3.6 3D Pythagoras and Trigonometry: 2-hour Pack

AQA 8365 · Calculator allowed · about 120 minutes

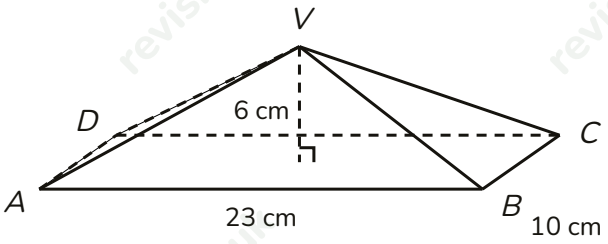
Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A pyramid has a rectangular base ABCD with AB = 23 cm and BC = 10 cm. The apex V is 6 cm vertically above the centre of the base. Calculate the length of the sloping edge VA. Give your answer correct to 3 significant figures.

Diagram NOT accurately drawn

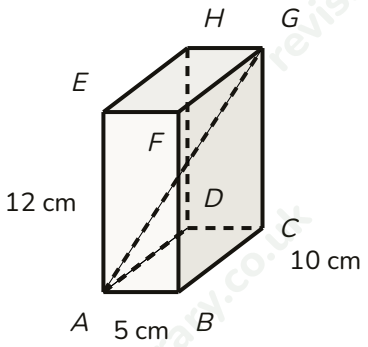


(Total for Question 1 is 3 marks)

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2 A cuboid ABCDEFGH has AB = 5 cm, AD = 10 cm and AE = 12 cm. Calculate the angle between the diagonal AG and the edge AD. Give your answer correct to 1 decimal place.

Diagram NOT accurately drawn

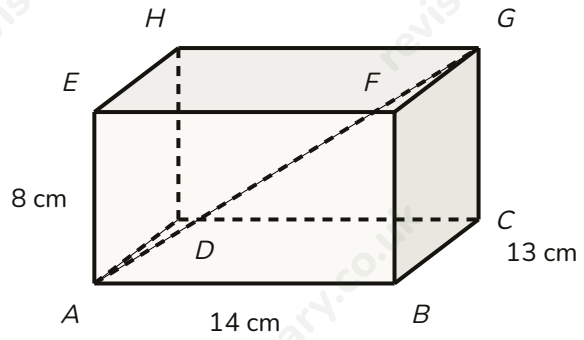


(Total for Question 2 is 3 marks)

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3 A cuboid ABCDEFGH has length AB = 14 cm, width BC = 13 cm and height AE = 8 cm. Calculate the length of the diagonal AG. Give your answer correct to 3 significant figures.

Diagram NOT accurately drawn

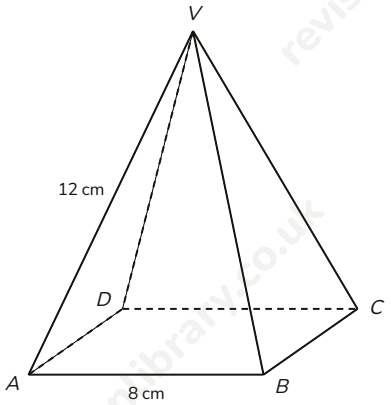


(Total for Question 3 is 3 marks)

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4 A pyramid has a square base ABCD with sides of length 8 cm. The apex V is directly above the centre of the base. Each sloping edge, such as VA, is 12 cm long. Calculate the height of the pyramid. Give your answer correct to 3 significant figures.

Diagram NOT accurately drawn

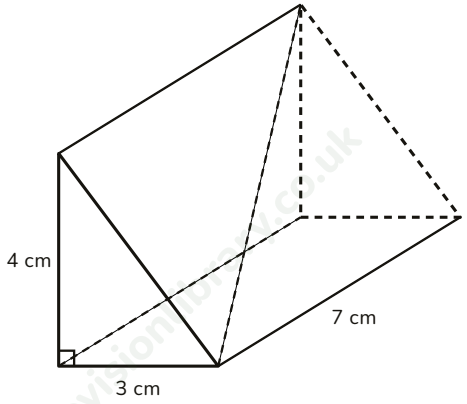


(Total for Question 4 is 3 marks)

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5 A triangular prism has a right-angled triangle as its cross-section, with perpendicular sides 3 cm and 4 cm. The length of the prism is 7 cm. Calculate the length of the longest straight line that can be drawn on or inside the prism, from one corner to the opposite corner. Give your answer correct to 3 significant figures.

