



4.15D Loci and Construction: Fluency and Exam Drill

EDEXCEL Edexcel 1MA1 · Calculator allowed · about 45 minutes

Total Marks

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Name: _____ Date: ____ / ____ / ____

Answer ALL questions. Show all your working.

- 1** Write down the locus of points that are exactly 3 cm from point A.

(Total for Question 1 is 1 mark)

- 2** Write down the locus of points that are equidistant from two distinct points A and B.

(Total for Question 2 is 1 mark)

- 3** Write down the locus of points that are exactly 4 cm from a given straight line l.

(Total for Question 3 is 1 mark)

- 4** Write down the locus of points equidistant from two intersecting straight lines.

(Total for Question 4 is 1 mark)

- 5** What is the locus of points at a distance less than or equal to 5 cm from point P?

(Total for Question 5 is 1 mark)

- 6** State a simple compass-and-straightedge construction to find the midpoint of segment AB.

(Total for Question 6 is 1 mark)

7 AB = 8 cm. M is the midpoint of AB. Work out the length AM.

(Total for Question 7 is 2 marks)

8 An angle measures 74 degrees. A student bisects it with ruler and compass. Work out the size of each angle after bisecting.

(Total for Question 8 is 2 marks)

9 Two towns S and T are 7 km apart. The region within 3 km of S and the region within 5 km of T are considered. Is there any point that lies in both regions? Give a reason.

(Total for Question 9 is 2 marks)

10 True or false? The locus of points equidistant from a fixed point F and a fixed line d is a parabola that can be constructed exactly using only compass and straightedge. Give a brief reason for your answer.

(Total for Question 10 is 2 marks)

11 Two circles have centres A and B which are 10 cm apart. One circle has radius 6 cm (centre A) and the other has radius 8 cm (centre B). Find the coordinates of their intersection points if AB lies along the x-axis with A at (0, 0) and B at (10, 0).

(Total for Question 11 is 3 marks)

- 12** Describe the steps to construct, with compass and straightedge, the perpendicular from a point P to a given line l (P not on l). Explain briefly why the construction produces a right angle.

(Total for Question 12 is 3 marks)

- 13** Triangle ABC has side lengths $AB = 7$ cm, $BC = 8$ cm and $AC = 9$ cm. Construct the perpendicular bisectors of two sides to find the circumcentre. Calculate the circumradius (to 3 significant figures).

(Total for Question 13 is 3 marks)

- 14** AB is a horizontal segment of length 10 cm with A at (0,0) and B at (10,0).
(a) Find the coordinates of the midpoint M of AB. (1 mark)
(b) Write down the equation of the perpendicular bisector of AB. (1 mark)
(c) A circle centre A has radius 6 cm. Find the intersection points of this circle with the perpendicular bisector. Give exact values where possible and decimal approximations to 3 s.f. (3 marks)

(a) Find the coordinates of the midpoint M of AB. (1)

(b) Write down the equation of the perpendicular bisector of AB. (1)

(c) Find the intersection points of circle centre A(0,0) radius 6 with the line $x = 5$. (3)
Give exact values where possible and decimal approximations to 3 s.f.

(Total for Question 14 is 5 marks)

- 15** Two straight paths cross at right angles at O. A designer needs a point on the angle bisector between the two paths that is exactly 5 m from O. If we take the bisector as the line $y = x$, find the coordinates of the point in the first quadrant that is 5 m from O. Give your answer to 3 significant figures.

(Total for Question 15 is 4 marks)

- 16** A school S is at (0, 0), a hospital H is at (12, 0) and a landmark P is at (6, 8). Find all points that are both equidistant from S and H and exactly 5 units from P. Give coordinates and justify your method. (8 marks)

(Total for Question 16 is 8 marks)

Mark scheme · 4.15D Loci and Construction: Fluency and Exam Drill

Question 1

- **B1** circle centre A radius 3 cm cao
- **Answer: A circle centre A radius 3 cm (the circle itself).**

Question 2

- **B1** the perpendicular bisector of AB cao
- **Answer: The perpendicular bisector of AB.**

Question 3

- **B1** two lines parallel to l, each 4 cm from l, cao
- **Answer: Two lines parallel to l, each 4 cm from l (one on each side).**

Question 4

- **B1** the two angle bisectors of the lines (both bisectors) cao
- **Answer: The two angle bisectors of the intersecting lines.**

Question 5

- **B1** the circle centre P radius 5 cm including the interior (a filled disc) cao
- **Answer: The circle centre P radius 5 cm together with its interior (the disc).**

Question 6

- **B1** draw equal-radius arcs from A and B to make two intersection points; join intersections; intersection with AB is midpoint cao
- **Answer: With compass draw equal-radius arcs from A and B to make two intersections; join those intersections with a straight line; where that line meets AB is the midpoint.**

Question 7

- **M1** take half of 8 or state $AM = AB/2$
- **A1** 4 cm cao
- **Answer: 4 cm**

Question 8

- **M1** divide 74 by 2
- **A1** 37 degrees cao
- **Answer: 37 degrees and 37 degrees**

