3.4 Trigonometric Graphs and Transformations: 2-hour Pack

Total Marks

AQA 8365 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 For $0 \leq x \leq 360$, the curve $y = 2 \sin(x - 60) + 1$ has a maximum point. Write down the coordinates of the maximum point.

(Total for Question 1 is 3 marks)

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2 The graph of $y = 1 \tan(6x)$ is drawn for $0 < x < 180$. Find the equations of the vertical asymptotes. Give the values of x.

(Total for Question 2 is 3 marks)

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3 The graph of $y = -\cos(3x) + 2$ is drawn for $0 \leq x \leq 360$. Write down the maximum value of y, the minimum value of y and the period of the graph.

(Total for Question 3 is 3 marks)

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4 The graph of $y = -2\cos(6x) + 2$ is drawn for $0 \leq x \leq 360$. Write down the maximum value of y, the minimum value of y and the period of the graph.

(Total for Question 4 is 3 marks)

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5 The graph of $y = a\sin(bx) + c$, where a and b are positive integers and c is an integer, is drawn for $0 \leq x \leq 360$. The maximum value of y is 7, the minimum value of y is -5 and the period is 180 degrees. Find the values of a, b and c.

(Total for Question 5 is 4 marks)

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6 The graph of $y = 4\tan(5x)$ is drawn for $0 < x < 180$. Find the equations of the vertical asymptotes. Give the values of x.

(Total for Question 6 is 3 marks)

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7 The graph of $y = a\cos(x) + 2$ passes through the point (120, -2). Find the value of a.

(Total for Question 7 is 3 marks)

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8 The graph of $y = -4\cos(5x) - 1$ is drawn for $0 \leq x \leq 360$. Write down the maximum value of y, the minimum value of y and the period of the graph.

(Total for Question 8 is 3 marks)

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9 For $0 \leq x \leq 360$, the curve $y = 5\cos(x - 40) - 4$ has a minimum point. Write down the coordinates of the minimum point.

(Total for Question 9 is 3 marks)

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10 Find the values of x between 0 and 360 inclusive where the graph of $y = 5\sin(6x)$ crosses the x-axis.

(Total for Question 10 is 3 marks)

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11 The graph of $y = a\sin(x) + 2$ passes through the point (210, 1). Find the value of a.

(Total for Question 11 is 3 marks)

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12 For $0 \leq x \leq 360$, the curve $y = \cos(x - 90) - 0$ has a minimum point. Write down the coordinates of the minimum point.

(Total for Question 12 is 3 marks)

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13 The graph of $y = a\cos(bx) + c$, where a and b are positive integers and c is an integer, is drawn for $0 \leq x \leq 360$. The maximum value of y is 6, the minimum value of y is -4 and the period is 360 degrees. Find the values of a, b and c.

(Total for Question 13 is 4 marks)

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14 The graph of $y = a\cos(x) - 5$ passes through the point (0, -11). Find the value of a.

(Total for Question 14 is 3 marks)

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15 Find the values of x between 0 and 360 inclusive where the graph of $y = 4\cos(x)$ crosses the x-axis.

(Total for Question 15 is 3 marks)

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16 The graph of $y = a\cos(bx) + c$, where a and b are positive integers and c is an integer, is drawn for $0 \leq x \leq 360$. The maximum value of y is 5, the minimum value of y is -5 and the period is 90 degrees. Find the values of a, b and c.

(Total for Question 16 is 4 marks)

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17 Find the values of x between 0 and 360 inclusive where the graph of $y = 3\sin(10x)$ crosses the x-axis.

(Total for Question 17 is 3 marks)

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18 The graph of $y = a\cos(bx) + c$, where a and b are positive integers and c is an integer, is drawn for $0 \leq x \leq 360$. The maximum value of y is 11, the minimum value of y is -1 and the period is 120 degrees. Find the values of a, b and c.

(Total for Question 18 is 4 marks)

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19 Solve $\cos(x) = -0.6$ for $0 \leq x \leq 360$, giving your answers correct to 1 decimal place.