Diagram NOT accurately drawn

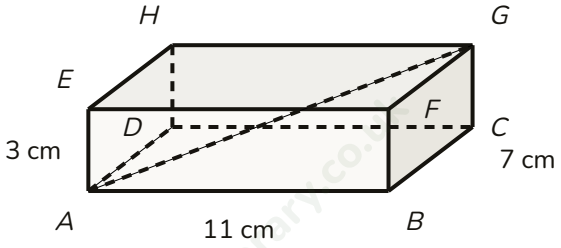


(Total for Question 5 is 3 marks)

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6 A cuboid ABCDEFGH has length AB = 11 cm, width BC = 7 cm and height AE = 3 cm. Calculate the length of the diagonal AG. Give your answer correct to 3 significant figures.

Diagram NOT accurately drawn

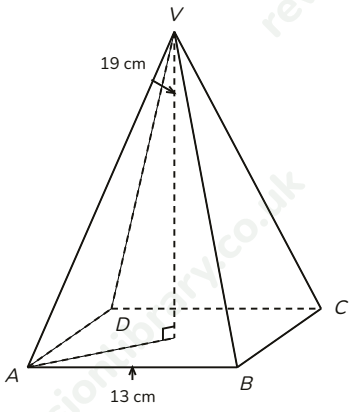


(Total for Question 6 is 3 marks)

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7 A pyramid has a square base ABCD with sides of length 13 cm. The apex V is 19 cm directly above the centre of the base. Calculate the angle between the sloping edge VA and the base ABCD. Give your answer correct to 1 decimal place.

Diagram NOT accurately drawn

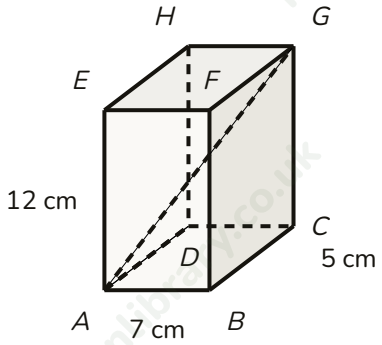


(Total for Question 7 is 3 marks)

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8 A cuboid ABCDEFGH has AB = 7 cm, AD = 5 cm and AE = 12 cm. Calculate the angle between the diagonal AG and the edge AD. Give your answer correct to 1 decimal place.

Diagram NOT accurately drawn

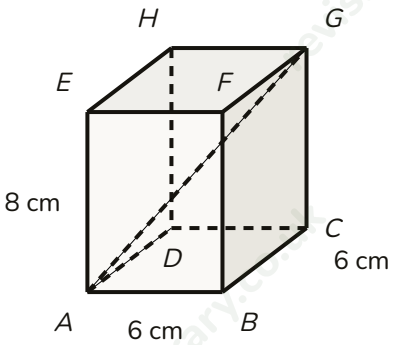


(Total for Question 8 is 3 marks)

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9 A cuboid ABCDEFGH has length AB = 6 cm, width BC = 6 cm and height AE = 8 cm. Calculate the length of the diagonal AG. Give your answer correct to 3 significant figures.

Diagram NOT accurately drawn

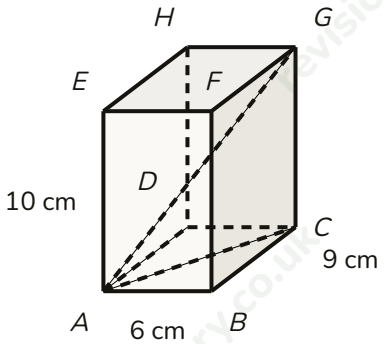


(Total for Question 9 is 3 marks)

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10 A cuboid ABCDEFGH has AB = 6 cm, BC = 9 cm and AE = 10 cm. Calculate the angle between the diagonal AG and the base ABCD. Give your answer correct to 1 decimal place.

