3.3 Further Trigonometric Identities and Equations: 1-hour Pack

AQA 8365 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Solving trigonometric equations.

(a) Solve $5\sin(x) = 3$ for $0 \leq x \leq 360$, giving your answers to 1 decimal place. **(3)**

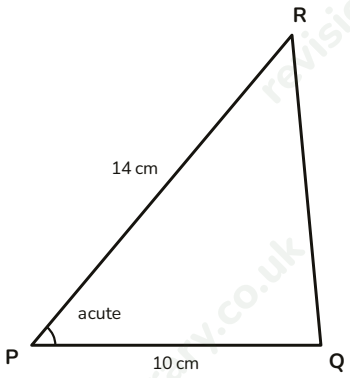
.....

(Total for Question 1 is 3 marks)

---

2 Triangle PQR has PQ = 10 cm and PR = 14 cm. The area of triangle PQR is 55 cm², and angle QPR is acute.

Diagram NOT accurately drawn



(a) Calculate the size of angle QPR, giving your answer correct to 1 decimal place. **(3)**

.....

(Total for Question 2 is 3 marks)

© Revision Library | revisionlibrary.co.uk

3 Solve $5\cos(x) = 2$ for $0 \leq x \leq 360$, giving your answers correct to 1 decimal place.

(Total for Question 3 is 3 marks)

---

4 Solve $\tan(x) = 1$ for $0 \leq x \leq 180$.

(Total for Question 4 is 1 mark)

---

5 Solve $\sin(x) = 0.5$ for $0 \leq x \leq 360$, giving all solutions.

(Total for Question 5 is 2 marks)

---

6 Solve $\cos(x) = -0.5$ for $0 \leq x \leq 360$, giving all solutions.

(Total for Question 6 is 2 marks)

---

7 Solving trigonometric equations.

(a) Solve $2\sin^2(x) - 1 = 0$ for $0 \leq x \leq 360$, giving all solutions. **(4)**

(Total for Question 7 is 4 marks)

© Revision Library | revisionlibrary.co.uk

8 Solve $\tan x = -1.7$ for $0 \leq x \leq 360$. Give your answers correct to 1 decimal place.

(Total for Question 8 is 4 marks)

9 Solve $\tan x = 0$ for $0 \leq x \leq 360$. Do not use a calculator.

(Total for Question 9 is 3 marks)

10 Solve $2 \sin x \cos x = -\sqrt{3} \sin x$ for $0 \leq x \leq 360$.

(Total for Question 10 is 4 marks)

11 Angle x is acute and $\sin x = 24/25$. Find the exact values of $\cos x$ and $\tan x$.

(Total for Question 11 is 2 marks)

12 Solve $2 \cos^2 x - 2 \sin x = 2$ for $0 \leq x \leq 360$.

(Total for Question 12 is 5 marks)

© Revision Library | revisionlibrary.co.uk

13 Simplify $4(\sin x + \cos x)^2 - 8 \sin x \cos x$.

(Total for Question 13 is 3 marks)

---

14 Given that $\sin(\theta) = 0.6$, where θ is obtuse $(90 < \theta < 180)$, use $\sin^2(\theta) + \cos^2(\theta) = 1$ to find the value of $\cos(\theta)$.

(Total for Question 14 is 3 marks)

© Revision Library | revisionlibrary.co.uk

15 x is an angle measured in degrees.

(a) Prove that $(\sin(x) + \cos(x))^2 = 1 + 2\sin(x)\cos(x)$ for all values of x. **(3)**

(b) Given that $\sin(x) + \cos(x) = 1.2$, use your result from part (a) to find the value of $\sin(x)\cos(x)$. **(2)**

.....

(c) Given also that $(\sin(x) - \cos(x))^2 = 1 - 2\sin(x)\cos(x)$, find the value of $(\sin(x) - \cos(x))^2$. **(2)**

.....

