5.2 Applications of Differentiation: Gradients, Tangents and Turning Points: Test Pack

AQA 8365 · Calculator allowed · about 65 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A curve has equation $y = x^3 + 2x^2 - 5x + 7$.

(a) Find dy/dx. **(2)**

.....

(b) Find the gradient of the curve at the point where $x = 2$. **(2)**

.....

(Total for Question 1 is 4 marks)

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2 A curve has equation $y = x^3 + 2x$. Find the equation of the normal to the curve at the point where $x = 0$. Give your answer in the form $ax + by = c$, where a, b and c are integers.

(Total for Question 2 is 5 marks)

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3 A square piece of card of side 30 cm has a square of side x cm cut from each corner. The sides are folded up to make an open box. Show that the volume is $V = x(30 - 2x)^2$ and find the value of x that gives the maximum volume. Find this maximum volume.

(Total for Question 3 is 6 marks)

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4 A curve has equation $y = -x^2 + 14x - 58$. Find the coordinates of the turning point of the curve and state whether it is a maximum or a minimum.

(Total for Question 4 is 4 marks)

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5 State, without further calculation, whether the curve $y = -x^2 + 4x + 1$ has a minimum or a maximum turning point.

(Total for Question 5 is 1 mark)

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6 A curve has equation $y = x^3 - 6x^2 - 15x + 3$.

(a) Find dy/dx. **(1)**

(b) Find the x-coordinates of the stationary points of the curve. **(3)**

(c) Find the corresponding y-coordinates of each stationary point. **(2)**

(d) Use the second derivative to determine the nature of each stationary point. **(3)**

(Total for Question 6 is 9 marks)

7 A curve has equation $y = x^2 - 4x + 1$. Calculate the gradient of the curve at the point where $x = 5$.

(Total for Question 7 is 2 marks)

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8 A curve has equation $y = x^3 - 9x^2 + 24x - 5$.

(a) Find dy/dx. **(1)**

(b) Find the coordinates of the stationary points of the curve. **(4)**

(c) Use the second derivative to determine the nature of each stationary point. **(3)**

(d) Find the range of values of x for which the curve is a decreasing function. **(2)**

(Total for Question 8 is 10 marks)

9 A curve has equation $y = x^3 - 27x + 10$. Find the coordinates of the stationary points of the curve and use the second derivative to determine the nature of each.

(Total for Question 9 is 4 marks)

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10 A curve has equation $y = x^3 - 3kx + 2$, where k is a non-zero constant.

(a) Find dy/dx in terms of k. **(2)**

(b) Show that the curve has two distinct stationary points only when $k > 0$, and find their x-coordinates in terms of k. **(4)**

(Total for Question 10 is 6 marks)

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11 A firework rocket is launched vertically. Its height above the ground, s metres, t seconds after launch is modelled by $s = 24t - 4t^2$, for $0 \leq t \leq 6$.

(a) Find ds/dt. **(2)**

(b) Find the maximum height reached by the rocket, and the time at which it occurs. **(2)**

(Total for Question 11 is 4 marks)

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Mark scheme · 5.2 Applications of Differentiation: Gradients, Tangents and Turning Points: Test Pack

A correct answer earns full marks unless the question asks for working, a method, proof or explanation.

Question 1

(a) differentiates at least two terms correctly (e.g. $3x^2$ and 4x seen) **M1**

(a) $dy/dx = 3x^2 + 4x - 5$ cao **A1**

(a) Answer: $dy/dx = 3x^2 + 4x - 5$

(b) substitutes x = 2 into their dy/dx, ft from part (a) **M1**

(b) gradient = 15 cao **A1**

(b) Answer: 15

Question 2

differentiates **M1**

gradient of tangent = 2 **A1**

gradient of normal = -1/(2) **M1**

line through (0, 0) with the normal gradient **M1**

x + 2y = 0 oe cao **A1**

Answer: x + 2y = 0

Question 3

base is (30 - 2x) by (30 - 2x) and height x **M1**

expands: $V = 4x^3 - 120x^2 + 900x$ and differentiates **M1**

sets dV/dx = 0 and solves **M1**

x = 5 (rejecting x = 15) **A1**

V = 2000 **A1**

maximum confirmed **B1**

Answer: x = 5, maximum volume = 2000 cm^3

Question 4

differentiates and sets equal to 0 **M1**

x = 7 **A1**

y = -9 **A1**

maximum **B1**

Answer: (7, -9) maximum

Question 5

maximum, since the coefficient of x^2 is negative **B1**

Answer: Maximum

Question 6

(a) $dy/dx = 3x^2 - 12x - 15$ cao **B1**

(a) Answer: $dy/dx = 3x^2 - 12x - 15$

(b) sets their dy/dx = 0 and simplifies, e.g. dividing by 3 to give $x^2 - 4x - 5 = 0$, ft from part (a) **M1**

