1.1 Higher Surds: Simplifying, Multiplying and Dividing: Test Pack

Total Marks

AQA 8365 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Simplify each surd fully, giving your answer in the form $k\sqrt{n}$ where n has no square factors.

(a) $\sqrt{50}$ **(1)**

(b) $\sqrt{72}$ **(1)**

(c) $\sqrt{45}$ **(1)**

(Total for Question 1 is 3 marks)

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2 Expand and simplify each expression, giving your answer in the form $a + b\sqrt{n}$.

(a) $(\sqrt{5} + 2)(\sqrt{5} + 3)$ **(3)**

(b) $(2\sqrt{3} - 1)(\sqrt{3} + 4)$ **(3)**

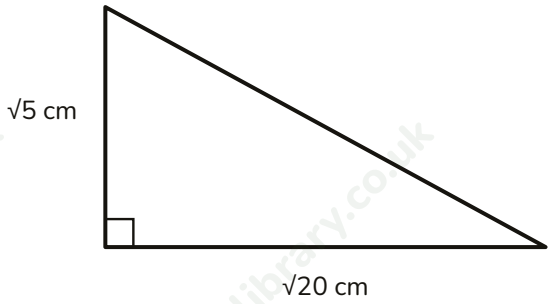
(c) $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$ **(3)**

(Total for Question 2 is 9 marks)

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3 A right-angled triangle has legs of length $\sqrt{20}$ cm and $\sqrt{5}$ cm.

Diagram NOT accurately drawn



(a) Show that the hypotenuse of the triangle has length 5 cm. **(3)**

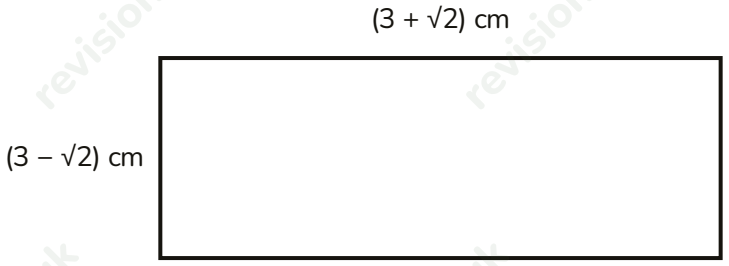
(b) Find the exact area of the triangle. **(3)**

(Total for Question 3 is 6 marks)

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4 A rectangle has sides of length $(3 + \sqrt{2})$ cm and $(3 - \sqrt{2})$ cm.

Diagram NOT accurately drawn



(a) Calculate the area of the rectangle, giving your answer as an integer. **(3)**

.....

(b) Find the perimeter of the rectangle. **(2)**

.....

(Total for Question 4 is 5 marks)

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5 Show that $(2 + \sqrt{3})^2 = 7 + 4\sqrt{3}$.

(Total for Question 5 is 3 marks)

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6 Which of the following is equivalent to $\sqrt{75}$?

- ☐ A) $3\sqrt{5}$
- ☐ B) $5\sqrt{3}$
- ☐ C) $15\sqrt{5}$
- ☐ D) $25\sqrt{3}$

(Total for Question 6 is 1 mark)

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7 Work out $(20\sqrt{45}) / (5\sqrt{3})$. Give your answer in the form $a\sqrt{b}$, where a is a fraction or integer.

(Total for Question 7 is 3 marks)

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8 Write $\sqrt{637}$ in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible.

(Total for Question 8 is 2 marks)

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9 Simplify $3\sqrt{45} + \sqrt{80} - 2\sqrt{20}$, giving your answer in the form $a\sqrt{b}$.

(Total for Question 9 is 3 marks)

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10 Expand and simplify $(6 - 2\sqrt{7})(-3 - 3\sqrt{7})$. Give your answer in the form $p + q\sqrt{s}$.

(Total for Question 10 is 3 marks)

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11 Show that $(3\sqrt{5} + \sqrt{7})^2$ can be written in the form $p + q\sqrt{r}$, and find p, q and r. Give your answer as $p + q\sqrt{r}$.

(Total for Question 11 is 3 marks)

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12 A is the point $(2\sqrt{3}, 1)$ and B is the point $(\sqrt{3}, 4)$. Find the exact length of AB, giving your answer in the form $k\sqrt{3}$.

(Total for Question 12 is 3 marks)

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13 Prove, using proof by contradiction, that $\sqrt{2}$ is an irrational number.

(Total for Question 13 is 5 marks)

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14 Given that $(a + b\sqrt{2})(3 - \sqrt{2}) = 11 + \sqrt{2}$, where a and b are integers, find the value of a and the value of b.

