3.6 3D Pythagoras and Trigonometry: Test Pack

AQA 8365 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A cuboid has length a, width b and height c. Which of the following gives the length of the space diagonal connecting two opposite vertices of the cuboid?

- ☐ A) $\sqrt{a^2 + b^2}$
- ☐ B) $\sqrt{a^2 + b^2 + c^2}$
- ☐ C) $a + b + c$
- ☐ D) $\sqrt{a \times b \times c}$

(Total for Question 1 is 1 mark)

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2 A cone has base radius r and slant height l. Which of the following gives the vertical height h of the cone?

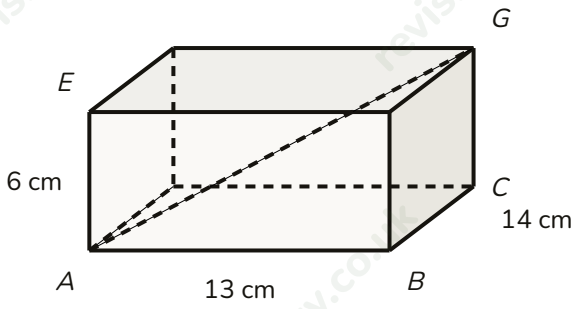
- ☐ A) $\sqrt{l^2 + r^2}$
- ☐ B) $\sqrt{l^2 - r^2}$
- ☐ C) $\sqrt{r^2 - l^2}$
- ☐ D) $l - r$

(Total for Question 2 is 1 mark)

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3 A cuboid ABCDEFGH has length AB = 13 cm, width BC = 14 cm and height AE = 6 cm. Calculate the length of the diagonal AG. Give your answer correct to 3 significant figures.

Diagram NOT accurately drawn

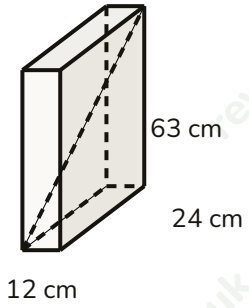


(Total for Question 3 is 3 marks)

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4 A cuboid has length 12 cm and width 24 cm. The length of its space diagonal is 63 cm. Work out the height of the cuboid.

Diagram NOT accurately drawn

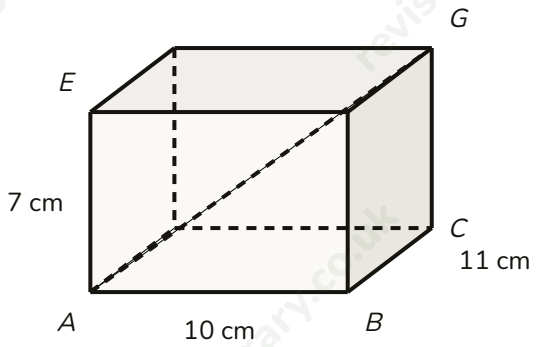


(Total for Question 4 is 3 marks)

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5 A cuboid ABCDEFGH has length AB = 10 cm, width BC = 11 cm and height AE = 7 cm. Calculate the length of the diagonal AG. Give your answer as a simplified surd.

Diagram NOT accurately drawn

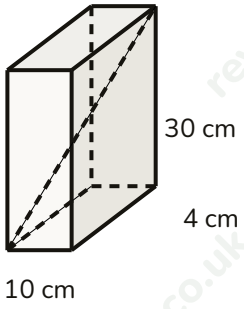


(Total for Question 5 is 3 marks)

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6 A cuboid has length 10 cm and width 4 cm. The length of its space diagonal is 30 cm. Work out the height of the cuboid.

Diagram NOT accurately drawn

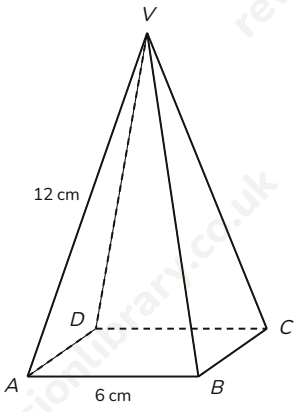


(Total for Question 6 is 3 marks)

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7 A pyramid has a square base ABCD with sides of length 6 cm. The apex V is directly above the centre of the base. Each sloping edge, such as VA, is 12 cm long. Calculate the height of the pyramid. Give your answer correct to 3 significant figures.

