3.3 Further Trigonometric Identities and Equations: 2-hour Pack

AQA 8365 · Calculator allowed · about 120 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Solve $\sin x = -0.37$ for $0 \leq x \leq 360$. Give your answers correct to 1 decimal place.

(Total for Question 1 is 4 marks)

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2 Solve $\sin x = 1/2$ for $0 \leq x \leq 360$. Do not use a calculator.

(Total for Question 2 is 3 marks)

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3 Solve $\sin(x - 90) = \sqrt{3}/2$ for $0 \leq x \leq 360$. Do not use a calculator.

(Total for Question 3 is 4 marks)

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4 Angle x is acute and $\sin x = 8/17$. Find the exact values of $\cos x$ and $\tan x$.

(Total for Question 4 is 2 marks)

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5 Solve $\cos x = 0.25$ for $0 \leq x \leq 360$. Give your answers correct to 1 decimal place.

(Total for Question 5 is 4 marks)

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6 Solve $\tan x = -2 \sin x$ for $0 \leq x \leq 360$.

(Total for Question 6 is 4 marks)

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7 Angle x is obtuse and $\sin x = 3/5$. Find the exact values of $\cos x$ and $\tan x$.

(Total for Question 7 is 2 marks)

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8 Solve $4 \cos^2 x = 3$ for $0 \leq x \leq 360$.

(Total for Question 8 is 5 marks)

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9 Solve $\sin x = 0.94$ for $0 \leq x \leq 360$. Give your answers correct to 1 decimal place.

(Total for Question 9 is 4 marks)

10 Solve $\cos(x + 45) = -1$ for $0 \leq x \leq 360$. Do not use a calculator.

(Total for Question 10 is 4 marks)

11 Simplify $4 \sin^2 x + 4 \cos^2 x$.

(Total for Question 11 is 3 marks)

12 Solve $\sin^2 x + \sin x = 0$ for $-180 \leq x \leq 180$.

(Total for Question 12 is 5 marks)

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13 Solve $\cos x = 0.31$ for $0 \leq x \leq 360$. Give your answers correct to 1 decimal place.

(Total for Question 13 is 4 marks)

14 Solve $2 \sin x \cos x = -\sin x$ for $0 \leq x \leq 360$.

(Total for Question 14 is 4 marks)

15 Angle x is acute and $\sin x = 9/41$. Find the exact values of $\cos x$ and $\tan x$.

(Total for Question 15 is 2 marks)

16 Simplify $(3 \sin^2 x)/(1 - \sin^2 x)$.

(Total for Question 16 is 3 marks)

17 Solve $4 \cos^2 x - 2 \sin x = 4$ for $0 \leq x \leq 360$.

(Total for Question 17 is 5 marks)

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18 Solve $\tan x = 1.0$ for $0 \leq x \leq 360$. Give your answers correct to 1 decimal place.

(Total for Question 18 is 4 marks)

19 Solve $\sin(x + 30) = 1/2$ for $0 \leq x \leq 360$. Do not use a calculator.

(Total for Question 19 is 4 marks)

20 Angle x is acute and $\sin x = 21/29$. Find the exact values of $\cos x$ and $\tan x$.

(Total for Question 20 is 2 marks)

21 Simplify $7(1 + \tan^2 x)$.

(Total for Question 21 is 3 marks)

22 Solve $\sin^2 x + \sin x = 0$ for $0 \leq x \leq 360$.

(Total for Question 22 is 5 marks)

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23 Solve $\tan x = 3.0$ for $0 \leq x \leq 360$. Give your answers correct to 1 decimal place.

(Total for Question 23 is 4 marks)

24 Solve $2 \sin x \cos x = \sin x$ for $0 \leq x \leq 360$.

(Total for Question 24 is 4 marks)

25 Simplify $7 - 7 \sin^2 x$.

(Total for Question 25 is 3 marks)

26 Solve $2 \cos^2 x - \sin x = 2$ for $0 \leq x \leq 360$.

(Total for Question 26 is 5 marks)

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27 A ship sails from port A on a bearing of 072 degrees for 15 km to reach point B. It then changes course and sails on a bearing of 155 degrees for 21 km to reach point C.

