1.2 Rationalising Denominators and Surd Equations: 1-hour Pack

Total Marks

AQA 8365 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Show that $1 / (\sqrt{5} - 2) = \sqrt{5} + 2$.

(Total for Question 1 is 4 marks)

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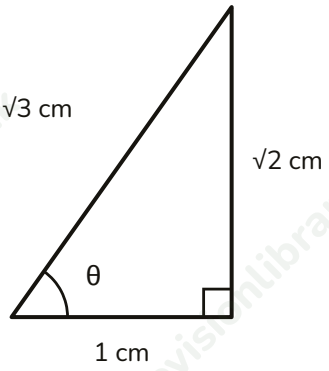
2 Solve $\sqrt{x + 4} = x - 2$, showing clearly why any extraneous solution is rejected.

(Total for Question 2 is 6 marks)

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3 A right-angled triangle has two shorter sides of length 1 cm and $\sqrt{2}$ cm.

Diagram NOT accurately drawn



(a) Show that the hypotenuse has length $\sqrt{3}$ cm. **(2)**

(b) Theta is the angle between the side of length 1 cm and the hypotenuse. Find cos(θ), rationalising your answer. **(3)**

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(c) Find the exact area of the triangle. **(1)**

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(Total for Question 3 is 6 marks)

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4 Rationalise the denominator of $1/(\sqrt{10} + \sqrt{7})$. Give your answer in its simplest form.

(Total for Question 4 is 3 marks)

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5 Solve $\sqrt{2x + 3} = 11$.

(Total for Question 5 is 3 marks)

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6 Show that $(-1 - 2\sqrt{7})/(5 + 2\sqrt{7})$ can be written as $(p + q\sqrt{7})/r$, where p, q and r are integers, and find p, q and r. Give the answer as a single simplified expression.

(Total for Question 6 is 4 marks)

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7 Simplify $(4 + 7\sqrt{7})/\sqrt{7}$, writing your answer in the form $a + b\sqrt{7}$ with a and b in their simplest form.

(Total for Question 7 is 3 marks)

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8 Rationalise the denominator of $2/\sqrt{10}$. Give your answer in its simplest form.

(Total for Question 8 is 3 marks)

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9 Rationalise the denominator of $1/(\sqrt{7} + \sqrt{3})$. Give your answer in its simplest form.

(Total for Question 9 is 3 marks)

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10 Simplify $(9 + 8\sqrt{3})/\sqrt{3}$, writing your answer in the form $a + b\sqrt{3}$ with a and b in their simplest form.

(Total for Question 10 is 3 marks)

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11 Solve $\sqrt{3x + 6} = 6$.

(Total for Question 11 is 3 marks)

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12 $x = 7 + \sqrt{48}$. Show that $1/x = 7 - \sqrt{48}$ and hence find the exact value of $x + 1/x$.

(Total for Question 12 is 3 marks)

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13 Rationalise the denominator of $1/(\sqrt{10} + \sqrt{5})$. Give your answer in its simplest form.

(Total for Question 13 is 3 marks)

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14 Prove that $1/(\sqrt{n+1} + \sqrt{n}) + 1/(\sqrt{n+1} - \sqrt{n}) = 2\sqrt{n+1}$ for all positive integers n.

(Total for Question 14 is 6 marks)

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Mark scheme · 1.2 Rationalising Denominators and Surd Equations: 1-hour Pack

Question 1

multiplies numerator and denominator by the conjugate $(\sqrt{5}+2)$	**M1**
expands denominator correctly: $(\sqrt{5}-2)(\sqrt{5}+2) = 5 - 4$	**M1**
denominator simplifies to 1	**A1**
$\sqrt{5} + 2$ cso, printed result obtained with no errors	**A1**

Answer: $\sqrt{5} + 2$ (shown)

Question 2

squares both sides: $x + 4 = (x-2)^2$	**M1**
expands $(x-2)^2 = x^2 - 4x + 4$	**M1**
rearranges to $x^2 - 5x = 0$	**M1**
factorises correctly: $x(x-5) = 0$	**M1**
identifies both roots $x = 0$ and $x = 5$	**A1**
rejects $x = 0$ with valid reason (requires $x - 2 \geq 0$, so $x \geq 2$; at $x=0$, RHS = -2 which is negative and cannot equal a square root); final answer $x = 5$	**A1**