(Total for Question 19 is 3 marks)

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20 Find the values of x between 0 and 360 inclusive where the graph of $y = 3\sin(3x)$ crosses the x-axis.

(Total for Question 20 is 3 marks)

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21 Find the coordinates of the minimum point of $y = 3\cos(x) - 2$, for $0 \leq x \leq 360$.

(Total for Question 21 is 2 marks)

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22 Solve $\sin(x) = -0.7$ for $0 \leq x \leq 360$, giving your answers correct to 1 decimal place.

(Total for Question 22 is 3 marks)

23 Solve $2\cos(x) + 1 = 0$ for $0 \leq x \leq 360$, giving all solutions.

(Total for Question 23 is 3 marks)

24 Describe fully the single transformation that maps $y = \cos(x)$ onto $y = \cos(5x)$.

(Total for Question 24 is 1 mark)

25 Using the symmetry properties of the sine, cosine and tangent graphs, find the exact value of each of the following without using a calculator.

(a) $\sin(150)$ **(2)**

(b) $\cos(210)$ **(2)**

(c) $\tan(315)$ **(2)**

(Total for Question 25 is 6 marks)

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26 Given that $\sin(\theta) = 0.28$, where θ is obtuse ($90 < \theta < 180$), use the identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to find the value of $\cos(\theta)$.

(Total for Question 26 is 4 marks)

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27 The graph of $y = a*\cos(x) + b$, where $a > 0$, has a maximum point at (0, 7) and a minimum point at (180, -1).

(a) Find the values of a and b. **(4)**

(Total for Question 27 is 4 marks)

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28 The graph of $y = a*\sin(x) + b$, where $a > 0$, has a maximum value of 11 and a minimum value of -3. Find the values of a and b.

(Total for Question 28 is 3 marks)

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29 The graph of $y = \sin(x)$ undergoes a stretch of scale factor 1/2 parallel to the x-axis, followed by a translation by vector (0, -3).

(a) Write down the equation of the resulting graph. **(1)**

.....

(b) State the period, in degrees, of the resulting graph. **(1)**

(c) Find the coordinates of one minimum point of the resulting graph, for $0 \leq x \leq 360$. **(2)**

.....

(Total for Question 29 is 4 marks)

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30 Solve $\sin(2x - 30) = 0.5$ for $0 \leq x \leq 360$, giving all solutions.

(Total for Question 30 is 5 marks)

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31 The graph of $y = 2\sin(x) + k$, for $0 \leq x \leq 360$, has a minimum value of -5.

(a) Find the value of k. **(2)**

(b) Using your value of k, explain why the equation $2\sin(x) + k = 0$ has no solutions for $0 \leq x \leq 360$. **(1)**

(Total for Question 31 is 3 marks)

32 The graph of $y = p*\cos(qx)$, for $0 \leq x \leq 360$, where $p > 0$ and q is a positive integer, has amplitude 6 and completes exactly 4 full cycles in the range $0 \leq x \leq 360$.

(a) Find the value of p. **(1)**

(b) Find the value of q. **(2)**

(c) Hence find the coordinates of the first maximum point of the graph for $x > 0$. **(2)**

(Total for Question 32 is 5 marks)

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Mark scheme · 3.4 Trigonometric Graphs and Transformations: 2-hour Pack

Question 1

x-coordinate found from sin(x - 60) = 1	**M1**
x = 150	**A1**
y = 3	**B1**

Answer: (150, 3)

Question 2

6x = 90	**M1**
6x = 90 + 180n used	**M1**
x = 15, 45, 75, 105, 135, 165 cao	**A1**

Answer: x = 15, 45, 75, 105, 135, 165

Question 3

maximum 3	**B1**
minimum 1	**B1**
period 120 degrees	**B1**

Answer: maximum = 3, minimum = 1, period = 120 degrees

Question 4

maximum 4	**B1**
minimum 0	**B1**
period 60 degrees	**B1**

Answer: maximum = 4, minimum = 0, period = 60 degrees

Question 5

a = (7 - (-5))/2 or c = (7 + (-5))/2	**M1**
a = 6	**A1**
c = 1	**A1**
b = 360/180 = 2	**B1**

