4.15 Loci and Construction: Test Pack

EDEXCEL 1MA1 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Priya stands at point P. Shade the locus of points that are within 3 m of P on a garden plan where 1 cm represents 1 m. Describe the locus.

(Total for Question 1 is 1 mark)

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2 Work out the distance from a point to a line. A road is represented by a straight line L on a map. A house H is 5 cm from L on the map. The scale is 1 cm = 10 m. Calculate the shortest real distance from the house to the road in metres.

(Total for Question 2 is 2 marks)

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3 On a drawing, two shops S1 and S2 are 8 cm apart. Draw and describe the locus of points that are equidistant from S1 and S2.

(Total for Question 3 is 2 marks)

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4 A wall is represented by the straight line segment PQ, 6 cm long, using a scale of 1 cm to 2 m. Draw PQ. A safety zone extends exactly 3 m from every point on the wall, including its two ends. Using the same scale, draw the locus of the boundary of the safety zone.

(Total for Question 4 is 3 marks)

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5 The incentre of a triangle is the point equidistant from all three sides, found by constructing the bisectors of all three interior angles. Explain why this point must always lie inside the triangle, for any triangle.

(Total for Question 5 is 2 marks)

6 Describe the locus of points that are less than 7 cm from a fixed point P.

(Total for Question 6 is 1 mark)

7 A and B are two points 54 cm apart. The point P is exactly 123 cm from A and exactly 123 cm from B.
Work out the shortest distance from P to the line AB.

(Total for Question 7 is 3 marks)

8 Draw a straight line 10 cm long and mark a point M on the line, roughly in the middle. Using a ruler and compasses only, construct the perpendicular to the line at the point M. Show all your construction lines.

(Total for Question 8 is 2 marks)

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9 Two straight roads meet at point O at an angle of 50 degrees. Draw the two roads (each about 7 cm long) meeting at O with this angle between them. A new footpath is to be built so that every point on it is the same distance from both roads. Using a ruler and compasses only, construct the locus of the footpath.

(Total for Question 9 is 3 marks)

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10 Two straight paths cross at right angles at O. A designer needs a point on the angle bisector between the two paths that is exactly 5 m from O. If we take the bisector as the line $y = x$, find the coordinates of the point in the first quadrant that is 5 m from O. Give your answer to 3 significant figures.

(Total for Question 10 is 4 marks)

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11 A builder needs to place a lamppost L so that it is 15 m from a corner A of a rectangular playground and also lies on the perpendicular bisector of the diagonal from A to the opposite corner C. The playground is 24 m by 10 m with A at one corner and C at the opposite corner. On a scale 1 cm = 2 m.

(a) Find possible point(s) L that are 15 m from A and lie on the perpendicular bisector of AC. **(4)**

(b) Find the coordinates of L in metres using A as (0,0). Show working and give the final coordinates in metres. **(3)**

(Total for Question 11 is 7 marks)

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12 A museum M and a cafe C are 12 m apart. A location P must be at least 6 m from M and at least 6 m from C. On a plan 1 cm = 2 m.

(a) Find the distance between the two circle centres on the plan and compute the distance between their circumferences (smallest distance between the two circles). Show working. **(4)**

(b) Explain whether there are any points that are at least 6 m from both M and C. **(4)**

(Total for Question 12 is 8 marks)

13 Three non-collinear points A(0, 0), B(6, 0) and C(2, 4) are given.

(a) Find equations of perpendicular bisectors of AB and BC. **(2)**

(b) Find the circle centre and its equation passing through A, B and C. **(3)**

(Total for Question 13 is 5 marks)

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Mark scheme · 4.15 Loci and Construction: Test Pack

Question 1

filled disc (circle and interior) centre P radius 3 cm on the plan cao **B1**

Answer: The disc (circle and interior) centre P, radius 3 cm.

Question 2

scale conversion 5 cm * 10 m/cm or equivalent method **M1**

50 m cao **A1**

Answer: 50 m

Question 3

identify perpendicular bisector of segment S1S2 as the locus **M1**

description: the perpendicular bisector of S1S2, a line at right angles through the midpoint, cao **A1**

Answer: The perpendicular bisector of S1S2: the straight line at right angles through the midpoint.

Question 4

two straight lines parallel to PQ, each 1.5 cm (3 m) from PQ, one on each side, matching the length of PQ **C1**

two semicircular ends of radius 1.5 cm (3 m), centred on P and on Q, joining the parallel lines **C1**

complete continuous locus with no gaps, forming a single closed stadium (racetrack) shape **A1**

Answer: A stadium (racetrack) shape: two straight lines 1.5 cm (3 m) either side of PQ, joined by semicircles of radius 1.5 cm (3 m) at each end.

Question 5

each angle bisector lies between the two sides that form the interior angle it bisects, so it stays within the triangle's interior region near that vertex **B1**

since all three bisectors lie in the region common to all three interior angles, their point of intersection must lie inside the triangle **B1**

Answer: Because each bisector is confined to the interior of its own angle, the point where all three meet must lie inside the triangle.

Question 6

the inside of a circle with centre P and radius 7 cm **B1**

Answer: the inside of a circle with centre P and radius 7 cm

Question 7

P lies on the perpendicular bisector of AB so a right-angled triangle with hypotenuse 123 and side 27 **M1**

$123^2 - 27^2 = 14400$ **M1**

120 cm **A1**

Answer: 120 cm

Question 8

two arcs of equal radius, centred at M, cutting the line on each side of M **C1**

correct perpendicular line drawn through M using two further equal arcs from those points, within 2 degrees of 90 degrees to the original line **A1**

Answer: A straight line through M at 90 degrees to the original line.