(Total for Question 15 is 7 marks)

---

16 Solve $2\cos^2(x) + 3\cos(x) - 2 = 0$ for $0 \leq x \leq 360$, giving all solutions.

(Total for Question 16 is 4 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 3.3 Further Trigonometric Identities and Equations: 1-hour Pack

Question 1

(a) sin(x) = 0.6 (rearrange) **M1**

(a) x = 36.9 (awrt) **A1**

(a) x = 143.1 (ft: 180 - first value, awrt) **A1**

(a) Answer: x = 36.9 or x = 143.1

Question 2

(a) rearrange Area = 1/2 x PQ x PR x sin(QPR) to make sin(QPR) the subject **M1**

(a) sin(QPR) = 55/70 = 11/14 oe (0.786) **A1**

(a) QPR = 51.8 degrees awrt **A1**

(a) Answer: 51.8 degrees

Question 3

rearrange to cos(x) = 0.4 **M1**

x = 66.4 (awrt, principal value) **A1**

x = 293.6 (ft: 360 - previous value, awrt) **A1**

Answer: x = 66.4 or x = 293.6

Question 4

x = 45 cao (the next solution, 225, lies outside the given range) **B1**

Answer: x = 45

Question 5

arcsin(0.5) = 30 (principal value) **M1**

x = 30 and x = 150 (using 180 - 30), both required cao **A1**

Answer: x = 30, 150

Question 6

reference angle arccos(0.5) = 60, and recognises cosine is negative in the second and third quadrants **M1**

x = 120 and x = 240, both required cao **A1**

Answer: x = 120, 240

Question 7

(a) rearrange to $\sin^2(x) = 0.5$ **M1**

(a) $\sin(x) = +1/\sqrt{2}$ or $\sin(x) = -1/\sqrt{2}$ (awrt ±0.707) **M1**

(a) x = 45 and x = 135 (from positive root) **A1**

(a) x = 225 and x = 315 (from negative root) cao **A1**

(a) Answer: x = 45, 135, 225, 315

Question 8

$\tan^{-1}(-1.7)$ used **M1**

120.5 cao **A1**

uses symmetry or period to find a second angle **M1**

x = 120.5, 300.5 cao **A1**

Answer: x = 120.5, 300.5

Question 9

uses an exact value to find a principal angle **M1**

one correct solution in the range **A1**

x = 0, 180, 360 (all solutions, no extras) cao **A1**

Answer: x = 0, 180, 360

Question 10

moves everything to one side and factorises sin x **M1**

sin x = 0 and cos x = $-\sqrt{3}/2$ **A1**

principal values found **M1**

x = 0, 150, 180, 210, 360 cao **A1**

Answer: x = 0, 150, 180, 210, 360

Question 11

uses $\sin^2 x + \cos^2 x = 1$ or a right-angled triangle **M1**

cos x = 7/25, tan x = 24/7 cao **A1**

Answer: cos x = 7/25, tan x = 24/7

Question 12

replaces $\cos^2 x$ by $1 - \sin^2 x$ **M1**

rearranges to a quadratic in sin x **M1**

factorises or solves **M1**

sin x = -1 and sin x = 0 **A1**

x = 0, 180, 270, 360 cao **A1**

Answer: x = 0, 180, 270, 360

Question 13

uses $\sin^2 x + \cos^2 x = 1$ or tan x = sin x/cos x **M1**

correct algebraic manipulation **M1**

4 oe cao **A1**

Answer: 4

Question 14

correctly rearranges $\sin^2(\theta) + \cos^2(\theta) = 1$ to $\cos^2(\theta) = 1 - \sin^2(\theta)$ **M1**

$\cos^2(\theta) = 1 - 0.6^2 = 1 - 0.36 = 0.64$ **M1**

cos(θ) = -0.8 (negative root selected since θ is obtuse) cao **A1**

Answer: cos(θ) = -0.8

Question 15

(a) expand the bracket: $\sin^2(x) + 2\sin(x)\cos(x) + \cos^2(x)$ **M1**

(a) use $\sin^2(x) + \cos^2(x) = 1$ to simplify **M1**

(a) fully shown: = 1 + 2 sin(x) cos(x), cso **A1**

(a) Answer: 1 + 2 sin(x) cos(x) (shown)

(b) substitute 1.2 into the identity: $1.2^2 = 1 + 2\sin(x)\cos(x)$ **M1**

(b) sin(x) cos(x) = 0.22 cao **A1**

(b) Answer: 0.22

(c) substitute sin(x) cos(x) = 0.22 into 1 - 2 sin(x) cos(x) **M1**

(c) = 0.56 cao **A1**

(c) Answer: 0.56

Question 16

let c = cos(x) and form $2c^2 + 3c - 2 = 0$ **M1**

factorise (2c - 1)(c + 2) = 0, giving c = 0.5 or c = -2 **M1**

reject c = -2, since $-1 \le \cos(x) \le 1$ for all x **A1**

x = 60 and x = 300 (both required, from c = 0.5) cao **A1**

Answer: x = 60 or x = 300 (cos(x) = -2 is rejected, since cos(x) can never be less than -1)

© Revision Library | revisionlibrary.co.uk