Diagram NOT accurately drawn

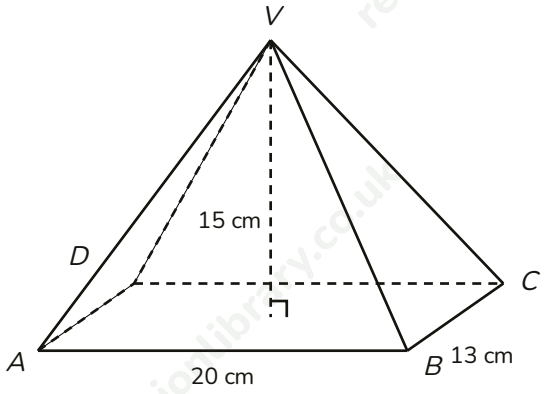


(Total for Question 10 is 3 marks)

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11 A pyramid has a rectangular base ABCD with AB = 20 cm and BC = 13 cm. The apex V is 15 cm vertically above the centre of the base. Calculate the length of the sloping edge VA. Give your answer correct to 3 significant figures.

Diagram NOT accurately drawn

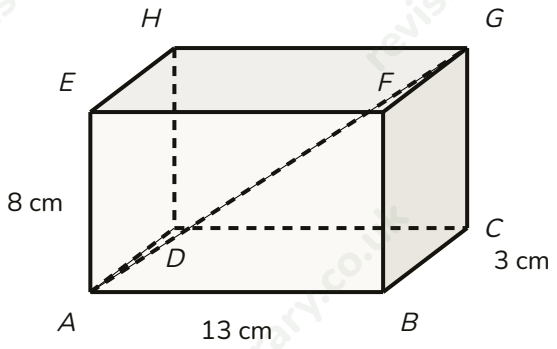


(Total for Question 11 is 3 marks)

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12 A cuboid ABCDEFGH has length AB = 13 cm, width BC = 3 cm and height AE = 8 cm. Calculate the length of the diagonal AG. Give your answer correct to 3 significant figures.

Diagram NOT accurately drawn

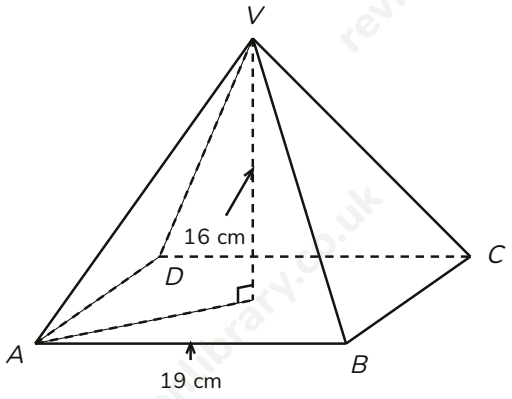


(Total for Question 12 is 3 marks)

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13 A pyramid has a square base ABCD with sides of length 19 cm. The apex V is 16 cm directly above the centre of the base. Calculate the angle between the sloping edge VA and the base ABCD. Give your answer correct to 1 decimal place.

Diagram NOT accurately drawn

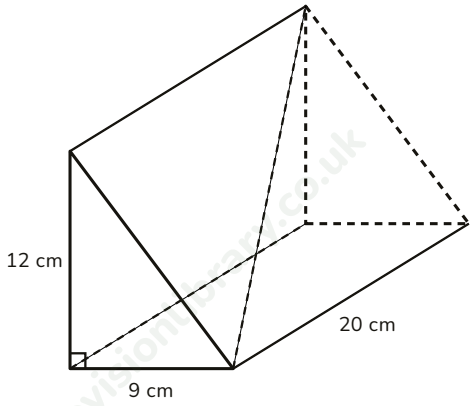


(Total for Question 13 is 3 marks)

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14 A triangular prism has a right-angled triangle as its cross-section, with perpendicular sides 9 cm and 12 cm. The length of the prism is 20 cm. Calculate the length of the longest straight line that can be drawn on or inside the prism, from one corner to the opposite corner. Give your answer correct to 3 significant figures.

Diagram NOT accurately drawn

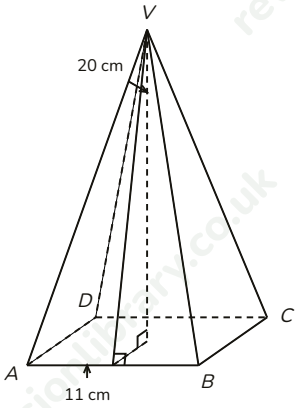


(Total for Question 14 is 3 marks)

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15 A pyramid has a square base ABCD with sides of length 11 cm. The apex V is 20 cm directly above the centre of the base. Calculate the angle between the sloping face VAB and the base ABCD. Give your answer correct to 1 decimal place.

