4.2 Matrices and Geometric Transformations: 2-hour Pack

Total Marks

AQA 8365 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Transformation S is a rotation of 90 degrees clockwise about the origin. Transformation T is an enlargement with scale factor -1 and centre the origin. Find the single matrix that represents S followed by T.

(Total for Question 1 is 3 marks)

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2 M = [[2, -1], [3, 4]] and v = [[5], [-2]].

(a) Calculate Mv. **(2)**

(b) The point (5, -2) is transformed using matrix M. Write down the coordinates of the image point. **(1)**

(Total for Question 2 is 3 marks)

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3 Write down the matrix that represents an enlargement with scale factor 3 and centre the origin.

(Total for Question 3 is 2 marks)

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4 The matrix [[1, 2], [-3, -3]] maps the points P(-5, -4) and Q(4, -5) to P' and Q'. Find the coordinates of P' and Q'.

(Total for Question 4 is 3 marks)

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5 The matrix [[2, -3], [1, -2]] maps triangle T, of area 4 square units, onto triangle T'. Work out the area of T'.

(Total for Question 5 is 4 marks)

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6 The matrix [[2, 3], [4, 5]] maps the point A to the point (5, 11). Find the coordinates of A.

(Total for Question 6 is 3 marks)

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7 A transformation maps (1, 0) to (-3, -3) and (0, 1) to (-1, -4). Write down the matrix of the transformation and find the image of the point (-1, 4).

(Total for Question 7 is 4 marks)

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8 Write down the matrix that represents a reflection in the line y = x.

(Total for Question 8 is 2 marks)

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9 The matrix [[-4, -1], [2, -4]] maps the points P(-1, -5) and Q(4, 5) to P' and Q'. Find the coordinates of P' and Q'.

(Total for Question 9 is 3 marks)

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10 Transformation S is a rotation of 90 degrees anticlockwise about the origin. Transformation T is an enlargement with scale factor 4 and centre the origin. Find the single matrix that represents S followed by T.

(Total for Question 10 is 3 marks)

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11 The matrix [[-1, -3], [-1, -4]] maps triangle T, of area 4 square units, onto triangle T'. Work out the area of T'.

(Total for Question 11 is 4 marks)

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12 The matrix [[0, 1], [2, 1]] maps the point A to the point (5, -7). Find the coordinates of A.

(Total for Question 12 is 3 marks)

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13 A transformation maps (1, 0) to (3, -2) and (0, 1) to (1, -3). Write down the matrix of the transformation and find the image of the point (1, 5).

(Total for Question 13 is 4 marks)

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14 Write down the matrix that represents a rotation of 90 degrees anticlockwise about the origin.

(Total for Question 14 is 2 marks)

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15 The matrix [[-2, -1], [5, 2]] maps the points P(5, 3) and Q(3, -1) to P' and Q'. Find the coordinates of P' and Q'.

(Total for Question 15 is 3 marks)

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16 Transformation S is a reflection in the line y = x. Transformation T is a reflection in the x-axis. Find the single matrix that represents S followed by T.

(Total for Question 16 is 3 marks)

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17 The matrix [[0, -3], [3, 3]] maps triangle T, of area 4 square units, onto triangle T'. Work out the area of T'.

(Total for Question 17 is 4 marks)

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18 Write down the matrix that represents a rotation of 180 degrees about the origin.

(Total for Question 18 is 2 marks)

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19 The matrix [[-1, 2], [0, 2]] maps the point A to the point (18, 14). Find the coordinates of A.

(Total for Question 19 is 3 marks)

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20 Write down the matrix that represents a rotation of 90 degrees clockwise about the origin.

(Total for Question 20 is 2 marks)

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21 The matrix [[1, -3], [2, -4]] maps the points P(-3, 2) and Q(5, -2) to P' and Q'. Find the coordinates of P' and Q'.

(Total for Question 21 is 3 marks)

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22 Transformation S is a reflection in the y-axis. Transformation T is a reflection in the line y = x. Find the single matrix that represents S followed by T.

(Total for Question 22 is 3 marks)

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23 A transformation maps (1, 0) to (-4, 4) and (0, 1) to (1, 1). Write down the matrix of the transformation and find the image of the point (2, 2).

(Total for Question 23 is 4 marks)

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24 The matrix [[1, -2], [2, -2]] maps the point A to the point (5, 2). Find the coordinates of A.