Answer: x = 5 (x = 0 rejected)

Question 3

(a) uses Pythagoras: $1^2 + (\sqrt{2})^2 = 1 + 2 = 3$	**M1**
(a) hypotenuse = $\sqrt{3}$ cso	**A1**

(a) Answer: $\sqrt{3}$ cm (shown)

(b) $\cos(\theta)$ = adjacent/hypotenuse = $1/\sqrt{3}$	**M1**
(b) rationalises by multiplying by $\sqrt{3}/\sqrt{3}$	**M1**
(b) $\sqrt{3}/3$ cao	**A1**

(b) Answer: $\sqrt{3}/3$

(c) $\sqrt{2}/2$ oe cao	**B1**

(c) Answer: $\sqrt{2}/2$ cm^2

Question 4

multiplies top and bottom by $\sqrt{10} - \sqrt{7}$	**M1**
denominator 10 - 7 = 3	**M1**
$(\sqrt{10} - \sqrt{7})/3$ cao	**A1**

Answer: $(\sqrt{10} - \sqrt{7})/3$

Question 5

squares both sides	**M1**
$2x + 3 = 121$	**M1**
$x = 59$ cao	**A1**

Answer: x = 59

Question 6

multiplies numerator and denominator by $5 - 2\sqrt{7}$	**M1**
expands numerator (at least 3 terms correct)	**M1**
denominator = -3	**M1**
$(-23 + 8\sqrt{7})/3$ cao	**A1**

Answer: $(-23 + 8\sqrt{7})/3$

Question 7

multiplies top and bottom by $\sqrt{7}$	**M1**
$4\sqrt{7} + 49$ over 7	**M1**
$7 + 4\sqrt{7}/7$ cao	**A1**

Answer: $7 + 4\sqrt{7}/7$

Question 8

multiplies numerator and denominator by $\sqrt{10}$	**M1**
$2\sqrt{10}/10$ or equivalent	**A1**
$\sqrt{10}/5$ cao	**A1**

Answer: $\sqrt{10}/5$

Question 9

multiplies top and bottom by $\sqrt{7} - \sqrt{3}$	**M1**
denominator 7 - 3 = 4	**M1**
$(\sqrt{7} - \sqrt{3})/4$ cao	**A1**

Answer: $(\sqrt{7} - \sqrt{3})/4$

Question 10

multiplies top and bottom by $\sqrt{3}$	**M1**
$9\sqrt{3} + 24$ over 3	**M1**
$8 + 3\sqrt{3}$ cao	**A1**

Answer: $8 + 3\sqrt{3}$

Question 11

squares both sides	**M1**
$3x + 6 = 36$	**M1**
$x = 10$ cao	**A1**

Answer: x = 10

Question 12

multiplies by $7 - \sqrt{48}$ over itself	**M1**
denominator 49 - 48 = 1 shown	**A1**
14 cao	**A1**

Answer: 14

Question 13

multiplies top and bottom by $\sqrt{10} - \sqrt{5}$	**M1**
denominator 10 - 5	**M1**
$(\sqrt{10} - \sqrt{5})/5$ cao	**A1**

Answer: $(\sqrt{10} - \sqrt{5})/5$

Question 14

combines the two fractions over a common denominator $(\sqrt{n+1}+\sqrt{n})(\sqrt{n+1}-\sqrt{n})$	**M1**
correctly obtains numerator $(\sqrt{n+1}-\sqrt{n}) + (\sqrt{n+1}+\sqrt{n}) = 2\sqrt{n+1}$	**M1**
correctly expands denominator to (n+1) - n	**M1**
denominator simplifies to 1	**A1**
reaches $2\sqrt{n+1}$ cso, printed result obtained with no errors	**A1**
recognises the simplification is independent of n, so the identity holds for all positive integers n	**B1**

Answer: $2\sqrt{n+1}$ (proven for all positive integers n)

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