Answer: a = 6, b = 2, c = 1

Question 6

5x = 90	**M1**
5x = 90 + 180n used	**M1**
x = 18, 54, 90, 126, 162 cao	**A1**

Answer: x = 18, 54, 90, 126, 162

Question 7

substitutes x = 120, y = -2	**M1**
cos(120) = -1/2 used	**M1**
a = 8 cao	**A1**

Answer: a = 8

Question 8

maximum 3	**B1**
minimum -5	**B1**
period 72 degrees	**B1**

Answer: maximum = 3, minimum = -5, period = 72 degrees

Question 9

x-coordinate found from cos(x - 40) = -1	**M1**
x = 220	**A1**
y = -9	**B1**

Answer: (220, -9)

Question 10

sin(6x) = 0	**M1**
6x = 0, 180, 360, ...	**M1**
x = 0, 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 330, 360 cao	**A1**

Answer: x = 0, 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 330, 360

Question 11

substitutes x = 210, y = 1	**M1**
sin(210) = -1/2 used	**M1**
a = 2 cao	**A1**

Answer: a = 2

Question 12

x-coordinate found from cos(x - 90) = -1	**M1**
x = 270	**A1**
y = -1	**B1**

Answer: (270, -1)

Question 13

a = (6 - (-4))/2 or c = (6 + (-4))/2	**M1**
a = 5	**A1**
c = 1	**A1**
b = 360/360 = 1	**B1**

Answer: a = 5, b = 1, c = 1

Question 14

substitutes x = 0, y = -11	**M1**
cos(0) = 1 used	**M1**
a = -6 cao	**A1**

Answer: a = -6

Question 15

cos(x) = 0	**M1**
x = 90, 270, 450, ...	**M1**
x = 90, 270 cao	**A1**

Answer: x = 90, 270

Question 16

a = (5 - (-5))/2 or c = (5 + (-5))/2	**M1**
a = 5	**A1**
c = 0	**A1**
b = 360/90 = 4	**B1**

Answer: a = 5, b = 4, c = 0

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Question 17

sin(10x) = 0 **M1**

10x = 0, 180, 360, ... **M1**

x = 0, 18, 36, 54, 72, 90, 108, 126, 144, 162, 180, 198, 216, 234, 252, 270, 288, 306, 324, 342, 360 cao **A1**

Answer: x = 0, 18, 36, 54, 72, 90, 108, 126, 144, 162, 180, 198, 216, 234, 252, 270, 288, 306, 324, 342, 360

Question 18

a = (11 - (-1))/2 or c = (11 + (-1))/2 **M1**

a = 6 **A1**

c = 5 **A1**

b = 360/120 = 3 **B1**

Answer: a = 6, b = 3, c = 5

Question 19

$\cos^{-1}(-0.6) = 126.9$ (awrt, principal value) **M1**

x = 126.9 (awrt 1 d.p.) **A1**

x = 233.1 (awrt 1 d.p., using 360 - 126.9) **A1**

Answer: x = 126.9, 233.1 (to 1 d.p.)

Question 20

sin(3x) = 0 **M1**

3x = 0, 180, 360, ... **M1**

x = 0, 60, 120, 180, 240, 300, 360 cao **A1**

Answer: x = 0, 60, 120, 180, 240, 300, 360

Question 21

minimum of 3cos(x) is -3, giving y = 3(-1) - 2 = -5 **M1**

(180, -5) cao **A1**

Answer: (180, -5)