(Total for Question 14 is 4 marks)

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Mark scheme · 1.1 Higher Surds: Simplifying, Multiplying and Dividing: Test Pack

Question 1

(a) $5\sqrt{2}$ cao **B1**

(a) Answer: $5\sqrt{2}$

(b) $6\sqrt{2}$ cao **B1**

(b) Answer: $6\sqrt{2}$

(c) $3\sqrt{5}$ cao **B1**

(c) Answer: $3\sqrt{5}$

Question 2

(a) all four terms of the expansion correct: $5 + 3\sqrt{5} + 2\sqrt{5} + 6$ **M1**

(a) collects rational terms (5+6=11) and surd terms ($3\sqrt{5}+2\sqrt{5}=5\sqrt{5}$) **M1**

(a) $11 + 5\sqrt{5}$ cao **A1**

(a) Answer: $11 + 5\sqrt{5}$

(b) all four terms correct: $6 + 8\sqrt{3} - \sqrt{3} - 4$ **M1**

(b) collects rational terms (6-4=2) and surd terms ($8\sqrt{3} - \sqrt{3} = 7\sqrt{3}$) **M1**

(b) $2 + 7\sqrt{3}$ cao **A1**

(b) Answer: $2 + 7\sqrt{3}$

(c) recognises difference of two squares **M1**

(c) 7 - 2 **M1**

(c) 5 cao **A1**

(c) Answer: 5

Question 3

(a) uses Pythagoras: $\text{hyp}^2 = (\sqrt{20})^2 + (\sqrt{5})^2$ **M1**

(a) simplifies to $\text{hyp}^2 = 20 + 5 = 25$ **M1**

(a) hyp = $\sqrt{25} = 5$ cso (answer given) **A1**

(a) Answer: 5 cm (given)

(b) Area = 1/2 x $\sqrt{20}$ x $\sqrt{5}$ **M1**

(b) combines surds to 1/2 x $\sqrt{100}$ **M1**

(b) 5 cm² cao **A1**

(b) Answer: 5 cm^2

Question 4

(a) sets up $(3+\sqrt{2})(3-\sqrt{2})$ **M1**

(a) uses difference of two squares to reach 9 - 2 **M1**

(a) 7 cm² cao **A1**

(a) Answer: 7 cm^2

(b) adds the four sides: $2(3+\sqrt{2}) + 2(3-\sqrt{2})$ oe **M1**

(b) 12 cm cao **A1**

(b) Answer: 12 cm

Question 5

writes $(2+\sqrt{3})(2+\sqrt{3})$ and expands to $4 + 2\sqrt{3} + 2\sqrt{3} + 3$ **M1**

collects rational terms (4+3=7) and surd terms ($2\sqrt{3}+2\sqrt{3}=4\sqrt{3}$) **M1**

$7 + 4\sqrt{3}$ cso (answer given, all steps must be shown) **A1**

Answer: $7 + 4\sqrt{3}$ (given)

Question 6

B cao **B1**

Answer: B) $5\sqrt{3}$

Question 7

$\sqrt{45} / \sqrt{3} = \sqrt{15}$ or equivalent **M1**

20/5 simplified or used as the coefficient **M1**

$4\sqrt{15}$ cao **A1**

Answer: $4\sqrt{15}$

Question 8

49 x 13 or a square factor of 637 identified **M1**

$7\sqrt{13}$ cao **A1**

Answer: $7\sqrt{13}$

Question 9

at least two surds simplified to a multiple of $\sqrt{5}$ **M1**

all surds expressed as multiples of $\sqrt{5}$ **M1**

$9\sqrt{5}$ cao **A1**

Answer: $9\sqrt{5}$

Question 10

at least 3 of the 4 terms correct **M1**

collects rational and surd parts **M1**

$24 - 12\sqrt{7}$ cao **A1**

Answer: $24 - 12\sqrt{7}$

Question 11

expands with at least 3 terms correct **M1**

simplifies $\sqrt{ab}$ term **M1**

$52 + 6\sqrt{35}$ cao **A1**

Answer: $52 + 6\sqrt{35}$

Question 12

finds the differences: x-difference = $\sqrt{3}$ (or $-\sqrt{3}$), y-difference = -3 (or 3) **M1**

uses AB = $\sqrt{(\sqrt{3})^2 + (-3)^2} = \sqrt{3+9} = \sqrt{12}$ **M1**

$2\sqrt{3}$ cao (units not required) **A1**

Answer: $2\sqrt{3}$

Question 13

assumes for contradiction that $\sqrt{2}$ is rational, so $\sqrt{2}$ = a/b where a and b are integers with no common factor and b is not 0 **B1**

squares both sides to obtain $2 = a^2/b^2$, so $a^2 = 2b^2$ **M1**

deduces a^2 is even, so a itself must be even, so a = 2k for some integer k **A1**

substitutes to get $4k^2 = 2b^2$, so $b^2 = 2k^2$, so b^2 is even and hence b is even **M1**

states that a and b are both even, contradicting the assumption that a/b was in a form with no common factor, so $\sqrt{2}$ cannot be written as a/b: $\sqrt{2}$ is irrational **A1**

Answer: $\sqrt{2}$ is irrational (proof by contradiction)

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Question 14

expands the brackets: $3a - a*\sqrt{2} + 3b*\sqrt{2} - 2b$	**M1**
collects rational and surd parts separately: $(3a - 2b) + (3b - a)\sqrt{2}$	**M1**
equates rational parts ($3a - 2b = 11$) and surd parts ($3b - a = 1$) and solves simultaneously	**M1**
$a = 5$ and $b = 2$ cao	**A1**

Answer: a = 5, b = 2

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