4.15H Loci and Construction: Higher Tier Practice

EDEXCEL Edexcel 1MA1 · Calculator allowed · about 75 minutes

Total Marks

Name: ______________________________ Date: ____ / ____ / ______

Answer ALL questions. Show all your working.

1 Draw the perpendicular bisector of the line segment joining points A(1, 3) and B(5, 7). Give the equation of the perpendicular bisector in the form y = mx + c.

(Total for Question 1 is 2 marks)

2 Describe the locus of points that are 4 cm from point P(0, 0) and also 3 cm from the line y = 5. In your answer, state the geometric place for each locus and then state how many intersection points there can be between them.

(Total for Question 2 is 3 marks)

3 On a coordinate grid, find the equation of the locus of points equidistant from the points A(2, 1) and B(8, 1). Then find the intersection of this locus with the line x = 5.

(a) Find the equation of the locus of points equidistant from A and B. **(2)**

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(b) Find the coordinate(s) of the intersection of this locus with the line x = 5. **(2)**

....

(Total for Question 3 is 4 marks)

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4 Circle C has centre (4, 2) and radius 5. Line l has equation y = 2. Work out the coordinates of the points on circle C that are closest to and farthest from line l. State distances from line l for both points.

(Total for Question 4 is 5 marks)

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5 Two points A(-5, 0) and B(5, 0) are 10 cm apart on a coordinate grid (1 unit = 1 cm). (a) Describe the locus of points that are equidistant from A and B, and give its equation. (b) A circle has centre A and radius 13 cm. Find the coordinates of the points where this circle meets the locus from part (a). Show your working.

(a) Describe the geometric locus in words and give its equation. **(2)**

(b) Find the coordinates of the points where the circle centre A radius 13 cm meets the locus x = 0. Show your working. **(4)**

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(Total for Question 5 is 6 marks)

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6 Construct, with ruler and compass, the locus of points equidistant from line m: y = 0 (the x-axis) and line n: x = 0 (the y-axis), for points in the region where x > 0 and y > 0. Describe the locus and give its equation in Cartesian form. State the steps of the construction briefly.

(Total for Question 6 is 4 marks)

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7 Points B(8, 0) and C(3, 6) define line BC. (a) Find the equation of line BC in the form y = mx + c. (b) Find the equation of the locus of points equidistant from B and C (the perpendicular bisector). Give the equation in the form y = mx + c and give the midpoint and slope used.

(a) Find the equation of line BC. **(2)**

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(b) Find the equation of the perpendicular bisector of BC (locus equidistant from B and C). Give midpoint and slope used. **(2)**

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(Total for Question 7 is 4 marks)

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8 Three non-collinear points A(0, 0), B(6, 0) and C(2, 4) are given. (a) Construct the perpendicular bisectors of AB and BC and state their equations. (b) Use their intersection to find the centre of the circle through A, B and C, then give the equation of that circle.

(a) Find equations of perpendicular bisectors of AB and BC. **(2)**

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(b) Find the circle centre and its equation passing through A, B and C. **(3)**

....

(Total for Question 8 is 5 marks)

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Mark scheme · 4.15H Loci and Construction: Higher Tier Practice

Question 1

- **M1** midpoint of AB found as (3, 5) or equivalent
- **A1** equation $y = -x + 8$ cao
- **Answer: $y = -x + 8$ (equivalently $y = -1x + 8$). Working: midpoint $((1+5)/2, (3+7)/2) = (3,5)$. Slope AB $= (7-3)/(5-1) = 1$ so perpendicular slope $= -1$. Line: $y - 5 = -1(x - 3)$ gives $y = -x + 8$. Recompute: midpoint (3,5), perpendicular slope -1, substitute gives $y = -x + 8$.**

Question 2

- **M1** identify locus 1 as a circle centre (0,0) radius 4 and locus 2 as the two lines parallel to $y = 5$ at distance 3 ($y = 8$ and $y = 2$)
- **M1** state that a circle and a horizontal line can intersect in 0, 1 or 2 points
- **A1** conclusion: there can be up to 2 intersection points cao
- **Answer: Locus 1: a circle centre (0,0) radius 4 cm. Locus 2: the set of points 3 cm from the line $y = 5$ are the two lines $y = 8$ and $y = 2$. A horizontal line and a circle can meet in 0, 1 or 2 points, so there can be up to 2 intersection points in total.**

Question 3

- (a) **M1** midpoint or perpendicular bisector method identified
- (a) **A1** equation $x = 5$ cao
- **(a) Answer: $x = 5$**
- (b) **M1** recognise that substituting $x = 5$ gives the same line so all y allowed
- (b) **A1** state intersection is the whole line $x = 5$: all points of form (5, y) cao
- **(b) Answer: All points (5, y) for any real y. Since the locus is the line $x = 5$, its intersection with $x = 5$ is the entire line.**