(b) factorises or solves the quadratic, e.g. (x-5)(x+1) = 0 **M1**

(b) x = 5 and x = -1 (both values) cao **A1**

(b) Answer: x = 5 and x = -1

(c) substitutes both x-values into $y = x^3 - 6x^2 - 15x + 3$, ft from part (b) **M1**

(c) (5, -97) and (-1, 11) (both required) cao **A1**

(c) Answer: (5, -97) and (-1, 11)

(d) finds the second derivative d2y/dx2 = 6x - 12 **M1**

(d) evaluates at both x = 5 (= 18) and x = -1 (= -18), ft from part (b) **A1**

(d) (5, -97) is a minimum (d2y/dx2 = 18 > 0); (-1, 11) is a maximum (d2y/dx2 = -18 < 0) **A1**

(d) Answer: Minimum at (5, -97); maximum at (-1, 11)

Question 7

differentiates to obtain dy/dx = 2x - 4 and substitutes x = 5 **M1**

gradient = 6 cao **A1**

Answer: gradient = 6

Question 8

(a) $dy/dx = 3x^2 - 18x + 24$ cao **B1**

(a) Answer: $dy/dx = 3x^2 - 18x + 24$

(b) sets their dy/dx = 0 and simplifies, e.g. dividing by 3 to give $x^2 - 6x + 8 = 0$, ft from part (a) **M1**

(b) factorises or solves the quadratic, e.g. (x-2)(x-4) = 0 **M1**

(b) x = 2 and x = 4 (both values) cao **A1**

(b) (2, 15) and (4, 11) (both coordinate pairs) cao **A1**

(b) Answer: (2, 15) and (4, 11)

(c) finds the second derivative d2y/dx2 = 6x - 18 **M1**

(c) evaluates at both x = 2 (= -6) and x = 4 (= 6), ft from part (b) **A1**

(c) (2, 15) is a maximum (d2y/dx2 = -6 < 0); (4, 11) is a minimum (d2y/dx2 = 6 > 0) **A1**

(c) Answer: Maximum at (2, 15); minimum at (4, 11)

(d) tests the sign of dy/dx = 3(x-2)(x-4) between the stationary values **M1**

(d) 2 < x < 4 oe **A1**

(d) Answer: 2 < x < 4

Question 9

differentiates and sets $3x^2 - 27 = 0$ to obtain x = 3 and x = -3 **M1**

y-coordinates found: (3, -44) and (-3, 64) (both required, ft) **A1**

finds the second derivative d2y/dx2 = 6x and evaluates it at both x-values **M1**

(3, -44) is a minimum (d2y/dx2 = 18 > 0); (-3, 64) is a maximum (d2y/dx2 = -18 < 0) **A1**

Answer: Minimum at (3, -44); maximum at (-3, 64)

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Question 10

(a) differentiates the term -3kx correctly **M1**

(a) $dy/dx = 3x^2 - 3k$ cao **A1**

(a) Answer: $dy/dx = 3x^2 - 3k$

(b) sets their $dy/dx = 0$ to obtain $x^2 = k$, ft from part (a) **M1**

(b) considers the cases $k > 0$, $k = 0$ and $k < 0$ for real solutions of $x^2 = k$ **M1**

(b) correct reasoning: $k < 0$ gives no real solutions; $k = 0$ gives one repeated solution $x = 0$; $k > 0$ gives two distinct real solutions, so two distinct stationary points require $k > 0$, cso **A1**

(b) $x = \sqrt{k}$ and $x = -\sqrt{k}$ **A1**

(b) Answer: Two distinct stationary points exist only when $k > 0$, at $x = \sqrt{k}$ and $x = -\sqrt{k}$

Question 11

(a) differentiates at least one term correctly **M1**

(a) $ds/dt = 24 - 8t$ cao **A1**

(a) Answer: $ds/dt = 24 - 8t$

(b) sets $ds/dt = 0$ and solves to find $t = 3$, ft from part (a) **M1**

(b) maximum height = 36 m at $t = 3$ seconds **A1**

(b) Answer: Maximum height 36 m at $t = 3$ seconds

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