(Total for Question 24 is 3 marks)

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25 A = [[4, 2], [3, 5]] and B = [[6, 3], [4, 2]].

(a) Calculate the determinant of A. **(2)**

(b) Calculate the determinant of B and hence state, with a reason, whether B is singular. **(3)**

(Total for Question 25 is 5 marks)

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26 R = [[0, -1], [1, 0]] represents a rotation of 90 degrees anticlockwise about the origin. S = [[1, 0], [0, -1]] represents a reflection in the x-axis.

(a) Find the single matrix that represents a reflection in the x-axis followed by a rotation of 90 degrees anticlockwise about the origin. **(3)**

(b) Describe fully the single transformation represented by your answer to part (a). **(2)**

(Total for Question 26 is 5 marks)

27 Triangle T has area 6 cm² and is transformed by matrix A = [[2, 1], [3, 4]].

(a) Find det(A). **(2)**

(b) Find the area of the image of T after the transformation. **(2)**

(c) A second triangle is transformed by matrix B, where det(B) = -3. The image has area 45 cm². Find the area of the original triangle. **(2)**

(Total for Question 27 is 6 marks)

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28 Under a transformation represented by matrix T = [[p, q], [r, s]], the point (2, 1) maps to (7, 4) and the point (1, 3) maps to (6, 2).

(a) Form two pairs of simultaneous equations and find the values of p, q, r and s. **(4)**

(b) Using your matrix, find the image of the point (4, -2). **(2)**

(c) Find T^{-1} and use it to find the point that maps to the image (13, 6) under transformation T. **(5)**

(Total for Question 28 is 11 marks)

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Mark scheme · 4.2 Matrices and Geometric Transformations: 2-hour Pack

Question 1

matrix of S = [[0, 1], [-1, 0]] and matrix of T = [[-1, 0], [0, -1]] **B1**

multiplies in the correct order: TS, not ST **M1**

[[0, -1], [1, 0]] cao **A1**

Answer: [[0, -1], [1, 0]]

Question 2

(a) correct method (row by column), at least one entry correct **M1**

(a) [[12], [7]] cao **A1**

(a) Answer: [[12], [7]]

(b) (12, 7) ft from part (a) **B1**

(b) Answer: (12, 7)

Question 3

first column shows the image of (1, 0) **B1**

[[3, 0], [0, 3]] cao **B1**

Answer: [[3, 0], [0, 3]]

Question 4

multiplies the matrix by a position vector **M1**

P' correct **A1**

Q' correct **A1**

Answer: P' = (-13, 27), Q' = (-6, 3)

Question 5

uses the determinant as the area scale factor **M1**

determinant = -1 **A1**

|-1| x 4 **M1**

4 square units cao **A1**

Answer: 4 square units

Question 6

finds the inverse matrix or sets up simultaneous equations **M1**

det = -2 or one equation correct **A1**

(4, -1) cao **A1**

Answer: (4, -1)

Question 7

columns are the images of (1, 0) and (0, 1) **B1**

[[-3, -1], [-3, -4]] **B1**

multiplies matrix by position vector **M1**

(-1, -13) **A1**

Answer: [[-3, -1], [-3, -4]], image (-1, -13)

Question 8

first column shows the image of (1, 0) **B1**

[[0, 1], [1, 0]] cao **B1**

Answer: [[0, 1], [1, 0]]

Question 9

multiplies the matrix by a position vector **M1**

P' correct **A1**

Q' correct **A1**

Answer: P' = (9, 18), Q' = (-21, -12)

Question 10

matrix of S = [[0, -1], [1, 0]] and matrix of T = [[4, 0], [0, 4]] **B1**

multiplies in the correct order: TS, not ST **M1**

[[0, -4], [4, 0]] cao **A1**

Answer: [[0, -4], [4, 0]]

Question 11

uses the determinant as the area scale factor **M1**

determinant = 1 **A1**

|1| x 4 **M1**

4 square units cao **A1**

Answer: 4 square units

Question 12

finds the inverse matrix or sets up simultaneous equations **M1**

det = -2 or one equation correct **A1**

(-6, 5) cao **A1**

Answer: (-6, 5)