Diagram NOT accurately drawn

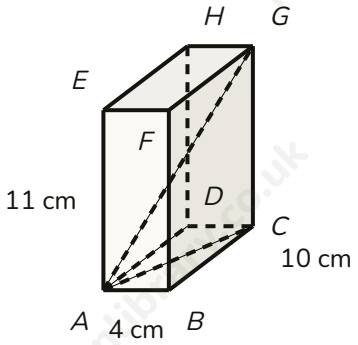


(Total for Question 7 is 3 marks)

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8 A cuboid ABCDEFGH has AB = 4 cm, BC = 10 cm and AE = 11 cm. Calculate the angle between the diagonal AG and the base ABCD. Give your answer correct to 1 decimal place.

Diagram NOT accurately drawn

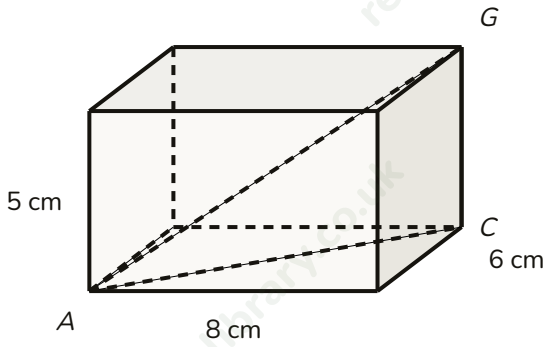


(Total for Question 8 is 3 marks)

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9 A cuboid has a rectangular base with length 8 cm and width 6 cm, and height 5 cm. AC is a diagonal of the base and AG is the space diagonal from the same base vertex A to the opposite top vertex G.

Diagram NOT accurately drawn



(a) Find the length of AC. **(2)**

.....

(b) Hence find the angle between AG and the base plane, giving your answer correct to 1 decimal place. **(2)**

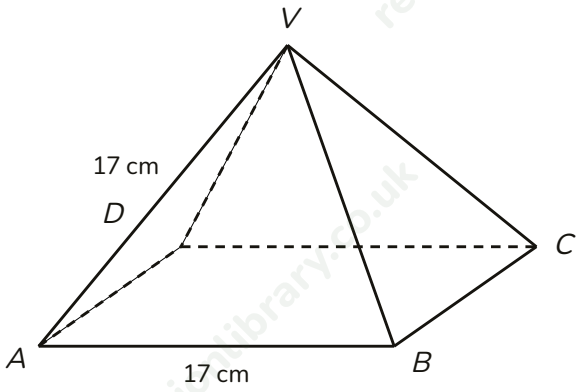
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(Total for Question 9 is 4 marks)

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10 A pyramid has a square base ABCD with sides of length 17 cm. The apex V is directly above the centre of the base. Each sloping edge, such as VA, is 17 cm long. Calculate the height of the pyramid. Give your answer correct to 3 significant figures.

Diagram NOT accurately drawn



(Total for Question 10 is 3 marks)

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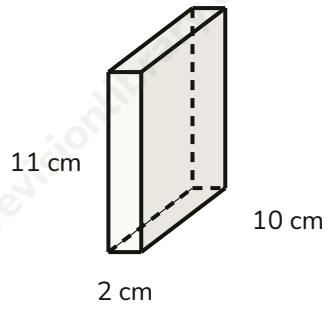
11 State Pythagoras' theorem for a right-angled triangle with hypotenuse h and shorter sides x and y.

(Total for Question 11 is 1 mark)

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12 A cuboid has length 2 cm, width 10 cm and height 11 cm. Calculate the length of the space diagonal connecting two opposite vertices of the cuboid.

Diagram NOT accurately drawn

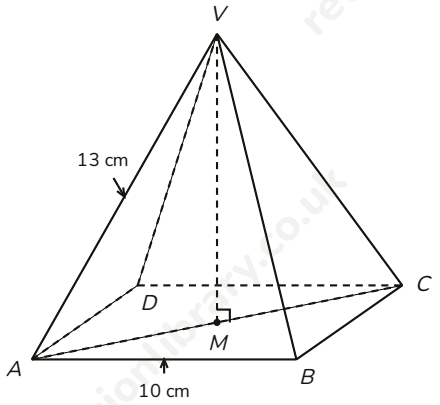


(Total for Question 12 is 2 marks)

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13 A square-based pyramid has a square base ABCD with side length 10 cm. The apex, V, is directly above the centre of the base, M. Each sloping edge VA, VB, VC and VD has length 13 cm.

Diagram NOT accurately drawn



(a) Find the length of the diagonal AC of the base, giving your answer in exact surd form. **(2)**

.....