Question 9

angle of 50 degrees drawn at O, within 2 degrees, with arms at least 6 cm long **B1**

equal-radius arcs from O crossing both roads, followed by equal-radius arcs from those crossing points intersecting each other **C1**

bisector ray drawn correctly from O through the intersection, splitting the 50 degree angle into two 25 degree angles (within 2 degrees) **A1**

Answer: The bisector of the 50 degree angle at O, giving a footpath at 25 degrees from each road.

Question 10

substitute $y = x$ into $x^2 + y^2 = 5^2$ so $2x^2 = 25$ **M1**

solve $x^2 = 12.5$ so $x = \sqrt{12.5}$ **M1**

x approx 3.5355339 so to 3 s.f. 3.54 **A1**

point is (3.54, 3.54) cao **A1**

Answer: Point approx (3.54, 3.54). (Since $x = \sqrt{12.5}$ approx 3.5355, to 3 s.f. 3.54.)

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Question 11

(a) find coordinates of C: (24, 10) and midpoint of AC as (12, 5) **M1**

(a) perpendicular bisector goes through (12, 5); direction vector perpendicular to AC: AC vector (24, 10) so perpendicular direction (-10, 24) or (10, -24) **M1**

(a) distance from A to midpoint is $\sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$ m, so use right triangle to find points at distance 15 from A on the perpendicular bisector **M1**

(a) state there are two possible points symmetric about the midpoint; give their distances along the bisector from the midpoint or coordinates cao **A1**

(a) Answer: There are two possible positions for L symmetric about the midpoint of AC, because the perpendicular bisector intersects the circle centre A radius 15 m twice (since 15 m is greater than AM = 13 m).

(b) set up equation: point on perpendicular bisector has form $(12,5) + t*(-10,24)/\sqrt{10^2+24^2}$ or using perpendicular direction and solve distance to A = 15 **M1**

(b) compute unit perpendicular vector length $\sqrt{100+576} = \sqrt{676} = 26$ so unit vector is $(-10/26, 24/26) = (-5/13, 12/13)$ and solve $(12 + t*(-5/13))^2 + (5 + t*(12/13))^2 = 15^2$ **M1**

(b) solutions $t = \pm 2*\sqrt{14}$ giving coordinates L1 = $(12 - 10*\sqrt{14}/13, 5 + 24*\sqrt{14}/13)$ and L2 = $(12 + 10*\sqrt{14}/13, 5 - 24*\sqrt{14}/13)$, with decimal forms approx (9.1218, 11.9077) and (14.8782, -1.9077) metres cao **A1**

(b) Answer: L1 = $(12 - 10*\sqrt{14}/13, 5 + 24*\sqrt{14}/13)$ approx (9.1218 m, 11.9077 m); L2 = $(12 + 10*\sqrt{14}/13, 5 - 24*\sqrt{14}/13)$ approx (14.8782 m, -1.9077 m).

Question 12

(a) convert centre distance: 12 m / 2 = 6 cm **M1**

(a) radius on plan for each circle: 6 m / 2 = 3 cm **M1**

(a) compute gap: centre distance 6 cm - 3 cm - 3 cm = 0 cm **M1**

(a) state smallest distance = 0 cm (circles touch) cao **A1**

(a) Answer: Smallest distance = 0 cm; the circles touch.

(b) points at least 6 m from M lie on or outside the circle of radius 6 m (3 cm on the plan) centred on M, and likewise for C **M1**

(b) the circles touch at one point, the midpoint of MC (centres 12 m apart = 6 m + 6 m), which is exactly 6 m from both **M1**

(b) so the only points ruled out are those inside either circle; every point on or outside both circles is at least 6 m from both **M1**

(b) clear conclusion: yes, there are points (infinitely many): any point on or outside both circles, including the touching point itself (exactly 6 m from both) and points further than 6 m from both, e.g. a point 10 m from each oe **A1**

(b) Answer: Yes. A point is at least 6 m from both if it lies on or outside both circles of radius 6 m centred on M and C. The circles touch at the midpoint of MC, which is exactly 6 m from both, and every point outside both circles is further than 6 m from both, so there are infinitely many such points.

Question 13

(a) find midpoints and slopes: midpoint AB = (3,0), slope AB = 0 => perp is vertical x = 3; midpoint BC = ((6+2)/2, (0+4)/2) = (4,2), slope BC = (4-0)/(2-6) = 4/(-4) = -1 so perp slope = 1 **M1**

(a) equations x = 3 and y - 2 = 1(x - 4) or y = x - 2 cao **A1**

(a) Answer: Perp bisector of AB: x = 3 (since AB is horizontal with midpoint (3,0)). Perp bisector of BC: midpoint (4,2), slope BC = -1 so perpendicular slope = 1, equation y - 2 = 1(x - 4) => y = x - 2.

(b) find intersection of x = 3 and y = x - 2 giving centre at (3,1) **M1**

(b) compute radius as distance from centre to A: $\sqrt{(3-0)^2 + (1-0)^2} = \sqrt{9+1} = \sqrt{10}$ **M1**

(b) final circle equation $(x - 3)^2 + (y - 1)^2 = 10$ cao **A1**

(b) Answer: Intersection of bisectors gives centre O = (3,1). Radius = distance OA = $\sqrt{(3-0)^2 + (1-0)^2} = \sqrt{10}$. Equation: $(x - 3)^2 + (y - 1)^2 = 10$.

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