Question 13

columns are the images of (1, 0) and (0, 1) **B1**

[[3, 1], [-2, -3]] **B1**

multiplies matrix by position vector **M1**

(8, -17) **A1**

Answer: [[3, 1], [-2, -3]], image (8, -17)

Question 14

first column shows the image of (1, 0) **B1**

[[0, -1], [1, 0]] cao **B1**

Answer: [[0, -1], [1, 0]]

Question 15

multiplies the matrix by a position vector **M1**

P' correct **A1**

Q' correct **A1**

Answer: P' = (-13, 31), Q' = (-5, 13)

Question 16

matrix of S = [[0, 1], [1, 0]] and matrix of T = [[1, 0], [0, -1]] **B1**

multiplies in the correct order: TS, not ST **M1**

[[0, 1], [-1, 0]] cao **A1**

Answer: [[0, 1], [-1, 0]]

Question 17

uses the determinant as the area scale factor **M1**

determinant = 9 **A1**

|9| x 4 **M1**

36 square units cao **A1**

Answer: 36 square units

Question 18

first column shows the image of (1, 0) **B1**

[[-1, 0], [0, -1]] cao **B1**

Answer: [[-1, 0], [0, -1]]

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Question 19

finds the inverse matrix or sets up simultaneous equations **M1**
det = -2 or one equation correct **A1**
(-4, 7) cao **A1**
Answer: (-4, 7)

Question 20

first column shows the image of (1, 0) **B1**
[[0, 1], [-1, 0]] cao **B1**
Answer: [[0, 1], [-1, 0]]

Question 21

multiplies the matrix by a position vector **M1**
P' correct **A1**
Q' correct **A1**
Answer: P' = (-9, -14), Q' = (11, 18)

Question 22

matrix of S = [[-1, 0], [0, 1]] and matrix of T = [[0, 1], [1, 0]] **B1**
multiplies in the correct order: TS, not ST **M1**
[[0, 1], [-1, 0]] cao **A1**
Answer: [[0, 1], [-1, 0]]

Question 23

columns are the images of (1, 0) and (0, 1) **B1**
[[-4, 1], [4, 1]] **B1**
multiplies matrix by position vector **M1**
(-6, 10) **A1**
Answer: [[-4, 1], [4, 1]], image (-6, 10)

Question 24

finds the inverse matrix or sets up simultaneous equations **M1**
det = 2 or one equation correct **A1**
(-3, -4) cao **A1**
Answer: (-3, -4)

Question 25

(a) 4(5) - 2(3) **M1**
(a) 14 cao **A1**
(a) Answer: 14
(b) 6(2) - 3(4) **M1**
(b) 0 cao **A1**
(b) dependent: B is singular because det(B) = 0 oe **B1**
(b) Answer: det(B) = 0, so B is singular

Question 26

(a) forms the product R x S, in this order **M1**
(a) correct multiplication attempt **M1**
(a) [[0, 1], [1, 0]] cao **A1**
(a) Answer: [[0, 1], [1, 0]]
(b) reflection **B1**
(b) in the line y = x **B1**
(b) Answer: Reflection in the line y = x

Question 27

(a) 2(4) - 1(3) **M1**
(a) 5 cao **A1**
(a) Answer: 5
(b) area x |det(A)| **M1**
(b) 30 cm² cao **A1**
(b) Answer: 30 cm^2
(c) area_image / |det(B)| **M1**
(c) 15 cm² cao **A1**
(c) Answer: 15 cm^2

Question 28

(a) forms 2p + q = 7 and p + 3q = 6, and solves for p and q **M1**
(a) p = 3, q = 1 **A1**
(a) forms 2r + s = 4 and r + 3s = 2, and solves for r and s **M1**
(a) r = 2, s = 0 **A1**
(a) Answer: T = [[3, 1], [2, 0]]
(b) correct multiplication **M1**
(b) (10, 8) cao **A1**
(b) Answer: (10, 8)
(c) det(T) = 3(0) - 1(2) = -2 **M1**
(c) forms the inverse using the formula (1/det) x [[s, -q], [-r, p]] **M1**
(c) T^{-1} = [[0, 0.5], [1, -1.5]] cao **A1**
(c) multiplies T^{-1} by [[13], [6]] **M1**
(c) (3, 4) cao **A1**
(c) Answer: T^{-1} = [[0, 0.5], [1, -1.5]]; the preimage of (13, 6) is (3, 4)

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