Diagram NOT accurately drawn

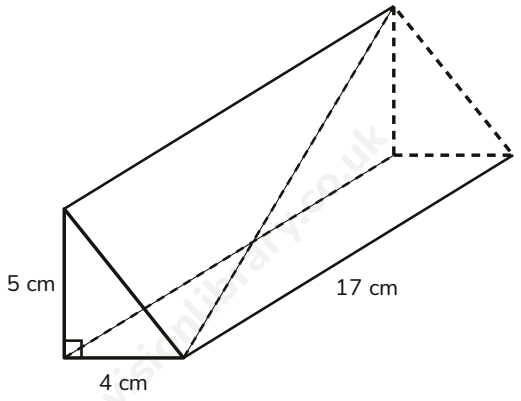


(Total for Question 15 is 3 marks)

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16 A triangular prism has a right-angled triangle as its cross-section, with perpendicular sides 4 cm and 5 cm. The length of the prism is 17 cm. Calculate the length of the longest straight line that can be drawn on or inside the prism, from one corner to the opposite corner. Give your answer correct to 3 significant figures.

Diagram NOT accurately drawn

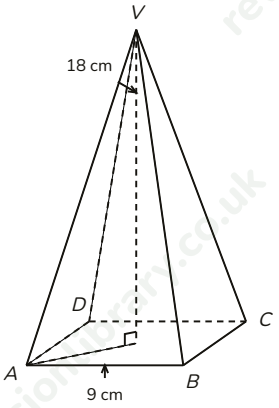


(Total for Question 16 is 3 marks)

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17 A pyramid has a square base ABCD with sides of length 9 cm. The apex V is 18 cm directly above the centre of the base. Calculate the angle between the sloping edge VA and the base ABCD. Give your answer correct to 1 decimal place.

Diagram NOT accurately drawn

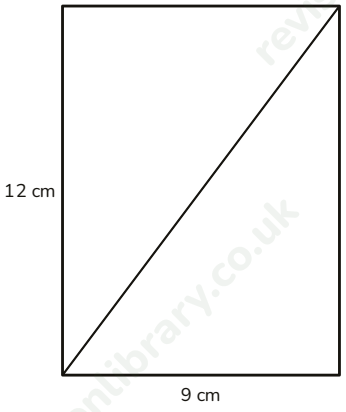


(Total for Question 17 is 3 marks)

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18 Calculate the length of the diagonal of a rectangle with length 9 cm and width 12 cm.

Diagram NOT accurately drawn

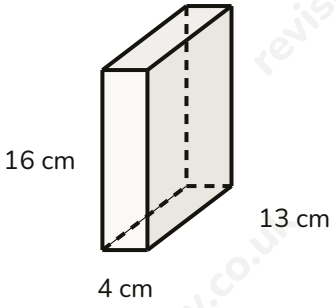


(Total for Question 18 is 2 marks)

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19 A cuboid has length 4 cm, width 13 cm and height 16 cm. Calculate the length of the space diagonal connecting two opposite vertices of the cuboid.

Diagram NOT accurately drawn



(Total for Question 19 is 2 marks)

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20 A vertical flagpole DF stands at the corner D of a horizontal rectangular courtyard ABCD, where AB = 18 m and AD = 24 m. The angle of elevation of the top of the flagpole, F, from corner B, is 32 degrees.

(a) Find the length BD (the diagonal of the courtyard). **(2)**

.....

(b) Hence calculate the height of the flagpole, DF, giving your answer correct to 3 significant figures. **(3)**

.....

(Total for Question 20 is 5 marks)

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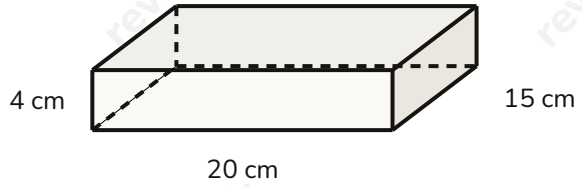
21 A vertical pole stands at corner A of a rectangular horizontal field ABCD, with AB = 9 m and AD = 12 m. Given that the diagonal AC = 15 m, and the angle of elevation of the top of the pole from corner C is 20 degrees, calculate the height of the pole, giving your answer correct to 3 significant figures.

