3.3 Further Trigonometric Identities and Equations: Test Pack

Total Marks

AQA 8365 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question tests recall of exact trigonometric values. Do not use a calculator for these, even though a calculator is allowed for the rest of this paper.

(a) Write down the exact value of sin 30 degrees. **(1)**

.....

(b) Write down the exact value of cos 60 degrees. **(1)**

.....

(c) Write down the exact value of tan 45 degrees. **(1)**

.....

(Total for Question 1 is 3 marks)

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2 Trigonometric identities.

(a) Which of the following is a correct trigonometric identity, true for all values of x? **(1)**

- ☐ A) $\sin^2(x) - \cos^2(x) = 1$
- ☐ B) $\sin^2(x) + \cos^2(x) = 1$
- ☐ C) $\sin(x) + \cos(x) = 1$
- ☐ D) $\tan(x) = \cos(x)/\sin(x)$

(Total for Question 2 is 1 mark)

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3 Simplify $\sin^2(x) + \cos^2(x) - 3$.

(Total for Question 3 is 1 mark)

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4 State the value of x, with $0 \leq x \leq 180$, for which $\sin(x) = 1$.

(Total for Question 4 is 1 mark)

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5 In triangle ABC, angle A = 52 degrees, angle B = 71 degrees, and side a (opposite angle A) = 8.4 cm.

Diagram NOT accurately drawn

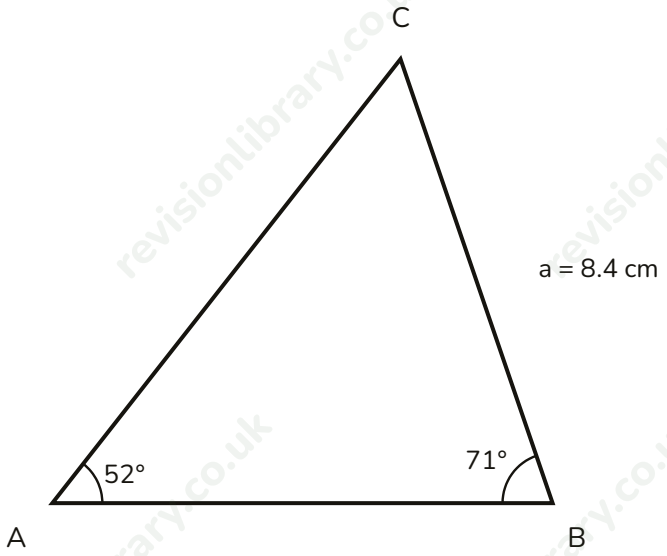


Diagram not drawn to scale

(a) Calculate the length of side b (opposite angle B), giving your answer correct to 3 significant figures. **(3)**

.....

(Total for Question 5 is 3 marks)

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6 Show that $(1 - \sin^2(x))/\cos(x) = \cos(x)$, for $\cos(x)$ not equal to 0.

(Total for Question 6 is 2 marks)

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7 The graph of $y = \sin(x)$ is transformed to give the graph of $y = 3\sin(x) - 2$.

(a) State the amplitude of $y = 3\sin(x) - 2$. **(1)**

(b) State the maximum and minimum values of $y = 3\sin(x) - 2$. **(2)**

(c) State the period of $y = 3\sin(x) - 2$, in degrees. **(1)**

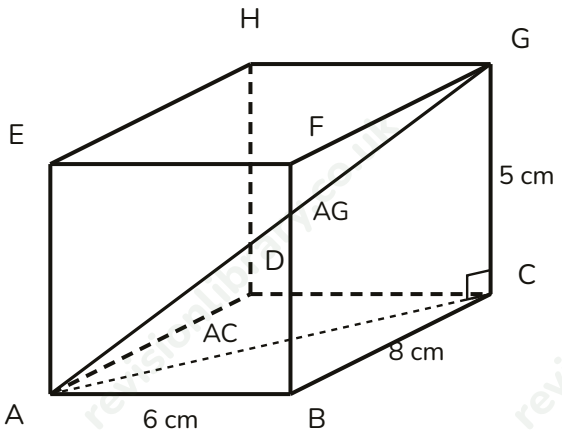
(Total for Question 7 is 4 marks)

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8 ABCDEFGH is a cuboid, with base ABCD and top EFGH. AB = 6 cm, BC = 8 cm and the vertical edge CG = 5 cm. AG is the diagonal from vertex A to vertex G.

