3.18H Area of Compound Shapes: Higher Tier Practice

Total Marks

EDEXCEL Edexcel 1MA1 · Calculator allowed · about 75 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A rectangle has length 12 cm and width 5 cm. A semicircle with diameter 12 cm is attached to one of the lengths, forming a single shape. Work out the area of the shape. Give your answer to 3 significant figures.

(Total for Question 1 is 2 marks)

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2 A right-angled triangle has legs 8 cm and 15 cm. A quarter-circle sector with radius 8 cm is attached to the shorter leg so that the right angle of the triangle and the centre of the sector coincide. Find the total area of the combined shape (triangle plus sector). Give your answer as an exact expression in terms of π and as a decimal to 2 decimal places.

(Total for Question 2 is 3 marks)

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3 A composite shape is formed by removing a semicircular notch from a square of side 10 cm. The diameter of the semicircle equals the side of the square. Calculate the area of the remaining shape. Give your answer in terms of π and as a decimal to 1 decimal place.

(Total for Question 3 is 4 marks)

4 A trapezium has parallel sides 14 cm and 6 cm and height 5 cm. A semicircle with diameter 10 cm is attached to the longer parallel side, so that the semicircle lies outside the trapezium. Calculate the area of the combined shape. Give your answer correct to 1 decimal place.

(Total for Question 4 is 4 marks)

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5 A shape consists of a right-angled isosceles triangle with legs of length x cm attached to a semicircle whose diameter is x cm. Express the total area A cm^2 of the shape in terms of x. Hence find the value of x for which A = 100 cm^2. Give answers exact where possible and a decimal to 2 d.p. for x.

(Total for Question 5 is 5 marks)

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6 A gardener makes a raised flower bed in the shape of a rectangle 6 m by 3 m. At each end she places a semicircular planting area whose diameters are the short side of the rectangle, so the semicircles extend beyond the rectangle. She covers the combined area with turf costing £4.50 per square metre. Work out the total cost of turf to the nearest pound. Show your working.

(Total for Question 6 is 6 marks)

7 A shape is made from a semicircle of radius 6 cm attached to the longer side of a rectangle 12 cm by 8 cm so that the diameter of the semicircle equals the longer side of the rectangle. A circular hole of radius 2 cm is cut out from the semicircular part. Calculate the area of the resulting shape. Give your answer to 1 decimal place.

(Total for Question 7 is 6 marks)

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8 Show that the area of a shape formed by a semicircle of radius r attached to a square of side r is $A = r^2 + (1/2)\pi r^2$. Hence, for r = 3 cm, calculate the percentage of the total area that is contributed by the semicircle. Give your answer to 1 decimal place.

(Total for Question 8 is 8 marks)

9 A compound shape consists of a rectangle 20 cm by 6 cm with a semicircle removed from the centre of one long side. The semicircle has radius 3 cm. The remaining outline is then rotated about the opposite long side to form a solid. Find the volume of the solid formed. Give your answer in terms of π and as a decimal to 2 decimal places. (Treat the semicircular notch as removed before rotation.)

(Total for Question 9 is 6 marks)

Mark scheme · 3.18H Area of Compound Shapes: Higher Tier Practice

Question 1

- **M1** area method: area rectangle + area semicircle, substitute values (oe)
- **A1** area = 12*5 + (1/2)*π*(6)^2 = 60 + 1/2*π*36 = 60 + 18pi, answer to 3 s.f. 60 + 18pi = 60 + 56.5487... = 116.549... -> 117
- **Answer:117 cm^2 (awrt, 3 s.f.) (117 cao)**
  Recompute: 18pi = 56.5487..., total 116.5487... rounds to 117 to 3 s.f..

Question 2

- **M1** area triangle = 1/2 * 8 * 15 = 60, area sector = (1/4) * π * 8^2 = 16pi (method)
- **A1** exact area = 60 + 16pi cao
- **A1** decimal: 60 + 16pi = 60 + 50.2655... = 110.27 cm^2 (to 2 d.p.)
- **Answer:Exact: 60 + 16pi cm^2. Decimal: 110.27 cm^2 (2 d.p.).**
  Working check: 16pi = 50.2655..., total 110.2655... -> 110.27.

Question 3

- **M1** area square = 10^2 = 100, area semicircle = 1/2 * π * (5)^2 = 12.5pi
- **M1** subtract: area = 100 - 12.5pi
- **A1** exact area 100 - 12.5pi cao
- **A1** decimal: 100 - 12.5pi = 100 - 39.2699... = 60.7 cm^2 (1 d.p.)
- **Answer:Exact: 100 - 12.5pi cm^2. Decimal: 60.7 cm^2 (1 d.p.).**
  Working check: 12.5pi = 39.2699..., 100 - 39.2699... = 60.730... -> 60.7 (1 d.p.).

Question 4

- **M1** area trapezium = 1/2*(14+6)*5 = 1/2*20*5 = 50
- **M1** area semicircle = 1/2*π*(5)^2 = 12.5pi
- **A1** sum = 50 + 12.5pi
- **A1** decimal: 50 + 12.5pi = 50 + 39.2699... = 89.3 cm^2 (1 d.p.)
- **Answer:89.3 cm^2 (1 d.p.).**
  Working check: trapezium 50, semicircle 12.5pi = 39.2699..., total 89.2699... -> 89.3.