(Total for Question 21 is 2 marks)

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22 A cuboid has a rectangular base with length 20 cm and width 15 cm, and height 4 cm.
Calculate the length of the diagonal connecting two opposite vertices of the cuboid, giving your answer correct to 3 significant figures.

Diagram NOT accurately drawn

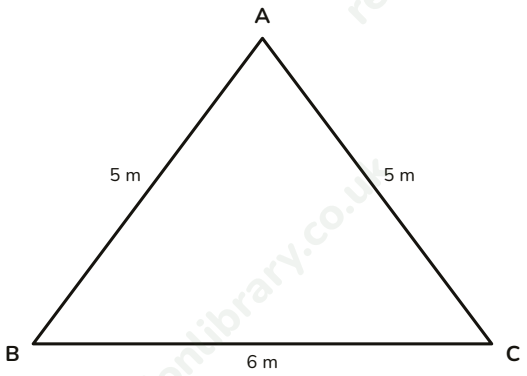


(Total for Question 22 is 3 marks)

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23 A tent has the shape of a triangular prism. The cross-section is an isosceles triangle with base BC = 6 m and each sloping edge AB = AC = 5 m, where A is the ridge point directly above the midpoint of BC.

Diagram NOT accurately drawn



(a) Find the perpendicular height of the ridge A above the base BC. **(3)**

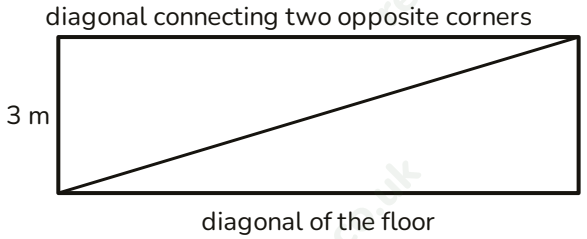
(b) Calculate the angle each roof panel makes with the horizontal ground, giving your answer correct to 1 decimal place. **(3)**

(Total for Question 23 is 6 marks)

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24 A room is in the shape of a cuboid, 6 m long, 8 m wide and 3 m high. Calculate the angle between the diagonal of the floor and the diagonal connecting two opposite corners of the room (one on the floor, one on the ceiling), giving your answer correct to 1 decimal place.

Diagram NOT accurately drawn

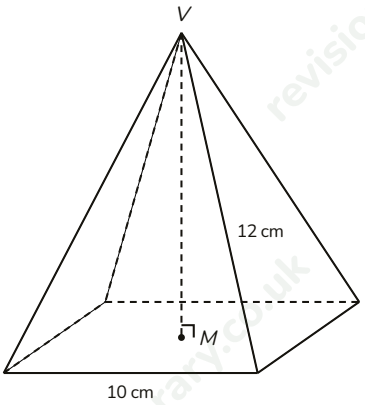


(Total for Question 24 is 3 marks)

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25 A square-based pyramid has base side 10 cm and vertical height 12 cm, with apex V directly above the centre M of the base. Calculate the length of a sloping edge VA, giving your answer correct to 3 significant figures.

Diagram NOT accurately drawn

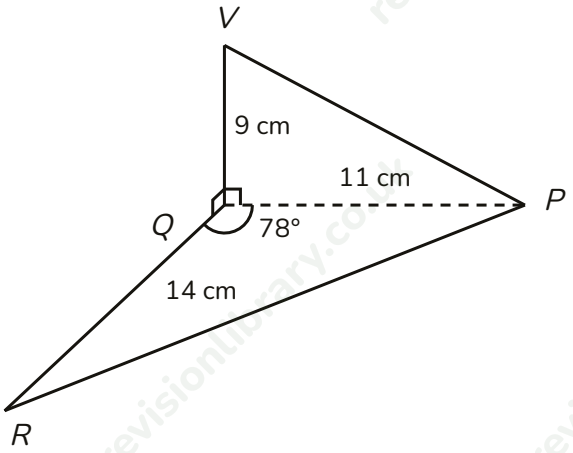


(Total for Question 25 is 3 marks)

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26 A pyramid has a triangular base PQR, with PQ = 11 cm, QR = 14 cm, and angle PQR = 78 degrees. The apex, V, is positioned such that VQ is perpendicular to the base plane PQR, with VQ = 9 cm.

Diagram NOT accurately drawn



(a) Use the cosine rule to find the length PR, giving your answer correct to 3 significant figures. **(3)**

.....