Diagram NOT accurately drawn

not drawn to scale



(a) Calculate the size of the angle between AG and the plane ABCD, giving your answer correct to 1 decimal place. **(3)**

.....

(b) Calculate the length of AG, giving your answer correct to 3 significant figures. **(2)**

.....

(Total for Question 8 is 5 marks)

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9 Solve $\sin x = -\sqrt{3}/2$ for $0 \leq x \leq 360$. Do not use a calculator.

(Total for Question 9 is 3 marks)

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10 Solve $2\cos^2 x + \cos x - 1 = 0$ for $0 \leq x \leq 360$.

(Total for Question 10 is 5 marks)

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11 Simplify $6(1 + \tan^2 x)$.

(Total for Question 11 is 3 marks)

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12 Angle x is acute and $\sin x = 12/13$. Find the exact values of $\cos x$ and $\tan x$.

(Total for Question 12 is 2 marks)

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13 Angle x is acute and $\cos x = 12/13$. Find the exact values of $\sin x$ and $\tan x$.

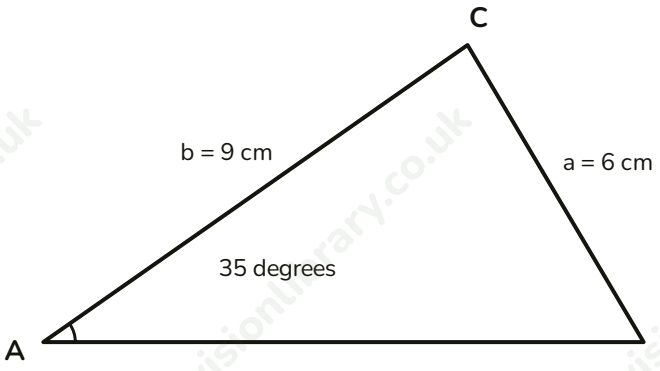
(Total for Question 13 is 2 marks)

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14 In triangle ABC, angle A = 35 degrees, a = 6 cm (opposite angle A) and b = 9 cm (opposite angle B).

Diagram NOT accurately drawn

Note: two triangles may be possible



(a) Find the two possible values of angle B, giving your answers correct to 1 decimal place. **(4)**

.....

(Total for Question 14 is 4 marks)

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15 Prove that $(1 + \sin(x))(1 - \sin(x)) = \cos^2(x)$ for all values of x.

(Total for Question 15 is 3 marks)

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16 Solving trigonometric equations.

(a) Solve $3\cos^2(x) + \cos(x) - 2 = 0$ for $0 \leq x \leq 360$, giving your answers to 1 decimal place where necessary. **(5)**

(Total for Question 16 is 5 marks)

17 A force vector makes an angle x with a reference axis, where $\sin(x) + \cos(x) = 7/5$.

(a) Show that $2\sin(x)\cos(x) = 24/25$. **(3)**

(b) Hence find the value of $(\sin(x) - \cos(x))^2$. **(2)**

(Total for Question 17 is 5 marks)

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Mark scheme · 3.3 Further Trigonometric Identities and Equations: Test Pack

Question 1

(a) 1/2 oe **B1**

(a) Answer: 1/2

(b) 1/2 oe **B1**

(b) Answer: 1/2

(c) 1 **B1**

(c) Answer: 1

Question 2

(a) B) $\sin^2(x) + \cos^2(x) = 1$ **B1**

(a) Answer: B) $\sin^2(x) + \cos^2(x) = 1$

Question 3

-2 cao **B1**

Answer: -2

Question 4

x = 90 cao **B1**

Answer: x = 90

Question 5

(a) correct sine rule stated: b/sin(B) = a/sin(A) **M1**

(a) correct substitution: b = 8.4 x sin(71) / sin(52) **M1**

(a) b = 10.1 cm awrt (3sf) **A1**

(a) Answer: 10.1 cm

Question 6

use $\sin^2(x) + \cos^2(x) = 1$ to write $1 - \sin^2(x) = \cos^2(x)$ **M1**

$\cos^2(x)/\cos(x) = \cos(x)$, fully shown, cso **A1**

Answer: cos(x) (shown)