(b) Hence find the vertical height VM of the pyramid, giving your answer correct to 3 significant figures. **(3)**

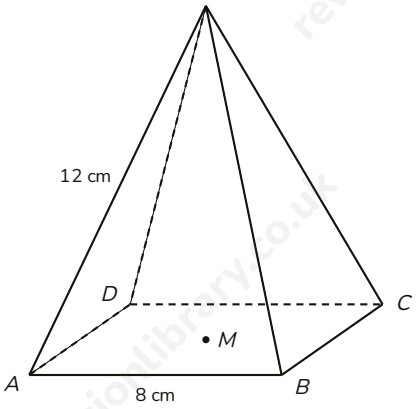
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(Total for Question 13 is 5 marks)

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14 A square-based pyramid has base ABCD with side 8 cm and centre M; each sloping edge has length 12 cm. Given that AM = $4\sqrt{2}$ cm, find the vertical height of the pyramid, giving your answer correct to 3 significant figures.

Diagram NOT accurately drawn

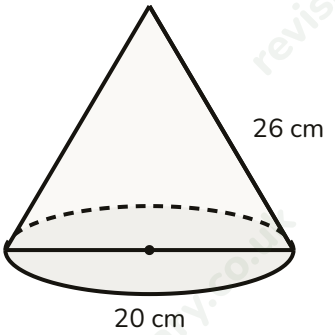


(Total for Question 14 is 2 marks)

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15 A cone has base diameter 20 cm and slant height 26 cm. Calculate the vertical height of the cone, giving your answer correct to 3 significant figures.

Diagram NOT accurately drawn

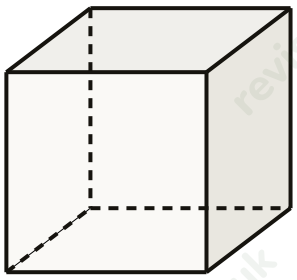


(Total for Question 15 is 3 marks)

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16 A cube has side length a. The angle between a vertical edge of the cube and the space diagonal from the same vertex is θ. Show that $\tan(\theta) = \sqrt{2}$.

Diagram NOT accurately drawn



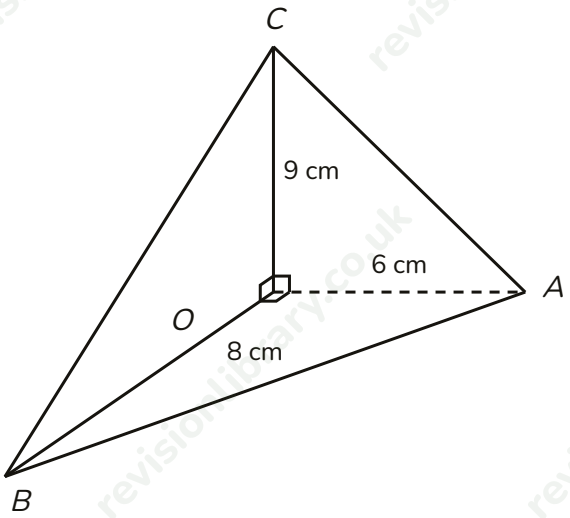
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(Total for Question 16 is 3 marks)

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17 O, A, B and C are points such that OA, OB and OC are mutually perpendicular edges meeting at O, with OA = 6 cm, OB = 8 cm and OC = 9 cm.

Diagram NOT accurately drawn



(a) Find the length AB. **(2)**

.....

(b) Find the length AC, giving your answer correct to 3 significant figures. **(2)**

.....

(c) Find the length BC, giving your answer correct to 3 significant figures. **(2)**

.....

(d) Using exact (unrounded) values, calculate the size of angle BAC, giving your answer correct to 1 decimal place. **(3)**

.....

(Total for Question 17 is 9 marks)

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Mark scheme · 3.6 3D Pythagoras and Trigonometry: Test Pack

A correct answer earns full marks unless the question asks for working, a method, proof or explanation.