Diagram NOT accurately drawn

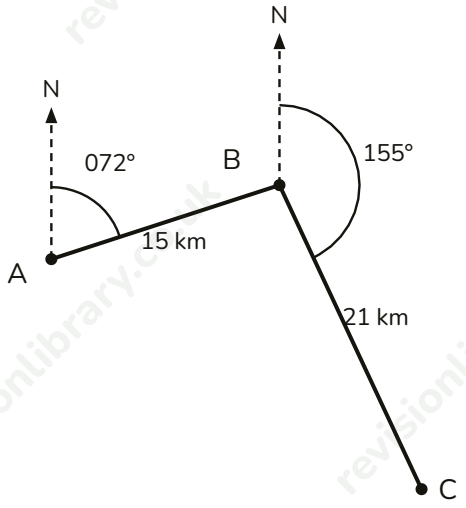


Diagram not drawn to scale

(a) Show that angle ABC = 97 degrees. **(2)**

(b) Calculate the distance AC, giving your answer correct to 3 significant figures. **(3)**

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(c) Find the bearing of C from A, giving your answer to the nearest degree. **(4)**

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(Total for Question 27 is 9 marks)

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Mark scheme · 3.3 Further Trigonometric Identities and Equations: 2-hour Pack

Question 1

$\sin^{-1}(-0.37)$ used	M1
201.7 cao	A1
uses symmetry or period to find a second angle	M1
x = 201.7, 338.3 cao	A1

Answer: x = 201.7, 338.3

Question 2

uses an exact value to find a principal angle	M1
one correct solution in the range	A1
x = 30, 150 (all solutions, no extras) cao	A1

Answer: x = 30, 150

Question 3

principal value found for x - 90	M1
extends the range of x - 90 to -90 to 270	M1
at least two correct solutions	A1
x = 150, 210 cao	A1

Answer: x = 150, 210

Question 4

uses $\sin^2 x + \cos^2 x = 1$ or a right-angled triangle	M1
cos x = 15/17, tan x = 8/15 cao	A1

Answer: cos x = 15/17, tan x = 8/15

Question 5

$\cos^{-1}(0.25)$ used	M1
75.5 cao	A1
uses symmetry or period to find a second angle	M1
x = 75.5, 284.5 cao	A1

Answer: x = 75.5, 284.5

Question 6

replaces tan x by sin x / cos x	M1
factorises sin x	M1
sin x = 0 and cos x = -1/2	A1
x = 0, 120, 180, 240, 360 cao	A1

Answer: x = 0, 120, 180, 240, 360

Question 7

uses $\sin^2 x + \cos^2 x = 1$ or a right-angled triangle	M1
cos x = -4/5, tan x = -3/4 cao	A1

Answer: cos x = -4/5, tan x = -3/4

Question 8

replaces $\cos^2 x$ by $1 - \sin^2 x$	M1
rearranges to a quadratic in sin x	M1
factorises or solves	M1
sin x = 1/2 and sin x = -1/2	A1
x = 30, 150, 210, 330 cao	A1

Answer: x = 30, 150, 210, 330

Question 9

$\sin^{-1}(0.94)$ used	M1
70.1 cao	A1
uses symmetry or period to find a second angle	M1
x = 70.1, 109.9 cao	A1

Answer: x = 70.1, 109.9

Question 10

principal value found for x + 45	M1
extends the range of x + 45 to 45 to 405	M1
x + 45 = 180 is the only value in the extended range	A1
x = 135 cao	A1

Answer: x = 135

Question 11

uses $\sin^2 x + \cos^2 x = 1$ or tan x = sin x/cos x	M1
correct algebraic manipulation	M1
4 oe cao	A1

Answer: 4

Question 12

factorises or solves the quadratic in sin x	M1
sin x = 0 and sin x = -1	A1
principal values found	M1
at least three correct solutions or all solutions of one root	A1
x = -180, -90, 0, 180 cao	A1

Answer: x = -180, -90, 0, 180

Question 13

$\cos^{-1}(0.31)$ used	M1
71.9 cao	A1
uses symmetry or period to find a second angle	M1
x = 71.9, 288.1 cao	A1

Answer: x = 71.9, 288.1

Question 14

moves everything to one side and factorises sin x	M1
sin x = 0 and cos x = -1/2	A1
principal values found	M1
x = 0, 120, 180, 240, 360 cao	A1