Question 7

(a) 3 **B1**

(a) Answer: 3

(b) maximum = 1 **B1**

(b) minimum = -5 **B1**

(b) Answer: maximum = 1, minimum = -5

(c) 360 degrees **B1**

(c) Answer: 360 degrees

Question 8

(a) AC = 10 cm (Pythagoras in right-angled triangle ABC: $\sqrt{6^2 + 8^2}$) **B1**

(a) tan(angle) = CG/AC = 5/10, using right-angled triangle ACG (right angle at C) **M1**

(a) angle = 26.6 degrees awrt **A1**

(a) Answer: 26.6 degrees

(b) AG = $\sqrt{AC^2 + CG^2} = \sqrt{10^2 + 5^2}$, or full 3D Pythagoras $\sqrt{6^2 + 8^2 + 5^2}$ **M1**

(b) AG = 11.2 cm awrt **A1**

(b) Answer: 11.2 cm

Question 9

uses an exact value to find a principal angle **M1**

one correct solution in the range **A1**

x = 240, 300 (all solutions, no extras) cao **A1**

Answer: x = 240, 300

Question 10

factorises or solves the quadratic in cos x **M1**

cos x = 1/2 and cos x = -1 **A1**

principal values found **M1**

at least three correct solutions or all solutions of one root **A1**

x = 60, 180, 300 cao **A1**

Answer: x = 60, 180, 300

Question 11

uses $\sin^2 x + \cos^2 x = 1$ or tan x = sin x/cos x **M1**

correct algebraic manipulation **M1**

$6/\cos^2 x$ oe cao **A1**

Answer: $6/\cos^2 x$

Question 12

uses $\sin^2 x + \cos^2 x = 1$ or a right-angled triangle **M1**

cos x = 5/13, tan x = 12/5 cao **A1**

Answer: cos x = 5/13, tan x = 12/5

Question 13

uses $\sin^2 x + \cos^2 x = 1$ or a right-angled triangle **M1**

sin x = 5/13, tan x = 5/12 cao **A1**

Answer: sin x = 5/13, tan x = 5/12

Question 14

(a) correct sine rule stated: sin(B)/b = sin(A)/a **M1**

(a) sin(B) = 9 x sin(35) / 6 = 0.860(4) oe **A1**

(a) B = 59.4 degrees awrt **A1**

(a) B = 120.6 degrees (180 - first value, awrt) **A1**

(a) Answer: B = 59.4 degrees or B = 120.6 degrees

Question 15

expand the LHS using difference of two squares: $1 - \sin^2(x)$ **M1**

use $\sin^2(x) + \cos^2(x) = 1$ to write $1 - \sin^2(x) = \cos^2(x)$ **M1**

fully shown: $= \cos^2(x)$, cso **A1**

Answer: $\cos^2(x)$ (shown)

Question 16

(a) let c = cos(x) and form $3c^2 + c - 2 = 0$ **M1**

(a) solve the quadratic (formula or factorisation) to get c = 2/3 or c = -1 **M1**

(a) x = 180 (from c = -1) **A1**

(a) x = 48.2 (from c = 2/3, awrt) **A1**

(a) x = 311.8 (ft: 360 - previous value, awrt) **A1**

(a) Answer: x = 48.2, 180, 311.8

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Question 17

(a) square both sides: $(\sin(x) + \cos(x))^2 = (7/5)^2$ **M1**

(a) expand LHS and use $\sin^2(x) + \cos^2(x) = 1$ to write $1 + 2\sin(x)\cos(x) = 49/25$ **M1**

(a) $2\sin(x)\cos(x) = 49/25 - 1 = 24/25$, cso **A1**

(a) Answer: $2\sin(x)\cos(x) = 24/25$ (shown)

(b) use $(\sin(x) - \cos(x))^2 = 1 - 2\sin(x)\cos(x)$, ft from part (a) **M1**

(b) = 1/25 oe (0.04) cao **A1**

(b) Answer: 1/25

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