Question 1

B cao **B1**

Answer: B) $\sqrt{a^2 + b^2 + c^2}$

Question 2

B cao **B1**

Answer: B) $\sqrt{l^2 - r^2}$

Question 3

$AC^2 = 13^2 + 14^2$ (face diagonal) seen **M1**

$AG^2 = AC^2 + 6^2 = 401$ **M1**

20.0 cm cao **A1**

Answer: 20.0 cm

Question 4

$12^2 + 24^2 + h^2 = 63^2$ **M1**

$h^2 = 3249$ **M1**

57 cm cao **A1**

Answer: 57 cm

Question 5

$AC^2 = 10^2 + 11^2$ (face diagonal) seen **M1**

$AG^2 = AC^2 + 7^2 = 270$ **M1**

$3\sqrt{30}$ cm cao **A1**

Answer: $3\sqrt{30}$ cm

Question 6

$10^2 + 4^2 + h^2 = 30^2$ **M1**

$h^2 = 784$ **M1**

28 cm cao **A1**

Answer: 28 cm

Question 7

$AC = \sqrt{6^2 + 6^2}$ or half-diagonal = 4.243 **M1**

$h^2 = 12^2$ - (half diagonal)^2 = 126.00 **M1**

11.2 cm cao **A1**

Answer: 11.2 cm

Question 8

$AC = \sqrt{4^2 + 10^2} = 10.770$ **M1**

tan (angle GAC) = 11 / AC **M1**

45.6 degrees cao **A1**

Answer: 45.6 degrees

Question 9

(a) $AC = \sqrt{8^2 + 6^2}$ **M1**

(a) AC = 10 cm cao **A1**

(a) Answer: 10 cm

(b) sets up tan(angle) = height / their AC = 5/10 **M1**

(b) angle = 26.6 degrees (1dp) cao **A1**

(b) Answer: 26.6 degrees (1dp)

Question 10

$AC = \sqrt{17^2 + 17^2}$ or half-diagonal = 12.021 **M1**

$h^2 = 17^2$ - (half diagonal)^2 = 144.50 **M1**

12.0 cm cao **A1**

Answer: 12.0 cm

Question 11

$h^2 = x^2 + y^2$ oe **B1**

Answer: $h^2 = x^2 + y^2$

Question 12

$d = \sqrt{2^2 + 10^2 + 11^2}$ **M1**

d = 15 cm cao **A1**

Answer: 15 cm

Question 13

(a) $AC = \sqrt{10^2 + 10^2}$ **M1**

(a) AC = $10\sqrt{2}$ cm (exact) cao **A1**

(a) Answer: $10\sqrt{2}$ cm

(b) finds AM = half of their AC = $5\sqrt{2}$, the distance from a base vertex to the centre **M1**

(b) uses Pythagoras on triangle VMA: $VM = \sqrt{VA^2 - AM^2} = \sqrt{13^2 - (5\sqrt{2})^2}$ **M1**

(b) VM = 10.9 cm (3sf) cao **A1**

(b) Answer: 10.9 cm (3sf)

Question 14

height = $\sqrt{12^2 - (4\sqrt{2})^2} = \sqrt{144 - 32}$ **M1**

10.6 cm awrt (3sf) **A1**

Answer: 10.6 cm

Question 15

radius = 20/2 = 10 (half the diameter) **M1**

$h = \sqrt{26^2 - 10^2}$ **M1**

h = 24 cm cao (exact) **A1**

Answer: 24 cm

Question 16

finds the base diagonal using Pythagoras: diagonal$^2 = a^2 + a^2 = 2a^2$, so the base diagonal is $a\sqrt{2}$ **M1**

identifies the right-angled triangle formed by the vertical edge (length a), the base diagonal (length $a\sqrt{2}$) and the space diagonal, with θ the angle between the vertical edge and the space diagonal **M1**

$\tan(\theta)$ = base diagonal / vertical edge = $(a\sqrt{2})/a = \sqrt{2}$, as required **A1**

Answer: $\tan(\theta) = \sqrt{2}$ (shown)

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Question 17

(a) AB = $\sqrt{6^2 + 8^2}$ **M1**

(a) AB = 10 cm cao **A1**

(a) Answer: 10 cm

(b) AC = $\sqrt{6^2 + 9^2}$ **M1**

(b) AC = 10.8 cm (3sf) cao (exact value $\sqrt{117} = 3\sqrt{13}$) **A1**

(b) Answer: 10.8 cm (3sf)

(c) BC = $\sqrt{8^2 + 9^2}$ **M1**

(c) BC = 12.0 cm (3sf) cao (exact value $\sqrt{145}$) **A1**

(c) Answer: 12.0 cm (3sf)

(d) correct cosine rule set up using exact squared lengths: $\cos(BAC) = (AB^2 + AC^2 - BC^2)/(2 \times AB \times AC) = (100 + 117 - 145)/(2 \times 10 \times \sqrt{117})$ **M1**

(d) correct evaluation, $\cos(BAC) = 72/(20\sqrt{117}) = 0.3328$ (accept awrt) **M1**

(d) angle BAC = 70.6 degrees (1dp) cao **A1**

(d) Answer: 70.6 degrees (1dp)

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