Question 9

- **M1** use intersection test: radii sum $3 + 5 = 8$ which is > 7 and $|5 - 3| = 2$ which is < 7 so circles overlap
- **A1** Yes, there are points in both regions (the two discs intersect) cao
- **Answer: Yes. Because $3 + 5 = 8 > 7$ and $|5 - 3| = 2 < 7$, the two discs intersect so there are points in both regions.**

Question 10

- **B1** false, because the parabola is a quadratic locus and cannot be constructed exactly with a finite sequence of compass-and-straightedge steps (oe)
- **B1** give reason: construction of a full parabola requires plotting many points or higher-order methods, not a single compass-and-straightedge procedure oe
- **Answer: False. The parabola is a continuous quadratic locus and cannot be constructed exactly by a finite straightedge-and-compass procedure; it is usually drawn by plotting many points.**

Question 11

- **M1** use formula $x = (r_1^2 - r_2^2 + d^2)/(2d)$ or set up equations and subtract to find x
- **M1** substitute $r_1 = 6$, $r_2 = 8$, $d = 10$ to get $x = (36 - 64 + 100)/20 = 72/20 = 3.6$
- **A1** $y = \pm\sqrt{6^2 - 3.6^2} = \pm 4.8$ so intersections $(3.6, 4.8)$ and $(3.6, -4.8)$ cao
- **Answer: Intersection points: (3.6, 4.8) and (3.6, -4.8). (Checks: $3.6^2 + 4.8^2 = 12.96 + 23.04 = 36 = 6^2$.)**

Question 12

- **M1** construct two equal-radius arcs centred at P that meet line l at two points A and B (or from P draw arcs to meet l at A and B)
- **M1** from A and B draw equal-radius arcs on the same side of l to meet at X; join P to X or join A and B to get the perpendicular; method stated
- **A1** reason: triangles formed are congruent so the line is at equal distances and meets l at 90 degrees (or arcs give perpendicular bisector) cao
- **Answer: Steps: From P draw arcs to meet l at A and B. With centres A and B and a radius larger than AB/2 draw arcs that meet at X. Join P to X; PX is perpendicular to l. Reason: the construction makes congruent triangles so PX meets l at equal angles, giving a right angle.**

Question 13

- **M1** use Heron to find area: $s = 12$, area $= \sqrt{12(12-7)(12-8)(12-9)} = \sqrt{720}$
- **M1** use circumradius formula $R = abc/(4 \cdot \text{area})$ with $a=8$, $b=9$, $c=7$
- **A1** $R = 7 \cdot 8 \cdot 9 / (4 \cdot \sqrt{720})$ approx 4.70 cm awrt
- **Answer: Circumradius R approx 4.70 cm (to 3 s.f.). Computation: $s = 12$, area $= \sqrt{720}$ approx 26.8328, $R = 504 / (4 \cdot 26.8328)$ approx 4.698 $\rightarrow$ 4.70.**

Question 14

- (a) **B1** midpoint $M = (5, 0)$ cao
- (a) **Answer: $M = (5, 0)$**
- (b) **B1** equation $x = 5$ cao
- (b) **Answer: $x = 5$**
- (c) **M1** substitute $x = 5$ into $x^2 + y^2 = 36$ to get $y^2 = 36 - 25 = 11$
- (c) **M1** take square root to get $y = \pm\sqrt{11}$
- (c) **A1** intersection points $(5, \sqrt{11})$ and $(5, -\sqrt{11})$ approx $(5, 3.32)$ and $(5, -3.32)$ cao
- (c) **Answer: Intersection points: $(5, \pm\sqrt{11})$ approx $(5, 3.32)$ and $(5, -3.32)$.**

Question 15

- **M1** substitute $y = x$ into $x^2 + y^2 = 5^2$ so $2x^2 = 25$
- **M1** solve $x^2 = 12.5$ so $x = \sqrt{12.5}$
- **A1** x approx 3.5355339 so to 3 s.f. 3.54
- **A1** point is $(3.54, 3.54)$ cao
- **Answer: Point approx (3.54, 3.54). (Since $x = \sqrt{12.5}$ approx 3.5355, to 3 s.f. 3.54.)**

Question 16

- **M1** identify locus equidistant from S and H is the perpendicular bisector of SH; since S(0,0) and H(12,0), bisector is $x = 6$
- **M1** parametrise points on bisector as (6, y)
- **M1** apply distance condition to P(6,8): distance = $|y - 8| = 5$, set up equation $|y - 8| = 5$
- **M1** solve $|y - 8| = 5$ to get $y = 13$ or $y = 3$
- **M1** state candidate points (6, 13) and (6, 3)
- **M1** check each point lies on perpendicular bisector and so is equidistant from S and H (show distance to S equals distance to H for one point)
- **A1** both points satisfy the condition; give coordinates (6, 13) and (6, 3) *cao*
- **A1** give brief construction statement: on the map draw perpendicular bisector of SH and mark points on it at distance 5 from P (using compass) *cao*
- **Answer: Points are (6, 13) and (6, 3). Reason: equidistant locus is $x = 6$; distance to P(6,8) is $|y - 8| = 5$ so $y = 13$ or 3; both points lie on $x = 6$.**