(b) Calculate the length VP, giving your answer correct to 3 significant figures. **(2)**

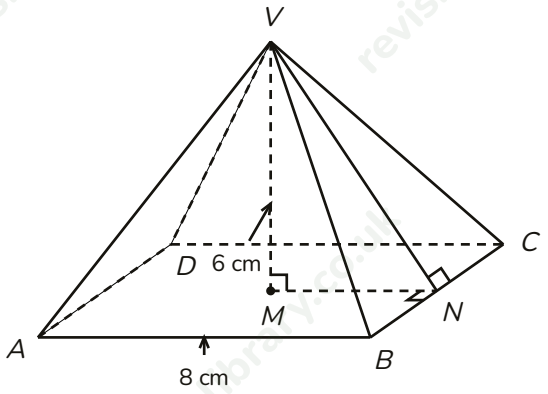
.....

(Total for Question 26 is 5 marks)

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27 A square-based pyramid has base ABCD with side length 8 cm, and apex V vertically above the centre M of the base, with height VM = 6 cm. N is the midpoint of BC.

Diagram NOT accurately drawn



(a) Find the length MN. **(1)**

.....

(b) Hence calculate the angle between the sloping face VBC and the base, giving your answer correct to 1 decimal place. **(3)**

.....

(Total for Question 27 is 4 marks)

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28 A vertical mast MT stands at corner M of a horizontal rectangular sports pitch MNPQ, with MN = 40 m and MQ = 30 m. The angle of elevation of the top T of the mast from corner P (diagonally opposite M) is 15 degrees.

(a) Find the length of the diagonal MP. **(1)**

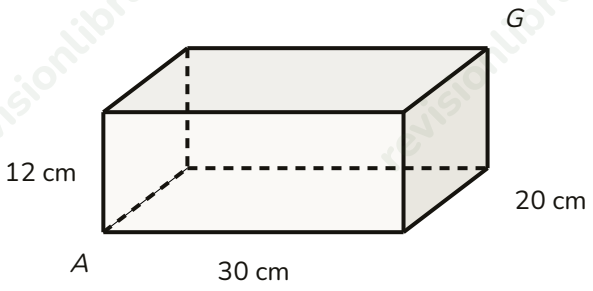
(b) Hence calculate the height of the mast, MT, giving your answer correct to 3 significant figures. **(3)**

(Total for Question 28 is 4 marks)

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29 A cuboid box has length 30 cm, width 20 cm and height 12 cm. A spider is at one bottom corner, A, and wants to crawl over the surface of the box to reach the opposite top corner, G. Calculate the shortest distance the spider must crawl, giving your answer correct to 3 significant figures.

Diagram NOT accurately drawn

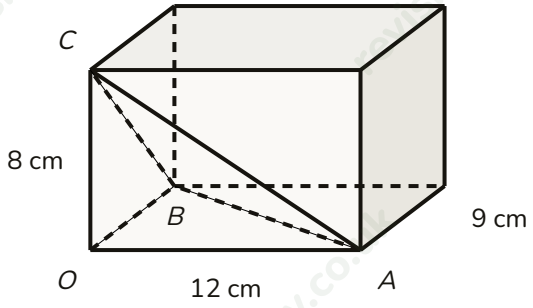


(Total for Question 29 is 5 marks)

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30 In a storage container, three edges OA, OB and OC meet at corner O and are mutually perpendicular, with OA = 12 cm, OB = 9 cm and OC = 8 cm.

Diagram NOT accurately drawn



(a) Find the length AB. **(1)**

.....

(b) Find the length AC, giving your answer correct to 3 significant figures. **(1)**

.....

(c) Find the length BC, giving your answer correct to 3 significant figures. **(1)**

.....

(d) Using exact (unrounded) values, calculate the size of angle BAC, giving your answer correct to 1 decimal place. **(2)**

.....

(Total for Question 30 is 5 marks)

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Mark scheme · 3.6 3D Pythagoras and Trigonometry: 2-hour Pack

A correct answer earns full marks unless the question asks for working, a method, proof or explanation.