Question 22

reference angle arcsin(0.7) = 44.4 (awrt) **M1**

x = 224.4 (awrt, using 180 + 44.4) **A1**

x = 315.6 (awrt, using 360 - 44.4) **A1**

Answer: x = 224.4 or x = 315.6

Question 23

rearrange to cos(x) = -0.5 **M1**

x = 120 cao **A1**

x = 240 cao **A1**

Answer: x = 120, 240

Question 24

stretch, scale factor 1/5, parallel to the x-axis oe **B1**

Answer: Stretch, scale factor 1/5, parallel to the x-axis

Question 25

(a) sin(150) = sin(180 - 150) = sin(30) **M1**

(a) 1/2 oe cao **A1**

(a) Answer: 1/2

(b) 210 is in the third quadrant, where cosine is negative; reference angle = 210 - 180 = 30 **M1**

(b) $-\sqrt{3}/2$ oe cao **A1**

(b) Answer: $-\sqrt{3}/2$

(c) 315 is in the fourth quadrant, where tangent is negative; reference angle = 360 - 315 = 45 **M1**

(c) -1 cao **A1**

(c) Answer: -1

Question 26

correctly quotes and rearranges $\sin^2(\theta) + \cos^2(\theta) = 1$ to $\cos^2(\theta) = 1 - \sin^2(\theta)$ **M1**

$\cos^2(\theta) = 1 - 0.28^2 = 1 - 0.0784 = 0.9216$ **M1**

$\sqrt{0.9216} = 0.96$ (awrt) **A1**

$\cos(\theta) = -0.96$ (negative root selected since θ is obtuse) cao **A1**

Answer: cos(θ) = -0.96

Question 27

(a) forms the equation a + b = 7 (from cos(0) = 1) **M1**

(a) forms the equation -a + b = -1 (from cos(180) = -1) **M1**

(a) b = 3 (by adding the two equations: 2b = 6) **A1**

(a) a = 4 cao **A1**

(a) Answer: a = 4, b = 3

Question 28

forms the equation a + b = 11 (from the maximum, sin(x) = 1) **M1**

forms the equation -a + b = -3 (from the minimum, sin(x) = -1) **M1**

a = 7 and b = 4 (both required) cao **A1**

Answer: a = 7, b = 4

Question 29

(a) y = sin(2x) - 3 oe cao **B1**

(a) Answer: y = sin(2x) - 3

(b) 180, ft from part (a) **B1**

(b) Answer: 180

(c) identifies a minimum of sin(2x) where 2x = 270, i.e. x = 135 (or x = 315), ft from part (a) **M1**

(c) (135, -4) oe, e.g. (315, -4), cao **A1**

(c) Answer: (135, -4)

Question 30

let u = 2x - 30, so the range for u is $-30 \leq u \leq 690$ **M1**

first two solutions for u: 30 and 150 (using $\sin^{-1}(0.5) = 30$ and 180 - 30) **M1**

further two solutions for u using the 360-degree period: 390 and 510 **M1**

x = 30 and x = 90 (from u = 30 and u = 150, using x = (u + 30)/2) **A1**

x = 210 and x = 270 (from u = 390 and u = 510) cao, all four values required **A1**

Answer: x = 30, 90, 210, 270

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Question 31

(a) the minimum of 2sin(x) is -2, so -2 + k = -5 **M1**

(a) k = -3 cao **A1**

(a) Answer: k = -3

(b) rearranges to sin(x) = 1.5 and states that this is impossible because sin(x) can only take values between -1 and 1 (1.5 is outside this range) oe ft from part (a) **B1**

(b) Answer: 2sin(x) - 3 = 0 gives sin(x) = 1.5, which is impossible since -1 ≤ sin(x) ≤ 1 for all x.

Question 32

(a) p = 6 cao **B1**

(a) Answer: p = 6

(b) 4 cycles over 360 degrees means the period is 360/4 = 90 **M1**

(b) q = 4, from period = 360/q = 90, cao **A1**

(b) Answer: q = 4

(c) cos(4x) is a maximum when 4x = 360 (the smallest positive solution, since 4x = 0 gives x = 0), ft from parts (a) and (b) **M1**

(c) (90, 6) cao **A1**

(c) Answer: (90, 6)

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