3.4 Trigonometric Graphs and Transformations: Test Pack

AQA 8365 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The graphs of y = sin(x), y = cos(x) and y = tan(x) are defined for $0 \leq x \leq 360$.

(a) Which of these correctly gives the amplitude of y = 4cos(x)? **(1)**

- ☐ A) 1
- ☐ B) 4
- ☐ C) 360
- ☐ D) 90

(b) State the period, in degrees, of y = sin(x). **(1)**

(c) State the period, in degrees, of y = tan(x). **(1)**

(d) State the amplitude of y = sin(x). **(1)**

(Total for Question 1 is 4 marks)

© Revision Library | revisionlibrary.co.uk

2 Each graph below is a transformation of y = sin(x). Describe fully the single transformation of y = sin(x) that produces each graph.

(a) y = sin(x) + 3 **(1)**

(b) y = sin(x - 60) **(1)**

(c) y = 5sin(x) **(1)**

(d) y = sin(4x) **(1)**

(Total for Question 2 is 4 marks)

---

3 Solve the following trigonometric equations for $0 \leq x \leq 360$.

(a) sin(x) = 0.5 **(3)**

(Total for Question 3 is 3 marks)

---

4 Describe fully the single transformation that maps y = sin(x) onto y = sin(x) - 4.

(Total for Question 4 is 1 mark)

© Revision Library | revisionlibrary.co.uk

5 The graph of $y = \sin(x)$ has a maximum point at (90, 1). Use this fact to find the coordinates of the stated point on each transformed graph.

(a) Find the coordinates of the maximum point of $y = 3\sin(x) + 2$. **(2)**

.....

(b) Find the coordinates of the maximum point of $y = \sin(x - 40) + 2$. **(2)**

.....

(c) Find the coordinates of the minimum point of $y = -2\sin(x)$. **(2)**

.....

(Total for Question 5 is 6 marks)

---

6 The height of the tide, H metres, in a small harbour on the Cornish coast is modelled by $H(t) = 6 + 2\sin(30t)$, where t is the number of hours after midnight and $0 \leq t \leq 24$.

(a) State the maximum and minimum heights of the tide predicted by the model. **(2)**

(b) Find the two smallest positive values of t at which the tide height is exactly 6 m. **(3)**

.....

(c) Calculate the height of the tide at $t = 4$, giving your answer correct to 2 decimal places. **(2)**

.....

(Total for Question 6 is 7 marks)

© Revision Library | revisionlibrary.co.uk

7 The graphs of $y = \sin(x)$ and $y = \cos(x)$ are both defined for $0 \leq x \leq 360$.

(a) Describe fully the single transformation that maps the graph of $y = \sin(x)$ onto the graph of $y = \cos(x)$. **(2)**

(b) Hence write $\sin(x)$ in terms of cos, in the form $\sin(x) = \cos(x - a)$ for some constant a. **(1)**

(Total for Question 7 is 3 marks)

---

8 The graph of $y = \sin(x)$ is transformed to give the graph of $y = \sin(x + 30)$. Describe fully the single transformation.

(Total for Question 8 is 2 marks)

---

9 The graph of $y = -5\cos(6x) + 1$ is drawn for $0 \leq x \leq 360$. Write down the maximum value of y, the minimum value of y and the period of the graph.

(Total for Question 9 is 3 marks)

---

10 Find the values of x between 0 and 360 inclusive where the graph of $y = 3\sin(x)$ crosses the x-axis.

(Total for Question 10 is 3 marks)

© Revision Library | revisionlibrary.co.uk

11 The graph of $y = a\cos(bx) + c$, where a and b are positive integers and c is an integer, is drawn for $0 \leq x \leq 360$. The maximum value of y is -2, the minimum value of y is -4 and the period is 60 degrees. Find the values of a, b and c.

(Total for Question 11 is 4 marks)

---

12 The depth of water, D metres, at the entrance to a marina is modelled by $D(t) = 5 + 3\sin(30t)$, where t is the number of hours after midnight and $0 \leq t \leq 24$. Find the maximum depth predicted by the model, and the smallest positive value of t at which this maximum occurs.

(Total for Question 12 is 3 marks)

---

13 The graph of $y = \tan(x)$ is translated by vector (45, 0) and is then stretched by scale factor 1/2 parallel to the y-axis, in that order.

(a) Write down the equation of the resulting graph. **(3)**

.....