Question 1

AC = $\sqrt{23^2 + 10^2}$ and half of AC = 12.540	**M1**
$VA^2 = 6^2 + (\text{half AC})^2$	**M1**
13.9 cm cao	**A1**

Answer: 13.9 cm

Question 2

AG = $\sqrt{269}$ = 16.401	**M1**
cos (angle) = 10 / AG (triangle A-D-G is right-angled)	**M1**
52.4 degrees cao	**A1**

Answer: 52.4 degrees

Question 3

$AC^2 = 14^2 + 13^2$ (face diagonal) seen	**M1**
$AG^2 = AC^2 + 8^2 = 429$	**M1**
20.7 cm cao	**A1**

Answer: 20.7 cm

Question 4

AC = $\sqrt{8^2 + 8^2}$ or half-diagonal = 5.657	**M1**
$h^2 = 12^2$ - (half diagonal)^2 = 112.00	**M1**
10.6 cm cao	**A1**

Answer: 10.6 cm

Question 5

hypotenuse of the cross-section = $\sqrt{3^2 + 4^2}$	**M1**
$d^2 = (3^2 + 4^2) + 7^2 = 74$	**M1**
8.60 cm cao	**A1**

Answer: 8.60 cm

Question 6

$AC^2 = 11^2 + 7^2$ (face diagonal) seen	**M1**
$AG^2 = AC^2 + 3^2 = 179$	**M1**
13.4 cm cao	**A1**

Answer: 13.4 cm

Question 7

right-angled triangle with the vertical height identified	**M1**
tan (angle) = 19 / 9.192	**M1**
64.2 degrees cao	**A1**

Answer: 64.2 degrees

Question 8

AG = $\sqrt{218}$ = 14.765	**M1**
cos (angle) = 5 / AG (triangle A-D-G is right-angled)	**M1**
70.2 degrees cao	**A1**

Answer: 70.2 degrees

Question 9

$AC^2 = 6^2 + 6^2$ (face diagonal) seen	**M1**
$AG^2 = AC^2 + 8^2 = 136$	**M1**
11.7 cm cao	**A1**

Answer: 11.7 cm

Question 10

AC = $\sqrt{6^2 + 9^2}$ = 10.817	**M1**
tan (angle GAC) = 10 / AC	**M1**
42.8 degrees cao	**A1**

Answer: 42.8 degrees

Question 11

AC = $\sqrt{20^2 + 13^2}$ and half of AC = 11.927	**M1**
$VA^2 = 15^2 + (\text{half AC})^2$	**M1**
19.2 cm cao	**A1**

Answer: 19.2 cm

Question 12

$AC^2 = 13^2 + 3^2$ (face diagonal) seen	**M1**
$AG^2 = AC^2 + 8^2 = 242$	**M1**
15.6 cm cao	**A1**

Answer: 15.6 cm

Question 13

right-angled triangle with the vertical height identified	**M1**
tan (angle) = 16 / 13.435	**M1**
50.0 degrees cao	**A1**

Answer: 50.0 degrees

Question 14

hypotenuse of the cross-section = $\sqrt{9^2 + 12^2}$	**M1**
$d^2 = (9^2 + 12^2) + 20^2 = 625$	**M1**
25.0 cm cao	**A1**

Answer: 25.0 cm

Question 15

right-angled triangle with the vertical height identified	**M1**
M is the midpoint of AB; tan (angle) = 20 / 5.5	**M1**
74.6 degrees cao	**A1**

Answer: 74.6 degrees

Question 16

hypotenuse of the cross-section = $\sqrt{4^2 + 5^2}$	**M1**
$d^2 = (4^2 + 5^2) + 17^2 = 330$	**M1**
18.2 cm cao	**A1**

Answer: 18.2 cm

Question 17

right-angled triangle with the vertical height identified	**M1**
tan (angle) = 18 / 6.364	**M1**
70.5 degrees cao	**A1**

Answer: 70.5 degrees

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Question 18

$d = \sqrt{9^2 + 12^2}$ **M1**

d = 15 cm cao **A1**

Answer: 15 cm

Question 19

$d = \sqrt{4^2 + 13^2 + 16^2}$ **M1**

d = 21 cm cao **A1**

Answer: 21 cm

Question 20

(a) $BD = \sqrt{18^2 + 24^2}$ **M1**

(a) BD = 30 m cao **A1**

(a) Answer: 30 m

(b) recognises that triangle FDB is right-angled at D, since the flagpole is vertical **M1**

(b) sets up DF = their BD x tan(32) **M1**

(b) DF = 18.7 m (3sf) cao **A1**

(b) Answer: 18.7 m (3sf)