Question 4

- **M1** recognise the centre (4,2) lies on the line $y = 2$, so line l actually crosses circle C (minimum distance is 0, not achieved by the vertically aligned points)
- **M1** find the closest points by solving $(x-4)^2 + (2-2)^2 = 25$, giving x = 4 plus or minus 5, i.e. $x = -1$ or $x = 9$
- **A1** closest points (-1, 2) and (9, 2), each at distance 0 from line l cao
- **M1** identify the farthest points as directly above and below the centre: (4, 2 + 5) and (4, 2 - 5)
- **A1** farthest points (4, 7) and (4, -3), each at distance 5 from line l cao
- **Answer: Line l: $y = 2$ passes through the centre (4,2) of circle C, so the circle actually crosses the line. Closest points (distance 0, where the circle meets l): (-1, 2) and (9, 2). Farthest points (distance 5, directly above/below the centre): (4, 7) and (4, -3).**

Question 5

- (a) **M1** identify locus as the perpendicular bisector of AB (the set of points equidistant from A and B)
- (a) **A1** state equation $x = 0$ (since AB is horizontal with midpoint (0,0)) cao
- **(a) Answer: The locus is the perpendicular bisector of AB: the straight line through the midpoint (0,0) at right angles to AB. Since AB lies along the x-axis, the perpendicular bisector is the y-axis, equation $x = 0$.**
- (b) **M1** set up equation of circle centre A(-5,0) radius 13: $(x+5)^2 + y^2 = 169$
- (b) **M1** substitute $x = 0$: $25 + y^2 = 169$, so $y^2 = 144$
- (b) **A1** $y = 12$ or $y = -12$
- (b) **A1** final coordinates (0, 12) and (0, -12) cao
- **(b) Answer: (0, 12) and (0, -12).**

Question 6

- **M1** identify locus as the bisector of the right angle formed by the two axes at the origin
- **M1** construction steps: draw an arc centred on the origin cutting both axes, then equal-radius arcs from those two points to intersect, and join the origin to that intersection
- **A1** equation derived: the bisector makes equal angles with both axes so it is the line $y = x$ (for $x > 0$)
- **A1** final clear statement: the locus is the line $y = x$, making a 45 degree angle with each axis, with justification cao
- **Answer: The locus is the angle bisector of the right angle between the x-axis and y-axis at the origin. Construction: draw an arc centred at the origin cutting the x-axis and y-axis at two points; from each of those points draw equal-radius arcs meeting at a point inside the angle; join the origin to that point. Because the bisector is equidistant from both axes, it makes an equal 45 degree angle with each, giving the equation $y = x$ (for $x > 0$).**

Question 7

- (a) **M1** calculate slope $m = (6 - 0)/(3 - 8) = 6/(-5) = -6/5$
- (a) **A1** equation $y = (-6/5)x + 48/5$ cao
- **(a) Answer: Slope $m = -6/5$. Equation $y = (-6/5)x + 48/5$. (Check: substituting $x = 8$ gives $y = (-6/5)*8 + 48/5 = -48/5 + 48/5 = 0$.)**
- (b) **M1** find midpoint $M = ((8+3)/2, (0+6)/2) = (11/2, 3)$ and perpendicular slope $m_{perp} = 5/6$
- (b) **A1** equation $y - 3 = (5/6)(x - 11/2)$ leading to $y = (5/6)x - 55/12 + 36/12 \Rightarrow y = (5/6)x - 19/12$ cao
- **(b) Answer: Midpoint $M = (11/2, 3)$. Perpendicular slope $= 5/6$. Equation: $y - 3 = (5/6)(x - 11/2)$ which simplifies to $y = (5/6)x - 19/12$.**

Question 8

- (a) **M1** find midpoints and slopes: midpoint AB = (3,0), slope AB = 0 => perp is vertical $x = 3$; midpoint BC = $((6+2)/2, (0+4)/2) = (4,2)$, slope BC $= (4-0)/(2-6) = 4/(-4) = -1$ so perp slope = 1
- (a) **A1** equations $x = 3$ and $y - 2 = 1(x - 4)$ or $y = x - 2$ cao
- **(a) Answer: Perp bisector of AB: $x = 3$ (since AB is horizontal with midpoint (3,0)). Perp bisector of BC: midpoint (4,2), slope BC = -1 so perpendicular slope = 1, equation $y - 2 = 1(x - 4) \Rightarrow y = x - 2$.**
- (b) **M1** find intersection of $x = 3$ and $y = x - 2$ giving centre at (3,1)
- (b) **M1** compute radius as distance from centre to A: $\sqrt{(3-0)^2 + (1-0)^2} = \sqrt{9+1} = \sqrt{10}$
- (b) **A1** final circle equation $(x - 3)^2 + (y - 1)^2 = 10$ cao
- **(b) Answer: Intersection of bisectors gives centre O = (3,1). Radius = distance OA = $\sqrt{(3-0)^2 + (1-0)^2} = \sqrt{10}$. Equation: $(x - 3)^2 + (y - 1)^2 = 10$.**

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