(Total for Question 13 is 3 marks)

---

14 Solve $3\sin(x) = 2\cos(x)$ for $0 \leq x \leq 360$, giving your answers correct to 1 decimal place.

(Total for Question 14 is 4 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 3.4 Trigonometric Graphs and Transformations: Test Pack

Question 1

(a) B) 4 **B1**

(a) Answer: B) 4

(b) 360 cao **B1**

(b) Answer: 360

(c) 180 cao **B1**

(c) Answer: 180

(d) 1 cao **B1**

(d) Answer: 1

Question 2

(a) translation by vector (0, 3) oe **B1**

(a) Answer: Translation by vector (0, 3)

(b) translation by vector (60, 0) oe **B1**

(b) Answer: Translation by vector (60, 0)

(c) stretch, scale factor 5, parallel to the y-axis oe **B1**

(c) Answer: Stretch, scale factor 5, parallel to the y-axis

(d) stretch, scale factor 1/4, parallel to the x-axis oe **B1**

(d) Answer: Stretch, scale factor 1/4, parallel to the x-axis

Question 3

(a) $\sin^{-1}(0.5) = 30$ (principal value) **M1**

(a) x = 30 **A1**

(a) x = 150 (using 180 - 30) cao **A1**

(a) Answer: x = 30, 150

Question 4

translation by vector (0, -4) oe **B1**

Answer: Translation by vector (0, -4)

Question 5

(a) 3(1) + 2 = 5, with x-coordinate unchanged at 90 **M1**

(a) (90, 5) cao **A1**

(a) Answer: (90, 5)

(b) x-coordinate shifts to 90 + 40 = 130 **M1**

(b) (130, 3) cao **A1**

(b) Answer: (130, 3)

(c) the original maximum of sin(x) becomes a minimum after reflection: -2(1) = -2 **M1**

(c) (90, -2) cao **A1**

(c) Answer: (90, -2)

Question 6

(a) maximum height = 8 m **B1**

(a) minimum height = 4 m **B1**

(a) Answer: Maximum = 8 m, minimum = 4 m

(b) sets up 6 + 2sin(30t) = 6, i.e. sin(30t) = 0 **M1**

(b) t = 6 (from 30t = 180) **A1**

(b) t = 12 (from 30t = 360) cao **A1**

(b) Answer: t = 6, t = 12

(c) substitutes t = 4 to give H = 6 + 2sin(120) **M1**

(c) H = 7.73 (awrt 2 d.p.) **A1**

(c) Answer: H = 7.73 m

Question 7

(a) translation **M1**

(a) by vector (-90, 0), i.e. 90 units in the negative x-direction (left) oe cao **A1**

(a) Answer: Translation by vector (-90, 0), i.e. 90 to the left

(b) sin(x) = cos(x - 90) oe cao **B1**

(b) Answer: sin(x) = cos(x - 90)

Question 8

translation (correct type) **B1**

full correct detail (vector, factor or axis) **B1**

Answer: translation by vector (-30, 0)

Question 9

maximum 6 **B1**

minimum -4 **B1**

period 60 degrees **B1**

Answer: maximum = 6, minimum = -4, period = 60 degrees

Question 10

sin(x) = 0 **M1**

x = 0, 180, 360, ... **M1**

x = 0, 180, 360 cao **A1**

Answer: x = 0, 180, 360

Question 11

a = (-2 - (-4))/2 or c = (-2 + (-4))/2 **M1**

a = 1 **A1**

c = -3 **A1**

b = 360/60 = 6 **B1**

Answer: a = 1, b = 6, c = -3

Question 12

maximum of 3sin(30t) is 3, giving D = 5 + 3 = 8 **M1**

maximum depth = 8 m **A1**

t = 3 (from 30t = 90) cao **A1**

Answer: Maximum depth = 8 m, at t = 3

Question 13

(a) applies the translation correctly to give tan(x - 45) **M1**

(a) applies the vertical stretch of scale factor 1/2 to give (1/2)tan(x - 45) **M1**

(a) y = 0.5tan(x - 45) oe cao **A1**

(a) Answer: y = 0.5tan(x - 45)

Question 14

divide both sides by cos(x) to form 3tan(x) = 2, hence tan(x) = 2/3 **M1**

$\tan^{-1}(2/3) = 33.7$ (awrt, principal value) **M1**

x = 33.7 (awrt 1 d.p.) **A1**

x = 213.7 (awrt 1 d.p., using 33.7 + 180) **A1**

Answer: x = 33.7, 213.7 (to 1 d.p.)

© Revision Library | revisionlibrary.co.uk