **A1**

minimum, since the coefficient of x^2 is positive (or d2y/dx2 = 2 > 0) **B1**

Answer: (4, 3), a minimum point

Question 17

differentiates correctly to obtain dy/dx = $3x^2$ - 6x - 9 **M1**

substitutes x = -2 into dy/dx **M1**

gradient = 15 fully shown cso **A1**

Answer: 15 (shown)

Question 18

finds the y-coordinate at x = 1 is 0 **M1**

differentiates to obtain dy/dx = 2x - 6 and finds the gradient at x = 1 is -4 **M1**

forms the tangent equation using y - 0 = -4(x - 1), dependent on both previous marks (dM1) **M1**

y = -4x + 4 oe cao **A1**

Answer: y = -4x + 4

Question 19

differentiates to obtain dy/dx = $6x^2$ + 6x - 36 **M1**

sets dy/dx = 0 and factorises to find the critical values x = -3 and x = 2, dependent on previous M1 (dM1) **M1**

-3 < x < 2 oe **A1**

Answer: -3 < x < 2

Question 20

(a) y = x^2/x + 6/x = x + 6/x, fully shown cso **B1**

(a) Answer: y = x + 6/x (shown)

(b) writes 6/x as $6x^{-1}$ and differentiates **M1**

(b) dy/dx = 1 - 6/x^2 oe cao **A1**

(b) Answer: dy/dx = 1 - 6/x^2

(c) sets their dy/dx = 0, giving x^2 = 6, ft from part (b) **M1**

(c) takes both square roots, x = $\sqrt{6}$ or x = -$\sqrt{6}$ **M1**

(c) x = $\sqrt{6}$ and x = -$\sqrt{6}$ (awrt 2.45 and -2.45) cao **A1**

(c) ($\sqrt{6}$, $2\sqrt{6}$) and (-$\sqrt{6}$, -$2\sqrt{6}$) (awrt (2.45, 4.90) and (-2.45, -4.90)) **A1**

(c) Answer: ($\sqrt{6}$, $2\sqrt{6}$) and (-$\sqrt{6}$, -$2\sqrt{6}$), i.e. approximately (2.45, 4.90) and (-2.45, -4.90)

(d) finds the second derivative d2y/dx2 = 12/x^3, ft from part (b) **M1**

(d) evaluates the sign at x = $\sqrt{6}$ (positive) and at x = -$\sqrt{6}$ (negative) **A1**

(d) ($\sqrt{6}$, $2\sqrt{6}$) is a minimum; (-$\sqrt{6}$, -$2\sqrt{6}$) is a maximum **A1**

(d) Answer: Minimum at ($\sqrt{6}$, $2\sqrt{6}$); maximum at (-$\sqrt{6}$, -$2\sqrt{6}$)

Question 21

(a) writes the base dimensions as (24 - 2x) and (15 - 2x), with height x **M1**

(a) expands V = x(24-2x)(15-2x) to obtain 360x - $78x^2$ + $4x^3$ (or equivalent unsimplified form) **M1**

(a) V = $4x^3$ - $78x^2$ + 360x, fully shown cso **A1**

(a) Answer: V = $4x^3$ - $78x^2$ + 360x (shown)

(b) differentiates at least two terms correctly, ft from part (a) **M1**

(b) dV/dx = $12x^2$ - 156x + 360 cao **A1**

(b) Answer: dV/dx = $12x^2$ - 156x + 360

(c) sets their dV/dx = 0 and simplifies, e.g. dividing by 12 to give x^2 - 13x + 30 = 0, ft from part (b) **M1**

(c) factorises or solves the quadratic, e.g. (x-3)(x-10) = 0 **M1**

(c) rejects x = 10 as it lies outside 0 < x < 7.5, so x = 3 **A1**

(c) substitutes x = 3 into V = $4x^3$ - $78x^2$ + 360x, dependent on previous M1 (dM1) **M1**

(c) maximum volume = 486 cm^3 **A1**

(c) Answer: x = 3 cm, maximum volume = 486 cm^3

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Question 22

(a) $dy/dx = 6x^2 - 30x + 24$ cao **B1**

(a) Answer: $dy/dx = 6x^2 - 30x + 24$

(b) sets their $dy/dx = 0$ and simplifies, e.g. dividing by 6 to give $x^2 - 5x + 4 = 0$, ft from part (a) **M1**

(b) factorises or solves the quadratic, e.g. $(x-1)(x-4) = 0$ **M1**

(b) $x = 1$ and $x = 4$ (both values) cao **A1**

(b) Answer: $x = 1$ and $x = 4$

(c) substitutes both x-values into $y = 2x^3 - 15x^2 + 24x + 6$, ft from part (b) **M1**

(c) (1, 17) and (4, -10) (both required) cao **A1**

(c) Answer: (1, 17) and (4, -10)

(d) finds the second derivative d2y/dx2 = 12x - 30 **M1**

(d) evaluates at both $x = 1$ (= -18) and $x = 4$ (= 18), ft from part (b) **A1**

(d) (1, 17) is a maximum (d2y/dx2 = -18 < 0); (4, -10) is a minimum (d2y/dx2 = 18 > 0) **A1**

(d) Answer: Maximum at (1, 17); minimum at (4, -10)

(e) tests the sign of $dy/dx = 6(x-1)(x-4)$ outside the stationary values **M1**

(e) $x < 1$ or $x > 4$ oe **A1**

(e) Answer: $x < 1$ or $x > 4$

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