Question 5

- **M1** area triangle (right isosceles) = 1/2 * x * x = x^2/2 and area semicircle = 1/2 * π * (x/2)^2 = π*x^2/8
- **M1** sum A = x^2/2 + π*x^2/8 = x^2*(1/2 + π/8)
- **A1** A = x^2*(4/8 + π/8) = x^2*((4+π)/8) cao
- **M1** set x^2*(4+π)/8 = 100 and rearrange x^2 = 800/(4+π)
- **A1** x = √800/(4+&π;) = decimal 10.58 cm (2 d.p.)
- **Answer:A = x^2*(4+π)/8. For A = 100: x^2 = 800/(4+π), so x = √800/(4+&π;) = 10.58 cm (2 d.p.).**
  Working check: 4+π = 7.141592654, 800/7.141592654 = 112.0198..., sqrt = 10.5839... -> 10.58 (2 d.p.).

Question 6

- **M1** area rectangle = 6*3 = 18 m^2
- **M1** diameter of each semicircle = 3 so radius = 1.5; area semicircle = 1/2*π*1.5^2 = 1.125pi each
- **M1** two semicircles combined = 2*1.125pi = 2.25pi
- **M1** total area = 18 + 2.25pi
- **A1** numeric area = 18 + 2.25pi = 18 + 7.068583... = 25.0686... m^2
- **A1** cost = 25.0686... * 4.5 = 112.8089... -> £113 to nearest pound
- **Answer:£113 (nearest pound).**
  Working check: total area 18 + 2.25pi = 25.0686..., cost 25.0686...*4.5 = 112.8089... -> 113.

Question 7

- **M1** area rectangle = 12*8 = 96
- **M1** area semicircle radius 6 = 1/2*π*6^2 = 18pi
- **M1** area hole = π*2^2 = 4pi; subtract from semicircle
- **M1** total area = 96 + 18pi - 4pi = 96 + 14pi
- **A1** evaluate: 14pi = 43.9823..., total = 139.9823...
- **A1** to 1 d.p. 140.0 cm^2
- **Answer:140.0 cm^2 (1 d.p.).**
  Working check: 14pi = 43.9823..., add 96 = 139.9823... -> 140.0.

Question 8

- **M1** area square = r^2 and area semicircle = 1/2*π*r^2 identified
- **A1** sum A = r^2 + 1/2*π*r^2 = r^2*(1 + π/2) cao
- **M1** substitute r = 3: area semicircle = 1/2*π*9 = 4.5pi, total area = 9 + 4.5pi
- **M1** compute numeric semicircle area and total area
- **A1** semicircle = 4.5pi = 14.1372..., total = 9 + 14.1372... = 23.1372...
- **M1** percentage = (semicircle/total)*100
- **A1** compute percentage = 14.1372.../23.1372... *100 = 61.1% (1 d.p.)
- **A1** final statement: 61.1% (1 d.p.) cao
- **Answer: Show that: A = r^2 + 1/2*π*r^2 = r^2*(1 + π/2). For r = 3: semicircle = 4.5pi = 14.1372..., total = 23.1372..., percentage = 14.1372.../23.1372...*100 = 61.1% (1 d.p.).**

Question 9

- **M1** interpret rotation: volume from rotating rectangle around long side gives cylinder radius 6 and length 20? Check correct axis: opposite long side -> rectangle height 6 rotates to make cylinder radius 6 and length 20. Volume cylinder = π*r^2*h = π*6^2*20 = 720pi
- **M1** semicircular notch radius 3 is centred on one long side; when rotated about opposite long side it generates a solid equal to a full sphere of radius 3? Need careful method: distance between the notch and axis = 6 (height) so profile generates a toroidal segment. Simpler: use Pappus centroid theorem or recognise shape is cylinder minus solid of revolution of semicircle around parallel axis distance 6. Use Pappus: volume = area of semicircle * distance travelled by its centroid
- **M1** area semicircle = 1/2*π*3^2 = 4.5pi; centroid of semicircle from its flat edge is 4r/(3pi) = 4*3/(3pi) = 4/π. The flat edge lies on the long side being rotated, at distance 6 from the axis, and the notch bulges inward (towards the axis), so the centroid is closer to the axis: distance from axis = 6 - 4/π
- **M1** distance travelled = 2pi*(6 - 4/π) = 12pi - 8
- **A1** volume removed = area * distance = 4.5pi * (12pi - 8) = 54pi^2 - 36pi
- **A1** final volume = 720pi - (54pi^2 - 36pi) = 720pi + 36pi - 54pi^2 = 756pi - 54pi^2. Decimal evaluate: 756pi = 2375.044...; 54pi^2 = 54*(9.8696044) = 532.959...; subtract = 1842.085... -> 1842.09 cm^3 (2 d.p.)
- **Answer:Volume = 756pi - 54pi^2 cm^3 (exact). Decimal: 1842.09 cm^3 (2 d.p.).**
  Working check: 756pi = 2375.044..., 54pi^2 = 532.959..., difference 1842.085... -> 1842.09.