Question 21

height = 15 * tan(20) **M1**

5.46 m awrt (3sf) **A1**

Answer: 5.46 m

Question 22

finds the base diagonal, $\sqrt{20^2 + 15^2} = \sqrt{625} = 25$ **M1**

combines with the height: $d = \sqrt{25^2 + 4^2}$ **M1**

25.3 cm awrt (3sf) **A1**

Answer: 25.3 cm

Question 23

(a) finds the midpoint distance, BM = 3 m (half of BC) **M1**

(a) uses Pythagoras, $AM = \sqrt{AB^2 - BM^2} = \sqrt{5^2 - 3^2}$ **M1**

(a) AM = 4 m cao **A1**

(a) Answer: 4 m

(b) identifies angle ABM as the angle between the sloping roof panel AB and the horizontal ground **M1**

(b) sets up tan(angle) = their AM / BM = 4/3 **M1**

(b) angle = 53.1 degrees (1dp) cao **A1**

(b) Answer: 53.1 degrees (1dp)

Question 24

floor diagonal = $\sqrt{6^2 + 8^2} = \sqrt{100} = 10$ **M1**

tan(angle) = 3/10, using the right-angled triangle formed by the height, floor diagonal and space diagonal **M1**

16.7 degrees awrt (1dp) **A1**

Answer: 16.7 degrees

Question 25

finds $AM^2 = (10/\sqrt{2})^2 = 50$, the square of half the base diagonal **M1**

$VA = \sqrt{12^2 + 50}$ **M1**

13.9 cm awrt (3sf) **A1**

Answer: 13.9 cm

Question 26

(a) correct cosine rule set up, $PR^2 = 11^2 + 14^2 - 2(11)(14)\cos(78)$ **M1**

(a) correct evaluation, $PR^2 = 317 - 308\cos(78) = 252.96$ (accept awrt) **M1**

(a) PR = 15.9 cm (3sf) cao **A1**

(a) Answer: 15.9 cm (3sf)

(b) recognises triangle VQP is right-angled at Q, since VQ is perpendicular to the base plane, and uses $VP = \sqrt{VQ^2 + QP^2}$ **M1**

(b) VP = 14.2 cm (3sf) cao **A1**

(b) Answer: 14.2 cm (3sf)

Question 27

(a) MN = 4 cm cao **B1**

(a) Answer: 4 cm

(b) recognises angle VNM as the angle between the sloping face VBC and the base (since VM is perpendicular to the base and both VN and MN are perpendicular to BC) **M1**

(b) sets up tan(angle) = VM / their MN = 6/4 **M1**

(b) angle = 56.3 degrees (1dp) cao **A1**

(b) Answer: 56.3 degrees (1dp)

Question 28

(a) MP = 50 m cao **B1**

(a) Answer: 50 m

(b) recognises triangle MPT is right-angled at M, since the mast is vertical **M1**

(b) MT = MP * tan(15), ft from part (a) **M1**

(b) 13.4 m awrt (3sf) **A1**

(b) Answer: 13.4 m

Question 29

recognises that the shortest surface path corresponds to unfolding two adjacent faces into a flat rectangle and finding the straight-line distance across the net using Pythagoras' theorem **M1**

considers more than one possible unfolding (combining two of the three dimensions with the third), e.g. $\sqrt{(30+20)^2 + 12^2}$ and $\sqrt{(30+12)^2 + 20^2}$ **M1**

calculates the third candidate, $\sqrt{(20+12)^2 + 30^2} = \sqrt{1024 + 900} = \sqrt{1924}$ **M1**

correctly identifies $\sqrt{1924}$ as the smallest of the three candidate distances **A1**

shortest distance = 43.9 cm (3sf) cao **A1**

Answer: 43.9 cm (3sf)

Question 30

(a) AB = 15 cm cao **B1**

(a) Answer: 15 cm

(b) 14.4 cm awrt (3sf), exact value $4\sqrt{13}$ **B1**

(b) Answer: 14.4 cm

(c) 12.0 cm awrt (3sf), exact value $\sqrt{145}$ **B1**

(c) Answer: 12.0 cm

(d) correct cosine rule using exact squared lengths: $\cos(BAC) = (AB^2 + AC^2 - BC^2)/(2*AB*AC) = (225 + 208 - 145)/(2*15*\sqrt{208})$ **M1**

(d) 48.3 degrees awrt (1dp) **A1**

(d) Answer: 48.3 degrees

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