Answer: x = 0, 120, 180, 240, 360

Question 15

uses $\sin^2 x + \cos^2 x = 1$ or a right-angled triangle	M1
cos x = 40/41, tan x = 9/40 cao	A1

Answer: cos x = 40/41, tan x = 9/40

Question 16

uses $\sin^2 x + \cos^2 x = 1$ or tan x = sin x/cos x	M1
correct algebraic manipulation	M1
$3\tan^2 x$ oe cao	A1

Answer: $3\tan^2 x$

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Question 17

replaces $\cos^2 x$ by $1 - \sin^2 x$	**M1**
rearranges to a quadratic in sin x	**M1**
factorises or solves	**M1**
sin x = -1/2 and sin x = 0	**A1**
x = 0, 180, 210, 330, 360 cao	**A1**

Answer: x = 0, 180, 210, 330, 360

Question 18

$\tan^{-1}(1.0)$ used	**M1**
45.0 cao	**A1**
uses symmetry or period to find a second angle	**M1**
x = 45.0, 225.0 cao	**A1**

Answer: x = 45.0, 225.0

Question 19

principal value found for x + 30	**M1**
extends the range of x + 30 to 30 to 390	**M1**
at least two correct solutions	**A1**
x = 0, 120, 360 cao	**A1**

Answer: x = 0, 120, 360

Question 20

uses $\sin^2 x + \cos^2 x = 1$ or a right-angled triangle	**M1**
cos x = 20/29, tan x = 21/20 cao	**A1**

Answer: cos x = 20/29, tan x = 21/20

Question 21

uses $\sin^2 x + \cos^2 x = 1$ or tan x = sin x/cos x	**M1**
correct algebraic manipulation	**M1**
$7/\cos^2 x$ oe cao	**A1**

Answer: $7/\cos^2 x$

Question 22

factorises or solves the quadratic in sin x	**M1**
sin x = -1 and sin x = 0	**A1**
principal values found	**M1**
at least three correct solutions or all solutions of one root	**A1**
x = 0, 180, 270, 360 cao	**A1**

Answer: x = 0, 180, 270, 360

Question 23

$\tan^{-1}(3.0)$ used	**M1**
71.6 cao	**A1**
uses symmetry or period to find a second angle	**M1**
x = 71.6, 251.6 cao	**A1**

Answer: x = 71.6, 251.6

Question 24

moves everything to one side and factorises sin x	**M1**
sin x = 0 and cos x = 1/2	**A1**
principal values found	**M1**
x = 0, 60, 180, 300, 360 cao	**A1**

Answer: x = 0, 60, 180, 300, 360

Question 25

uses $\sin^2 x + \cos^2 x = 1$ or tan x = sin x/cos x	**M1**
correct algebraic manipulation	**M1**
$7 \cos^2 x$ oe cao	**A1**

Answer: $7 \cos^2 x$

Question 26

replaces $\cos^2 x$ by $1 - \sin^2 x$	**M1**
rearranges to a quadratic in sin x	**M1**
factorises or solves	**M1**
sin x = 0 and sin x = -1/2	**A1**
x = 0, 180, 210, 330, 360 cao	**A1**

Answer: x = 0, 180, 210, 330, 360

Question 27

(a) back bearing from B to A = 072 + 180 = 252 degrees	**M1**
(a) angle ABC = 252 - 155 = 97 degrees, cso	**A1**

(a) Answer: 97 degrees (shown)

(b) correct cosine rule stated: $AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \times \cos(ABC)$	**M1**
(b) correct substitution: $AC^2 = 15^2 + 21^2 - 2 \times 15 \times 21 \times \cos(97)$	**M1**
(b) AC = 27.3 km awrt (3sf)	**A1**

(b) Answer: 27.3 km

(c) correct sine rule stated: sin(BAC)/BC = sin(ABC)/AC	**M1**
(c) angle BAC = 49.9 degrees awrt	**A1**
(c) correct strategy: bearing of C from A = bearing of B from A + angle BAC = 072 + BAC	**M1**
(c) bearing = 122 degrees cao (nearest degree)	**A1**

(c) Answer: 122 degrees

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